16.5 SIGNAL DISTORTION DUE TO INTERMODULATION PRODUCTS

Operating an amplifier under large-signal conditions causes distortions in the output signal. This distortion is primarily caused by deviation from linear operation, which causes new frequencies to appear at the output port, usually referred to as "intermodulation products."

DEFINITION-INTERMODULATION PRODUCTS: *The additional frequencies at the output of a nonlinear amplifier (or in general any nonlinear network) when two or more sinusoidal signals are applied at the input.*

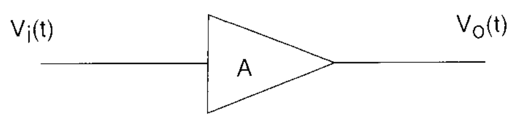
To illustrate this concept, let's consider the following input signal consisting of two frequencies, each with unity amplitude:

$$V_i(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t) \tag{16.22}$$

If $V_i(t)$ is applied to a nonlinear amplifier (see Figure 16.22) with the output/input voltage characteristic of:

$$V_o(t) = AV_i(t) + BV_i^2(t) + CV_i^3(t) \tag{16.23}$$

FIGURE 16.22 Nonlinear amplifier.



Then, the output signal $V_o(t)$ will contain not only the original frequencies f_1 and f_2, but also the following intermodulation products: DC, $2f_1$, $2f_2$, $3f_1$, $3f_2$, $f_1 \pm f_2$, $2f_1 \pm f_2$, and $2f_2 \pm f_1$.

We may classify these intermodulation products as follows:

Second harmonics: $2f_1, 2f_2$ (caused by V_i^2 term).

Third harmonics: $3f_1, 3f_2$ (caused by V_i^3 term).
Second-order intermodulation products: $f_1 \pm f_2$ (caused by V_i^2 term).
Third-order intermodulation products: $2f_1 \pm f_2$, $2f_2 \pm f_1$ (caused by V_i^3 term).

These are plotted on the frequency scale along with the original frequencies, as shown in Figure 16.23. From this figure, we can see that all additional frequencies can be filtered out except the intermodulation products $2f_1 - f_2$ and $2f_2 - f_1$, which are very close to f_1 and f_2 and fall within the amplifier bandwidth and cannot be filtered out. Thus, they are capable of signal distortions at the output.

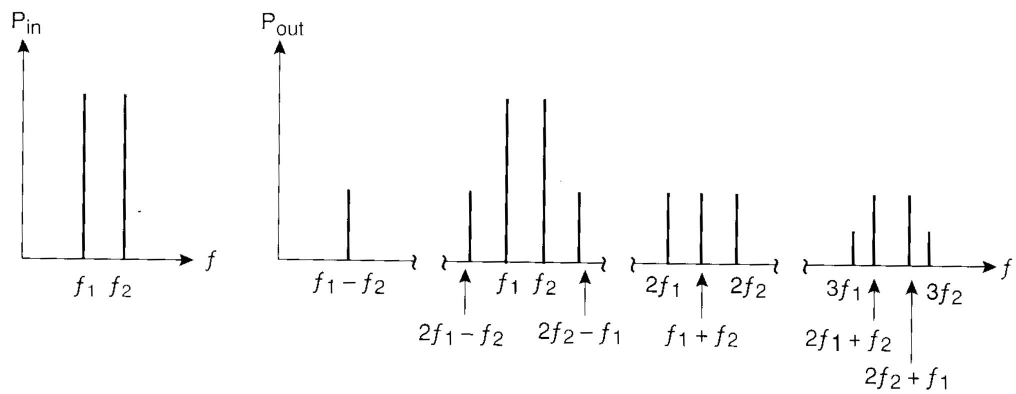


FIGURE 16.23 Input and output power spectrum.

Third-order two-tone intermodulation products $(2f_1 - f_2)$ and $(2f_2 - f_1)$ have special importance because they set the upper limit on the dynamic range or bandwidth of the amplifier.

A measure of the second- or third-order intermodulation distortion is given by two theoretical intercept points, as shown in Figure 16.24. As can be seen from Figure 16.24, the third-order product has a lower intercept point than the second-order product and thus is more significant in distortion analysis.

If the third-order product output power is measured versus the input power, then the third-order intercept (TOI) point can be theoretically obtained, as shown in Figure 16.25. The higher the value of power at TOI (P_{TOI} or P_{IP}), the larger the dynamic range of the amplifier will be.

The power at the third-order intercept point can be theoretically and experimentally obtained to be approximately given by:

$$P_{IP}(\text{dBm}) \approx P_{1\text{dB}}(\text{dBm}) + 10(\text{dB}) \tag{16.24}$$

Furthermore, the difference between the two curves $(P_{f1} - P_{2f1-f2})$ is a variable quantity and is maximum at $P_{o,mds}$ and zero at P_{IP}. It can be shown that:

$$P_{f_1} - P_{2f_1-f_2} = \frac{2}{3}(P_{IP} - P_{2f_1-f_2}) \ \ (\text{dBm}) \tag{16.25}$$

We now define the "spurious free dynamic range" (DR_f) to be the difference between two powers $P_{f_1} - P_{2f_1-f_2}$ when the third-order intermodulation product is equal to the minimum detectable signal. That is:

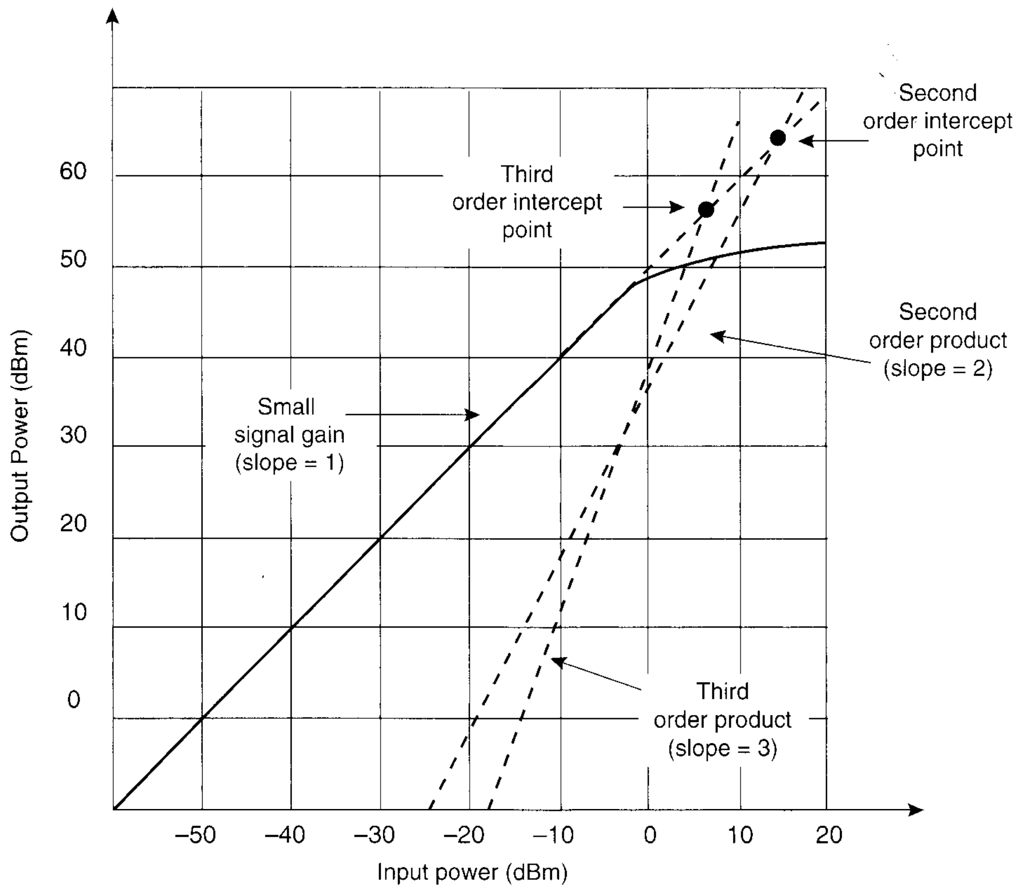


FIGURE 16.24 Second and third order intercept points.

$$DR_f = (P_{f_1} - P_{2f_1 - f_2}) \quad \text{when} \quad (P_{2f_1 - f_2} = P_{o,mds})$$

Thus, we can write DR_f as:

$$DR_f = (P_{f_1} - P_{o,mds}) = \frac{2}{3}(P_{IP} - P_{o,mds}) \quad \text{(dBm)} \qquad (16.26)$$

where from Equation 16.10,

$$P_{o,mds}(\text{dBm}) = -174 \text{ dBm} + 10\log_{10}\text{B} + G_A(\text{dB}) + F(\text{dB}) + X(\text{dB})$$

EXAMPLE 16.7

Calculate dynamic range (DR) and spurious free dynamic range (DR_f) for a microwave high-power/broadband amplifier that has a gain of 20 dB, a noise figure of 5 dB, a bandwidth of 250 MHz and can deliver a power of P_{1dB} = 30 dBm (assume X = 3 dB).

Solution:

$$P_{o,mds}(\text{dBm}) = -174 \text{ dBm} + 10\log_{10}B + G_A(\text{dB}) + F(\text{dB}) + X(\text{dB})$$

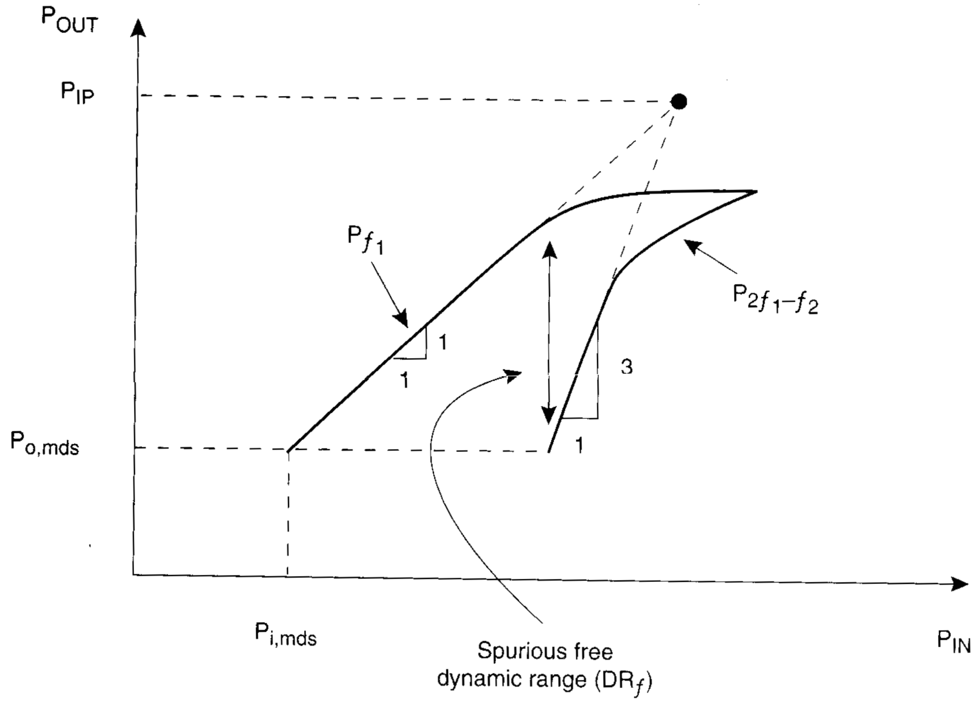


FIGURE 16.25 Third-order intercept point.

$$= -174 + 10\log_{10}(250 \times 10^6) + 5 + 20 + 3 = -62 \text{ dBm}$$

$$DR = 30 - (-62) = 92 \text{ dB}$$

$$DR_f = (2/3)(30 + 10 + 62) = 68 \text{ dB}$$

Two-Tone Measurement Technique. Third-order intercept power (P_{IP}) is a figure of merit for intermodulation product suppression. A high intercept point is a good indicator and signifies a high suppression of undesired intermodulation products. An experimental method for finding P_{IP} is by the use of a technique called "two-tone measurement technique."

In this technique, two signals of close but different frequencies that have equal magnitude are applied to the input of the amplifier, as shown in Figure 16.23. Using a spectrum analyzer, the outputs are examined (see Figure 16.23), and from a simple measurement of the difference in power between the main output (P_{f_1} in dBm) and the third-order intermodulation product ($P_{2f_1-f_2}$ in dBm), we can obtain P_{IP} (in dBm). To find P_{IP}, first let's define:

$$\Delta = P_{f_1} - P_{2f_1-f_2} \text{ (dB)} \tag{16.27a}$$

Then, substituting for $P_{2f_1-f_2}$ from Equation 16.27a in 16.25, we can write:

$$\Delta = \frac{2}{3}(P_{IP} - P_{2f_1 - f_2}) = \frac{2}{3}[P_{IP} - (P_{f_1} - \Delta)] \quad \text{(dB)} \tag{16.27b}$$

By rearranging terms in Equation 16.27b we obtain $(P_{IP})_3$ for the third-order harmonic as :

$$(P_{IP})_3 = P_{f_1} + \frac{\Delta}{2} \quad \text{(dBm)} \tag{16.28a}$$

Thus, by halving the difference (in dB) between the main output and one of the third-order intermodulation products and adding it to the main output, we can obtain the third-order intercept point (in dBm) as in dicated by Equation 16.28a.

EXAMPLE 16.8

If through measurement we find that P_{f_1} = 8 dBm and Δ = 40 dB for the third-order intermodulation product, what is the power at the third-order intercept point?

$(P_{IP})_3$ can easily be calculated to be:

$$(P_{IP})_3 = 8 + 40/2 = 28 \text{ dBm}$$

NOTE: *It can be shown that in general, for the n^{th} order intermodulation product ($n \neq 1$), Equation 16.28a can be generalized as:*

$$(P_{IP})_n = P_{f_1} + \frac{\Delta}{n-1} \quad \text{(dBm)} \tag{16.28b}$$

where Δ is the difference between the fundamental harmonic power and the undesired n^{th} intermodulation product power.

16.6 MULTISTAGE AMPLIFIERS: LARGE-SIGNAL DESIGN

As discussed in the last chapter, most practical transistor amplifiers usually consist of a number of stages connected in cascade forming a multistage amplifier. In a high-power amplifier, each stage should be designed for operation at maximum power such that the maximum power transfer condition is met. In the next several sections, we will present a detailed analysis of a multistage high-power amplifier.

16.6.1 Analysis

Consider a general N-stage amplifier configuration, as shown in Figure 16.26. To have a stable amplifier, the stability of the individual stages as well as overall stability must be checked.

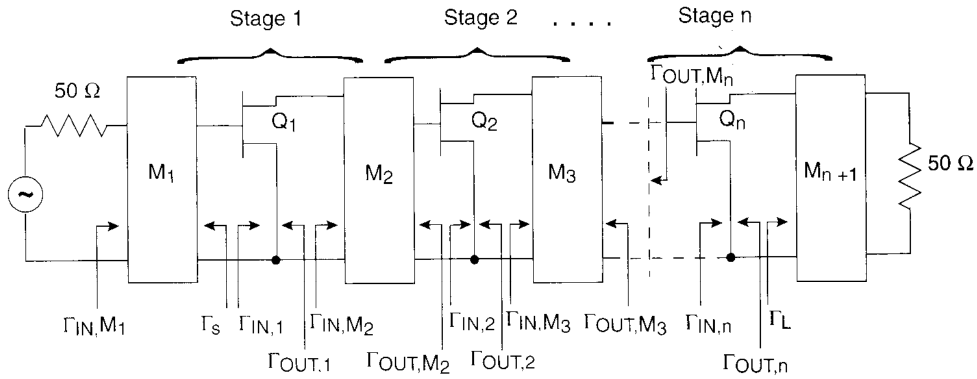


FIGURE 16.26 *N*-stage FET amplifier configuration.

In this type of amplifier, the goal is to produce the overall highest possible power. Thus, each stage must operate at or close to its 1-dB gain compression point under large-signal conditions. This means that using power contours we need to select Γ_{LP} at the point where $P_{OUT} = P_{max}$, (which is at the output port of each transistor) and then use conjugate matched condition for the input port to minimize *VSWR* and create the maximum power transfer condition, i.e.,

$$\Gamma_S = (\Gamma_{IN,1})^* \tag{16.29a}$$

$$\Gamma_{IN,M2} = \Gamma_{LP,1} \tag{16.29b}$$

$$\Gamma_{OUT,M2} = (\Gamma_{IN,2})^* \tag{16.29c}$$

$$\vdots$$

$$\vdots$$

$$\Gamma_{OUT,Mn} = (\Gamma_{IN,n})^* \tag{16.29d}$$

$$\Gamma_L = \Gamma_{LP,n} \tag{16.29e}$$

where $\Gamma_{LP,1}$, $\Gamma_{LP,2}....\Gamma_{LP,n}$ represent points on the power contours where P_{max} occurs for transistors $Q_1, Q_2, ...Q_n$, respectively.

16.6.2 Overall Third-Order Intercept Point Power

High-power amplifiers are designed not only to obtain large amounts of output power but also to have a high third-order intercept point (*TOI*). If each individual stage has a known power value at *TOI* (P_{TOI}), then assuming in-phase addition (of the P_{TOI} of each stage), overall P_{TOI} of the multistage power amplifier is given by:

$$\frac{1}{P_{TOI}} = \frac{1}{P_{TOI,n}} + \frac{1}{G_{Pn}P_{TOI,n-1}} + \dots + \frac{1}{G_{Pn}G_{Pn-1}\dots G_{P2}P_{TOI,1}} \tag{16.30}$$

where G_P is the power gain.

If all stages are identical, i.e.,

$$G_{Pk} = G_P,\ k = 1,2,\ldots,n \tag{16.31 a}$$

$$P_{TOI,k} = P,\ k = 1,2,\ldots,n \tag{16.31 b}$$

Then, Equation 16.22 reduces to:

$$\frac{1}{P_{TOI}} = \frac{1}{P}\left(1 + \frac{1}{G_P} + \frac{1}{G_P^2} + \ldots + \frac{1}{G_P^{n-1}}\right) \tag{16.32}$$

Using Equation 15.33, the geometric series identity, Equation 16.32 becomes:

$$\frac{1}{P_{TOI}} = \frac{1}{P}\left(\frac{1 - 1/G_P^n}{1 - 1/G_P}\right) \tag{16.33}$$

For an infinite chain of amplifier stages (i.e., $n \to \infty$), we can write Equation 16.32 as:

$$P_{TOI} = P(1 - 1/G_P) \tag{16.34}$$

In practice, n may be large (but is not infinite), thus Equation 16.34 gives a best case scenario for the amplifier's overall power at the third-order intercept point (P_{TOI}), which is a power amplifier's figure of merit—very similar to noise measure (M), which is a figure of merit for an LNA as discussed earlier.

16.6.3 Dynamic Range Considerations

As discussed earlier, the dynamic range of an amplifier is bound at the lower end by noise considerations ($P_{o,mds}$) and at the upper end by 1-dB gain compression point ($P_{1\text{dB}}$). Thus, for an n-stage amplifier, we can write:

a) Lower Limit of Dynamic Range.

$$(P_{o,mds})_{cas} = KT(\text{dBm}) + 10\log_{10}B + 10\log_{10}F_{cas} + G_{A,tot}(\text{dB}) + X(\text{dBm}) \tag{16.35}$$

where (see Equation 15.28):

$$G_{A,tot} = G_{A1}G_{A2}\ \ldots\ G_{An}, \tag{16.36}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1}G_{A2}} + \ldots + \frac{F_n - 1}{G_{A1}G_{A2}\ldots G_{An-1}} \tag{16.37}$$

Special Case: Identical Amplifiers For identical amplifiers and n very large ($n \to \infty$), Equations 16.36 and 16.37 simplify as:

$$G_{Ak} = G_A,\quad k = 1,2,\ldots,n \tag{16.38 a}$$

$$F_k = F,\quad k = 1,2,\ldots,n \tag{16.38 b}$$

$$G_{A,tot} = (G_A)^n \tag{16.39}$$

$$F_{cas} = 1 + \frac{F - 1}{1 - 1/G_A} = 1 + M \tag{16.40}$$

$$(P_{o,mds})_{cas} = KT(\text{dBm}) + 10\log_{10}B + 10\log_{10}(1 + M) + nG_A(\text{dB}) + X(\text{dB})\ \ (\text{dBm}) \tag{16.41}$$

where (see Equation 15.36):

$$M = \frac{F-1}{1-1/G_A} \tag{16.42}$$

NOTE: *$(P_{o,mds})_1$ for the first stage is given by:*

$$(P_{o,mds})_1(\text{dBm}) = KT(\text{dBm}) + 10\log_{10}B + 10\log_{10}F + G_A(\text{dB}) + X\,(\text{dB}) \tag{16.43}$$

Combining Equations 16.41 and 16.43 we can see that:

$$(P_{o,mds})_{cas}\,(\text{dBm}) = (P_{o,mds})_1\,(\text{dBm}) + \Delta P_{o,n} \tag{16.44a}$$

where

$$\Delta P_{o,n} = 10\log_{10}\left(\frac{1+M}{F}\right) + (n-1)G_A \quad (\text{dB}) \tag{16.44b}$$

Because $\Delta P_{o,n}$ is always positive, thus we can write:

$$(P_{o,mds})_{cas} \geq (P_{o,mds})_1 \tag{16.45}$$

Equation 16.45 shows an important consideration where the output minimum detectable signal for the whole cascade $(P_{o,mds})_{cas}$ is determined by and depends greatly on the minimum detectable signal of the first stage of the cascade, $(P_{o,mds})_1$. Thus, it is important to have the first stage operate at the lowest possible output noise level.

b) Upper Limit of Dynamic Range. At the upper limit, the total output power at 1-dB gain compression point ($P_{1dB,cas}$) can be shown to be of similar form to Equation 16.30:

$$\frac{1}{P_{1dB,cas}} = \frac{1}{P_{1dB,n}} + \frac{1}{G_{Pn}P_{1dB,n-1}} + \ldots + \frac{1}{G_{Pn}G_{Pn-1}\cdots G_{P2}P_{1dB,1}} \tag{16.46}$$

If all stages are identical, i.e.,

$$G_{Pk} = G_P,\ k = 1,2,\ldots,n \tag{16.47a}$$

$$P_{1dB,k} = P_{1dB},\ k = 1,2,\ldots,n \tag{16.47b}$$

then Equation 16.46 reduces to:

$$\frac{1}{P_{1dB,cas}} = \frac{1}{P_{1dB}}\left(1 + \frac{1}{G_P} + \frac{1}{G_P^2} + \ldots + \frac{1}{G_P^{n-1}}\right) \tag{16.48}$$

Using Equation 15.33, the geometric series identity, Equation 16.48 becomes:

$$\frac{1}{P_{1dB,cas}} = \frac{1}{P_{1dB}}\left(\frac{1-1/G_P^n}{1-1/G_P}\right) \tag{16.49}$$

For an infinite and idenfical chain of amplifier stages (i.e., $n\to\infty$) and very similar to Equation 16.34, we can write Equation 16.49 as:

$$P_{1dB,cas} = P_{1dB}(1 - 1/G_P) \tag{16.50}$$

NOTE: *Because $G_P \geq 1$ and $P_{1dB,n} = P_{1dB}$, we can see from Equation 16.50 that at all times:*

$$P_{1dB,cas} \leq P_{1dB,n} \tag{16.51}$$

That is, the signal at the output of the cascade is below or at the 1 dB gain compression point of the last-stage amplifier.

As can be seen from Equation 16.51, the power output at 1 dB gain compression point for the whole cascade ($P_{1dB,cas}$) is limited by the last stage 1 dB gain compression point power capability ($P_{1dB,n}$). Thus, we can conclude the following:

CONCLUSION: *It is crucial to have the last stage of the cascade be designed such that it has the highest power handling capability.*

16.6.4 Wide Dynamic Range Multi-stage Amplifier Design

For an n-stage amplifier, the actual dynamic range (DR) is given by:

$$DR = P_{1dB,cas}(\text{dBm}) - (P_{o,mds})_{cas}(\text{dBm}) \tag{16.52}$$

As noted from the previous section, however, for a wide dynamic range design we need to have the following considerations firmly in place:

a. From Equation 16.45 it can be concluded that the first stage sets the lower limit. Therefore, ideally we would like to have:

$$(P_{o,mds})_{cas} \approx (P_{o,mds})_1 \tag{16.53}$$

b. From Equation 16.51, we can observe that the upper limit is determined by the 1 dB gain compression point of the last stage and ideally, we would like to have:

$$P_{1dB,cas} \approx P_{1dB,n} \tag{16.54}$$

Thus, the maximum dynamic range or the **best estimate** of dynamic range (DR_{max}) that we can hope for, can be written as:

$$DR_{max} = P_{1dB,n}(\text{dBm}) - (P_{o,mds})_1(\text{dBm}) \tag{16.55a}$$

Thus, for n identical amplifiers ($n \to \infty$):

$$DR_{max} - DR = \Delta P_{o,n} + P_{1dB}/G_P \tag{16.55b}$$

POINT OF CAUTION: *From Equations 16.44 and 16.45 we can observe that increasing the number of stages (n) and/or available gain (G_A) of each stage will increase the overall gain but will reduce the effective dynamic range by increasing $(P_{o,mds})_{cas}$. Thus, there is a trade-off between the overall gain and the dynamic range of a multistage amplifier.*

resultant matrix to be used in the next multiplication is stored. Similarly, we start with the first noise matrix $\|C_1\|$, the second, $\|C_2\|$, and the first stage ABCD-matrix, $\|E_1\|$, and produce a resultant noise matrix, using Equation 3.17. The resultant noise matrix is then combined with that of the third stage, using the resultant ABCD-matrix of the first two stages to produce the three-stage resultant noise matrix, and so on.

It is not necessary in all cases to use the ABCD-matrix to calculate the overall performance. In some cases, a *Z*-matrix, *Y*-matrix, *S*-matrix, or other matrix may prove most desirable in some circumstances, in which case, it is necessary to develop appropriate formulas to replace Equation 3.17 to calculate the equivalent noise matrix. Also, the noise matrix based on the representation of Figure 3.2*b* may not prove best in all cases. A matrix based on the two-voltage representation of Figure 3.2*a*, or the two-current representation of Figure 3.2*b* may be most useful in particular cases. These noise matrices are expressed as follows:

$$\|C_{1v}\| = \|N_{1v}\| \times \|N_{1v}^*\|^T \equiv \left\| \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \right\| = \left\| \begin{vmatrix} V_{n1} \times V_{n1}^* & V_{n1} \times V_{n2}^* \\ V_{n2} \times V_{n1}^* & V_{n2} \times V_{n2}^* \end{vmatrix} \right\| \tag{3.18}$$

$$\|C_{1i}\| = \|N_{1i}\| \times \|N_{1i}^*\|^T \equiv \left\| \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \right\| = \left\| \begin{vmatrix} I_{n1} \times I_{n1}^* & I_{n1} \times I_{n2}^* \\ I_{n2} \times I_{n1}^* & I_{n2} \times I_{n2}^* \end{vmatrix} \right\| \tag{3.18a}$$

It is necessary, of course, to develop the detailed expressions for these matrices in terms of measured noise characteristics, analogous to Equation 3.16, in order to use them. Also, the transformations to single overall matrices must be developed, based on the specific multiple circuit interconnection configurations and the form of the circuit matrices.

We shall not pursue these matters further here, because for most applications considered in this book we are interested in cascaded interconnections. At the lower frequencies, the assumption of no correlation between the noise sources of a circuit suffices, so that a single noise voltage or current may be used, as in Figure 3.3.

3.4 Calculation of IP

Prediction of IM distortion is an important consideration in planning the receiver design. As indicated earlier, a good measure of performance for any order of IM is the IP for that order. Usually only second- and third-order IPs are calculated; however, the technique may be extended to any order.

Figure 3.5 shows a configuration of two amplifiers with their voltage gains G_v and second- and third-order IPs. If we assume that a signal traversing the amplifiers encounters no phase shift, the composite IM performance may be calculated by

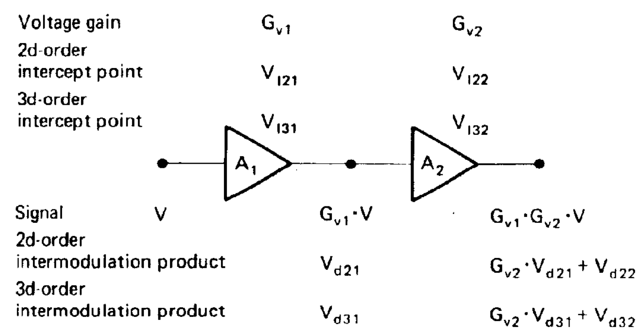


Figure 3.5 Block diagram of cascaded amplifiers with IM distortion.

assuming in-phase addition of the individual contributions. For example, the second-order product generated in amplifier A_1 is V_{d21} and that in A_2 is V_{d22}. Because V_{d21} is applied to the input of A_2, the overall IM product obtained at the output of A_2 is ($G_{v2}\,V_{d21} + V_{d22}$). The effect is the same as if an interfering signal of value

$$V_d = \frac{G_{v2}\,V_{d21} + V_{d22}}{G_{v1}\,G_{v2}} = \frac{V_{d21}}{G_{v1}} + \frac{V_{d22}}{G_{v1}\,G_{v2}} \tag{3.19}$$

were at the input. At the intercept point, this is equal to the input voltage V_{I2}. Generally (see Equation [2.5]), $V_{d2} = V^2/V_{I2}$, referred to the output of an amplifier. Thus, $V_{d2j} = V^2/V_{I2j}$ at the output of amplifier *j*. To place this discussion on a common footing, we can refer the signal level to the input, $V_{d2j} = (VG_{vj})^2/V_{I2j}$, and note that V_d can be expressed as V^2/V_{I2tot}. Collecting terms, we find:

$$\frac{1}{V_{I2tot}} = \frac{G_{v1}}{V_{I21}} + \frac{G_{v1}\,G_{v2}}{V_{I22}} \tag{3.20}$$

This may be extended to any number of amplifiers in cascade. It shows that the greater the gain to the indicated point, the more important it is to have a high IP. To reduce the problems associated with IM, selective filters should be provided as

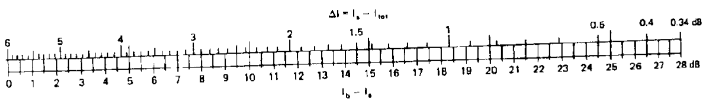


Figure 3.6 Nomogram for calculating the second-order IP for cascaded amplifiers.

near the front of the receiver as possible to reduce the gain to signals likely to cause IM.

While this formula is relatively easy to calculate, the IPs are generally available in dBm, which must first be converted to power before the formula may be used. The nomogram of Figure 3.6 allows the combination of values directly. For this, we rewrite Equation 3.20 as follows:

$$\frac{1}{V_I} = \frac{1}{V_a} + \frac{1}{V_b} \quad V_a \le V_b \tag{3.21}$$

The various V quantities are those referred to the receiver input. It is irrelevant from which amplifier V_a and V_b are derived, but if we choose V_a to be the smaller as indicated and express the other as a ratio of V_a,

$$\frac{V_I}{V_a} = \frac{1}{1 + V_a/V_b} \tag{3.22}$$

the denominator on the right remains less than 2. The resultant is a relationship between two voltage ratios. The values I_J in Figure 3.6 correspond to the equivalent intercept levels V_I in Equation 3.22, measured in dBm. The use of this tool is quite simple:

- Using the gains, recompute the IPs to the system input. ($I_b = I_{bO}/G_{bI}$, where I_{bO} is measured at the amplifier output and G_{bI} is the gain between system input and amplifier output.)
- Form the difference value between the two recalculated IPs [I_b (dBm) − I_a (dBm)].

- In the nomogram, determine the value I and subtract it from I_a to get I_{tot}.
- If there are more than two amplifiers, select the resultant I from the first two and combine similarly with the third, and so on, until all amplifiers have been considered.

The procedure to determine the third-order IP is analogous to that for the second-order, noting, however, that $V_{d3} = V^3/V_{I3}{}^2$. In this case, after manipulating the variables, we find:

$$\frac{1}{V_{I3tot}} = \left[\left(\frac{G_{v1}}{V_{I31}}\right)^2 + \left(\frac{G_{v1}\,G_{v2}}{V_{I32}}\right)^2\right]^{1/2} \tag{3.23}$$

This can be simplified analogously to Equation 3.22 as follows:

$$\frac{1}{V_I^2} = \frac{1}{V_a^2} + \frac{1}{V_b^2} \quad V_a \le V_b \tag{3.24}$$

or

$$\left(\frac{V_I}{V_a}\right)^2 = \frac{1}{1 + (V_a/V_b)^2} \tag{3.25}$$

Just as Figure 3.6 was used to evaluate Equation 3.22, so the nomogram in Figure 3.7 can be used to evaluate Equation 3.25.

All of these calculations need to be made with care. Some amplifiers invert the signal. In that case, the IM components can subtract rather than add. At RF, there are generally other phase shifts that occur either in the amplifiers or in their coupling circuits, so that without thorough analysis it is impossible to determine how the IM powers add vectorially.

Certainly the assumption of in-phase addition made in the preceding equations is the worst-case situation. In many practical cases, however, most of the IM distortion is confined to the stage prior to the selective filtering, so that the contributions of earlier stages may be neglected. Nonetheless, the matter needs careful attention.

```
220 DIM B$[15]
230 INPUT B$
240 DISP "IDENT NEXT STAGE NF,GA
    IN(DB)";
250 INPUT F1,G1
260 X=10^(F1/10)
270 U=10^(F2/10)
280 V=10^(G1/10)
290 W=10*LGT(X+(U-1)/V)
295 Y=INT(W*10+.5)/10
300 PRINT TAB(1);F1;TAB(7);G1;TA
    B(13);Y;TAB(22);B$
310 F2=W
320 GOTO 210
330 END
```

4.17 INTERMODULATION DISTORTION (IM)[4]

When two signals whose frequencies F_1 and F_2 are applied to a non-linear device, other frequencies are generated at:

$$F_1' = F_1 - F\Delta$$

and

$$F_2' = F_2 + F\Delta$$

where

$$F\Delta = F_2 - F_1 \text{ and } F_2 > F_1$$

This shown in Figure (4-20).

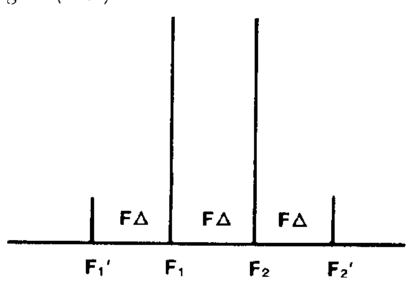


Fig. 4-20. Spectrum due to intermodulation distortion of signals at F_1 and F_2

Assume that a receiver is tuned to a frequency $F_3 = F_2'$. Whenever signals appear simultaneously at frequencies F_1 and F_2, an unwanted signal F_2' appears at F_3, which for all practical purposes appears to be a legitimate signal. This spurious signal may cause serious interference for a desired signal.

Consider a 1000 channel system with 25 kHz channel spacing. Intermodulation products can result for any pair of signals whose separation from each other is $N \cdot F\Delta$ and they are spaced $N \cdot F\Delta$ and $2NF\Delta$ respectively, from a desired channel, where $F\Delta$ = 25 kHz. The possibilities of interference resulting from intermodulation distortion are very great.

All signal processors are non-linear to some degree and will cause the generation of intermodulation distortion terms.

When two signals

$$[A_1 \cos w_1 t \text{ and } A_2 \cos (w_2 t + \delta)] = \rho_i \quad (4\text{-}49)$$

are simultaneously applied to a non-linear processor whose transfer function is of the form

$$K_1 \rho_i + K_2 \rho_i^2 + K_3 \rho_i^3 + \cdots Kn \rho_i^n \quad (4\text{-}50)$$

output terms result which include all possible products. The most significant of these are

$$K_1 [A_1 \cos w_1 t + A_2 \cos (w_2 t + \delta)] \quad (4\text{-}51)$$

which represents the two signals,

$$0.5 A_1^2 K_2 \cos 2w_1 t$$

$$0.5 A_2^2 K_2 \cos 2 (w_2 t + \delta)$$

$$A_1 A_2 K_2 \cos [w_1 t - w_2 (t + \delta)]$$

$$A_1 A_2 K_2 \cos [w_1 t + w_2 (t + \delta)]$$

representing the second order terms and

$$0.75 K_3 A_1^2 A_2 \cos [2w_1 t \pm w_2 (t + \delta)]$$

$$0.75 K_3 A_1 A_2^2 \cos [2w_2 (t + \delta) \pm w_1 t]$$

which represent the third order terms of interest.

Since these products have different slopes, they will cross at some point for a given input level. This intersection is termed the intercept point (Fig. (4-21)). Once this point is determined it is possible to predict the intermodulation product magnitudes, with good accuracy. The charts of Fig. (4-22) and (4-23) may be used to obtain the absolute or relative magnitudes of the second or third order products, given the second or third order intercept point, respectively.

It is also possible to calculate the distortion product magnitudes by noting that for a 0 dBm two-tone signal input, the third order intercept is equal to −1/2 of the third order product magnitude for a unity gain device. For example, given a third order intercept point of +20 dBm, the third order distortion product magnitudes would be 20 ÷ (−1/2) = −40 dBm, for a two-tone input signal with 0 dBm magnitude. This relationship may also be used in reverse. Given a third order product magnitude of −60 dBm for 0 dBm two-tone input signal and unity gain device, the intercept point is −60 x (−1/2) = 30 dBm.

All intercept information is referenced to the output unless otherwise specified. This includes the two-tone magnitude as well as the distortion product level. Assume an amplifier has gain of 10 dB and the two-tone input magnitude is −10 dBm. The amplifier output will consist of the two-tone signals whose magnitude is now 0 dBm; assuming that the third order products are −40 dBm, the third order output intercept point is +20 dBm. To relate this output intercept value to the input, simply subtract the gain.

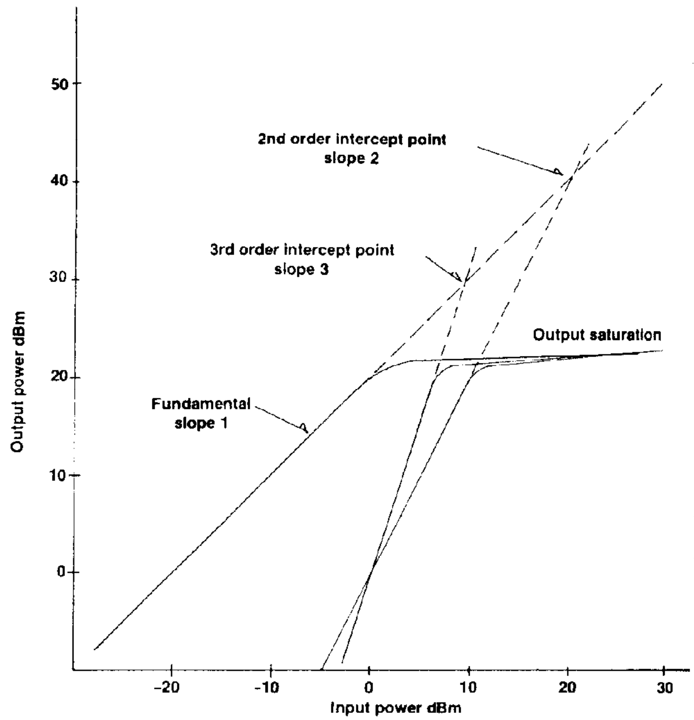


Fig. 4-21. Device outputs showing the fundamental and 2nd and 3rd order distortion products together with the extrapolated respective intercept points.

When using other than 0 dBm two-tone signals, normalize the levels to 0 dBm remembering that the third order distortion products increase 3 dB for every dB increase in the two-tone level.

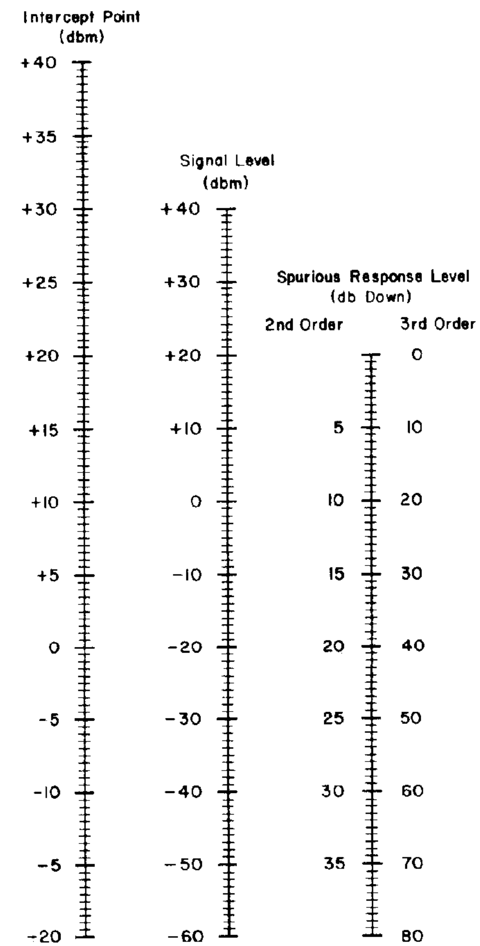


Fig. 4-22. Relative level spurious response nomograph. (Based on nomographs from Avantek, Inc., Santa Clara, California.)

For a two-tone level of −30 dBm, at the input of an amplifier whose gain is 20 dB, we have an output two-tone level of −10 dBm. Assume that the third order distortion products have a magnitude of −50 dBm. To normalize the two-tone output signal level of −10 dBm, add 10 dBm. Also add 3 • 10 dBm to the third order products (−50 + 3 • 10 = −20 dBm). The output third order intercept point

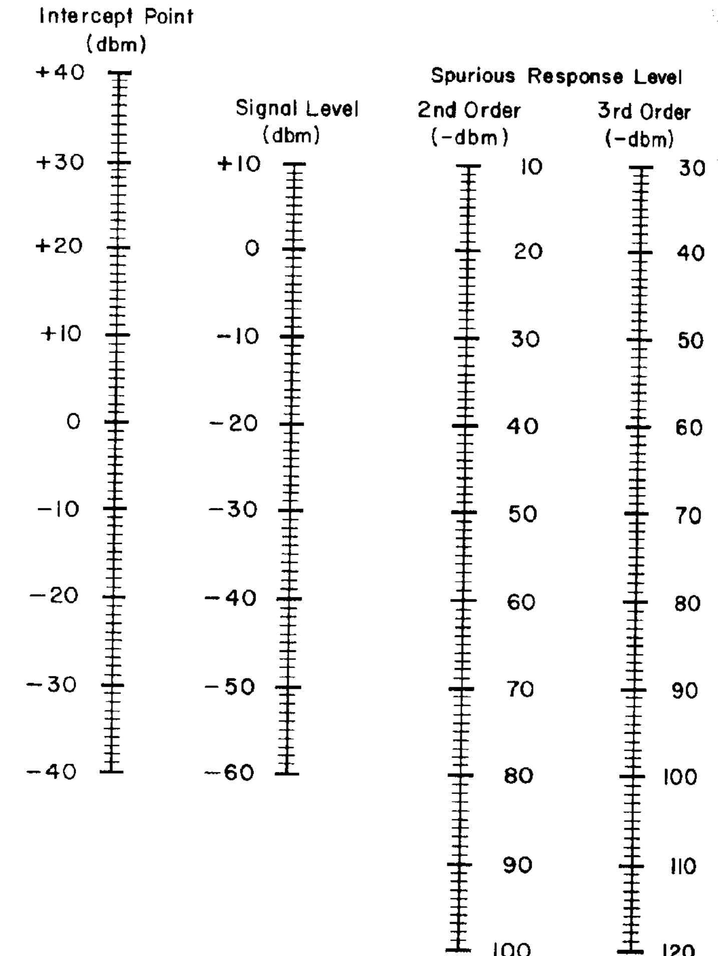


Fig. 4-23. Absolute level spurious response nomograph. (Based on nomographs from Avantek, Inc., Santa Clara, California.)

is −20 dBm • (−1/2) = 10 dBm.

When dealing with relative magnitudes, proceed as above, except note that there is a 2 dB/ dBm relationship between the output two-tone signals and the

distortion products.

For the case where the two-tone signals are unequal in magnitude, simply subtract 1/3 the difference between them from the larger.

Given:

Signal 1 +18 dBm
Signal 2 0 dBm

The equivalent equal magnitude two-tone signal has a power level of + 18 dBm − 1/3 (18 dBm − 0 dBm) = 12 dBm. Then proceed as before, using either the charts of calculation.

Second order intercepts are seldom considered because those products are generally farther removed from the desired frequency. Should it be of interest, second order terms may be related by the intercept point, as before.

4.17.1 Cascade Intercept Point

The system designer may be called upon to predict the intermodulation performance of many stages in cascade. One such application may involve a receiver where the intermodulation performance is specified. Assume that the specification reads: "The intermodulation distortion products resulting form a two-tone −30 dBm signal, seperated 2 and 4 MHz (respectively) from the desired frequency, and on the same side, shall not exceed −100 dBm." From Fig. (4-22) or (4-23), or by calculation, the third order intercept point must not be less than +5 dBm at the receiver input.

Further, assume that the receiver of Fig. (4-24) is being considered for this application. Since the two tones require 4 MHz of bandwidth, they will be processed by all stages, up to the first IF filter, which has a 50 kHz bandwidth and does not allow them to pass. This is the intermodulation distortion truncation point. All calculations must include all stages between the antenna and this filter which heavily attenuates the two-tone signal. Further intermodulation distortion contributions are negligible.

Norton's equation (4-52) may now be applied successively stage by stage, beginning at the antenna, up to the determined truncation point and the output intercept point determined. The gain to this point must be subtracted from this value to obtain the input intercept point, which may now be compared to the requirements.

$$\overset{3}{I}_t = \overset{3}{I}_2 - 10\log\left[1 + \frac{1}{g_2}\cdot\frac{\overset{3}{I}_2}{\overset{3}{I}_1}\right] \qquad (4\text{-}52)$$

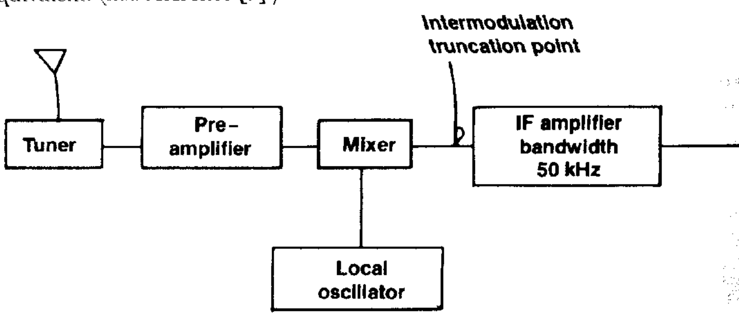
where

$\overset{3}{I_t}$ is the third order cascade output intercept point (dBm),

$\overset{3}{I_2}$ is the second stage third order output intercept point (dBm),
g_2 is the power gain of the second stage, and

$\overset{3}{I_1}$ is the first stage third order output intercept point.

Note: The terms in the brackets are not in dB notation. Use the numerical equivalent. (See reference [5]).



	Output Intercept Point dBm	Gain dB	Cascade Output Intercept Point
Tuner	N/A	-3.0	—
Preamp	20	15.0	20.0
Mixer	15	-7.0	10.875
		5.0	Input intercept point = 10.875 - 5.0 = 5.875 dBm

Fig. 4-24. Receiver example showing the input intercept point calculation.

The previous simplified example illustrates the cascade calculation method which indicates an input intercept point of 10.875 − 5 = 5.875 dBm. This numerical value barely meets the requirement. However, it must be kept in mind that Norton's equation assumes coherence of the intermodulation products through the various stages, which is not likely. The calculation is therefore considered pessimistic.

The second order intercept point may be computed in similar fashion using the equation:

$$\overset{2}{I_t} = \overset{2}{I_2} - 20\log\left[1+\sqrt{\frac{1}{g_2}\cdot\frac{\overset{2}{I_2}}{\overset{2}{I_1}}}\right] \qquad (4\text{-}53)$$

$\overset{2}{I_t}$ is the second order cascade output intercept point (dBm)
$\overset{2}{I_2}$ is the second stage second order output intercept point (dBm)
g_2 is the power gain of the second stage

$\overset{2}{I_1}$ is the first stage second order output intercept point

Note: the terms in the brackets are not in dB notation.

The procedure is to begin at the input as the first stage and compute the cascade with the following stage. This value becomes the first stage and the next stage (third in this case) becomes the next or second stage, *et cetera.*

The input intercept point becomes the final cascade intercept point minus the preceding gain in dB notation, as shown in Fig. (4-24). This computation can be tedious for complex systems and it is suggested that the program of Table 4-6 be utilized for that purpose.

Table 4-6.
Computer Program for the Calculation of Cascade Intercept Point

```
10 PRINT "  CASCADE INTERCEPT"
20 PRINT "  COMPUTES DEGRADATIO
   N OF THE INTERCEPT POINT DUE
    TO A PRECEDING STAGE"
30 PRINT
40 PRINT "  THE INPUT INT PT IS
    THE OUTPUT INT PT-GAIN"
50 DISP "CHOOSE 2ND (2) OR 3RD
   (3) ORDER";
60 INPUT N
70 PRINT "CASCADE INTERCEPT POI
   NT"
80 IF N=2 THEN 100
90 IF N=3 THEN 120
100 PRINT "SECOND ORDER"
110 GOTO 130
120 PRINT "THIRD ORDER"
130 PRINT "*********************
   ***********"
140 PRINT "IPT  G  CAS I  GT  IN
    IPT  STAGE"
150 PRINT "DBM  DB  DBM  DB  D
   BM"
```

```
160 PRINT
170 DISP "TITLE 1ST STAGE(5CHAR)
    "
180 DIM A$[5]
190 INPUT A$
200 DISP "ENTER 1ST STAGE INT PT
     (DBM)"
210 INPUT X
220 DISP "ENTER 1ST STAGE GAIN(D
    B)";
230 INPUT P
240 PRINT TAB(1);X;TAB(6);P;TAB(
    26);A$
250 PRINT
260 DISP "TITLE 2ND STAGE(5 CHAR
    )";
270 DIM B$[5]
280 INPUT B$
290 DISP "ENTER 2ND INT PT(DBM),
    GAIN(DB)",
300 INPUT Y,Z
310 K=20/(N-1)
320 L=10^(Z/10)
330 M=10^(Y/10)
340 J=10^(X/10)
350 D=P+Z
360 C=INT((Y-K*LGT(1+(1/L*M/J)^(
    1/(4-N))))*10+.5)/10
370 E=C-D
380 PRINT TAB(1);Y;TAB(6);Z;TAB(
    10);C;TAB(15);D;TAB(20);E;TA
    B(28);B$
390 PRINT
400 X=C
410 P=D
420 GOTO 260
430 END
```

The most frequent error in the computation of the second order intercept point is the determination of the truncation point. To determine this point, it is first necessary to develop an understanding of the second order distortion terms. These have been shown to be of the form:

Second harmonic terms, $2F_1$, $2F_2$

Sum and difference terms:

$F_1 \pm F_2$

Where

F_1 and F_2 are the two frequencies which cause the second order distortion.

There can be two cases which must be considered.

These are:

In band and
Out of band

The in band case results when F_1 and F_2 are within the receiver's tuning range. The out of band case results when these two signals are out of the receiver's tuning range.

The only barriers to second order distortion generation are the preselector attenuation, the conversion of the first mixer, and the use of high second order intercept components.

Many of the distortion products will not be converted by the first mixer and the remainder will suffer preselector attenuation. Where the conversion is the truncator the designer need not look beyond this point for further additive distortion. The attenuation of the preselector of the signal F_1 and (or) F_2 serves to increase the systems intercept point, which permits the use of lower intercept point components, if desired. Seldom is it ever necessary to extend the intercept analysis to the detector.

The following is a good approach to second order analysis:

Determine the range of the signals involved
Analyze the effect of the preselector on these signals

Examine the conversion of these distortion terms coordinated with preselector tuning.

A good design will attenuate the signals F_1 and (or) F_2 to livable levels and also reject through conversion the remainder.

Note: A simple relationship permits the determination of intercept point knowing the level of the distortion products.

This relationship is as follows:

$$\overset{m}{I} = S + \frac{\overset{m}{R}}{(m-1)}$$

where

m is the order
S is the signal (dBm)
$\overset{m}{R}$ is the intermodulation ratio (dB)

Example 4-7:

The requirements call for a suppression of second order products of 60 dB, resulting from signals of −50 dBm.

Then

$$\overset{m}{I} = -50 + 60/(2-1) = 10 \text{ dBm}$$

The equation is equally applicable to the third order case.

4.18 DESENSITIZATION

Desensitization of a receiver is the reduction of gain of that receiver to a desired frequency by a second unwanted signal. This can result from the presence of a stronger signal within the IF bandwidth of an FM receiver, the compression of stages in an AM receiver or from AGC takeover in AM and FM receivers, if used for the latter case.

Poorly designed receivers are subject to AGC takeover if the final selectivity is placed before the AGC detector, instead of at the front of the IF amplifier. An example of this is a multibandwidth multifunction receiver which must provide simultaneous outputs at several bandwidths. The filter following the final IF converter must be wider than the widest final filter bandwidth preceding the detectors. The IF strip is exposed to signals in bandwidth window possibly wider than that of the AGC system. The result of this is potential blocking or overload of the receiver.

A second possibility is AGC derived in a wide bandwidth with narrow band detection. Here a strong unwanted signal developes AGC, resulting in a weak desired output. To reduce desensitization effects, the designer must provide proper filtering throughout the receiver and eliminate sources of broadband noise (external and internal) to the receiver.

4.19 COMPRESSION

All linear systems, when given a sufficiently strong input signal, depart from a linear relationship between input and output. Such a system is said to be going into compression or saturation. An example of this situation would be an amplifier whose performance is shown in Fig. (4-25). The performance is linear up to an input signal level in excess of 0 dBm. Beyond this point, the output falls below the linearly extrapolated input/output characteristic and the amplifier is no longer linear.

Compression, or the −1 dB compression point, is generally taken as that point of −1 dB departure from linearity. In the example this is 10 dBm input or 19 dBm output. The input signal level which causes this departure is the value of compression usually specified. Where the output compression point is given, the gain between input and output (in dB) is subtracted from the output, to secure the input compression point.

Non-linear processes which provide a linear output amplitude, such as a mixer, are subject to compression as well. In this case the mixer is operating with a fixed local oscillator drive level and the input signal is increased while monitoring the output. Care must be exercised in this measurement to exclude all but the desired converted signal component. This may be readily accomplished by using a spectrum analyzer and examining the desired spectral line, or by using a RF voltmeter preceded by an appropriate IF filter. As a rule of thumb for double balanced diode mixers, the output compression point is the conversion loss below the local oscillator drive level. The input compression point is the output compression point increased by the amount of the conversion loss through the mixer.

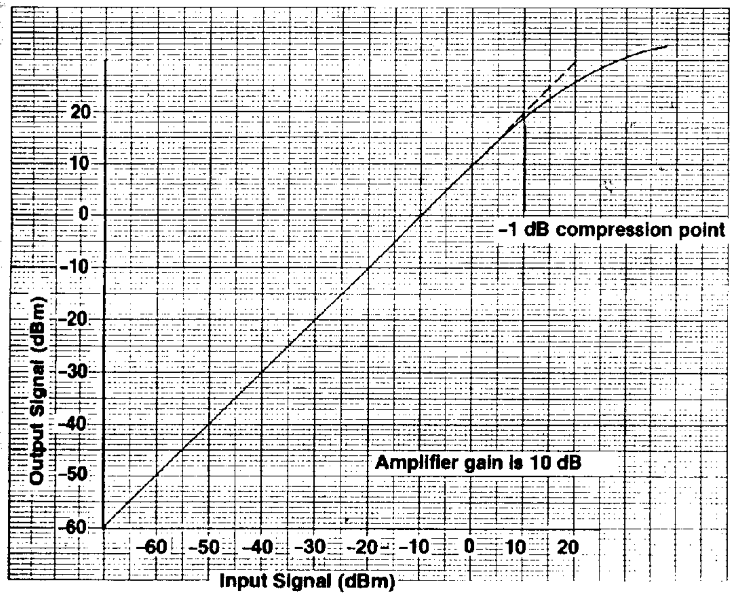


Fig. 4-25. An example showing the compression of a linear amplifier. Where the departure from linearity is −1 dB, this is the −1 dB compression point.

Example 4-8:

Given an output compression point of 9.5 dBm and a conversion loss of 6.5 dB, the input compression point is

9.5 + 6.5 = 16 dBm

4.20 CROSS MODULATION

When two signals appear simultaneously at a receiver's input, and one is modulated and the second is not, non-linearities in the receiver will impart modulation to the unmodulated signal from the modulated one. This process is called cross modulation and it is related to the third order intercept point by

$$M_c = \frac{\overset{3}{I}}{4P_2} + \frac{1}{2} \qquad (4\text{-}54)$$

where

$\overset{3}{I}$ is the receiver third order intercept point
P_2 is the power level of the stronger modulated signal

Note that the signal strength of the lesser unmodulated signal does not enter into the computation.

In most cases specifications define M_c and P_2; then the designer must solve for the required intercept point $\overset{3}{I}$ from

$$\overset{3}{I} = \left(M_c - \frac{1}{2}\right) 4P_2 \qquad (4\text{-}55)$$

Example 4-9:

Given:
Cross modulation must not exceed 20 dB
The interfering signal P_2 is −10 dBm
Find the required intercept point.

$$\overset{3}{I} = \left(\text{anl}\ \frac{20}{20} - \frac{1}{2}\right) 4\ \text{anl}\ \frac{-10}{10}$$

$= 5.79$ dB

For convenience, a chart of the cross modulation ratio and the intercept point minus the power of the stronger modulated signal is given in Fig. (4-26) and is based upon:

$$\frac{\overset{3}{I}}{P_2} = 4\left(M_c - \frac{1}{2}\right) \quad \text{Where } M_c = \frac{m}{m_1} \qquad (4\text{-}56)$$

m is the modulation index
m_1 is the effective modulation index

Example 4-11:

Find the solution to the previous problem using the chart of Fig. (4-26). Enter the chart at $m/m_1 = 20$ dB and read $\overset{3}{I} - P_2 = 16$. Since $P_2 = -10$, $\overset{3}{I} = 6$ dBm. References [6] and [4] contain additional information on this subject.

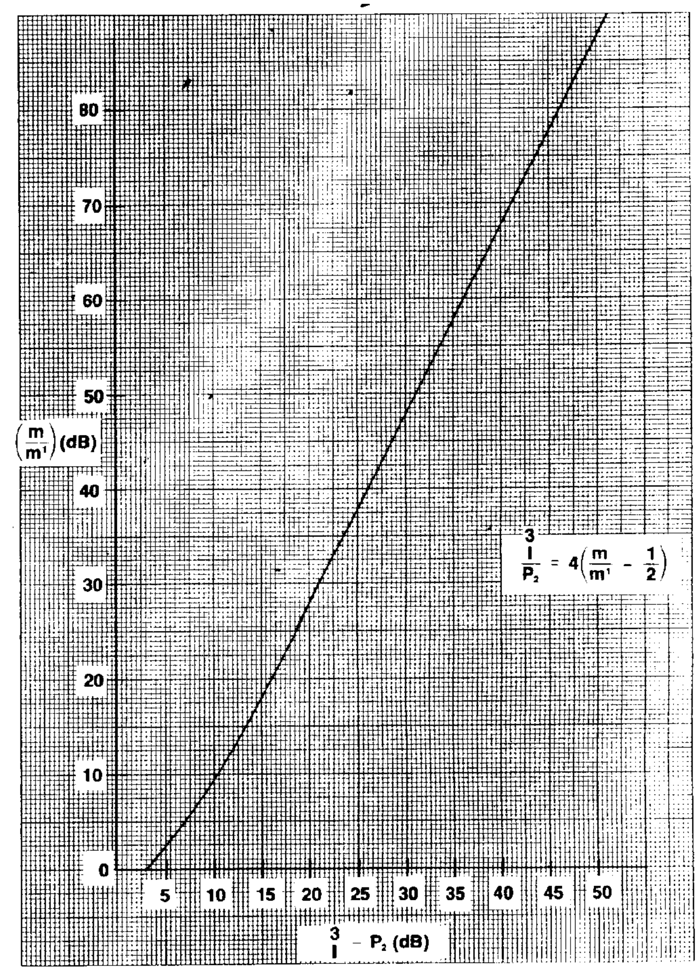


Fig. 4-26. Graph of cross modulation *versus* intercept point [6].

4.21 SPURIOUS FREE DYNAMIC RANGE

It is informative to determine the dynamic signal operation range of a system which is free of spurious signals resulting from third order intermodulation products. In this definition spurious free means that these spurious signals are equal to the noise level. The third order intermodulation product signal levels are related to the level of the two signals (producing them, respectively, on a three for one dB relationship). The spurious free dynamic range is related to the third order system intercept point by the following relationship:

$$SFDR\ (\text{dB}) = 0.67\,(\overset{3}{I} - kT/\text{MHz} - 10\log B - NF) \qquad (4\text{-}57)$$

where

SFDR is the spurious free dynamic range

$\overset{3}{I}_i$ is the system third order input intercept point
kT is the thermal noise level in a 1 MHz bandwidth
= −114 dBm / MHz
B is bandwidth in MHz
NF is the system noise figure (dB)

An example of the application of this relationship follows.

Example 4-11:

Given:

Third order input intercept point = 10 dBm (Note: Given the output intercept point, the input intercept point is the output intercept point minus the gain in dB notation.)
Bandwidth is 10 kHz
Noise figure is 5 dB

Find the spurious free dynamic range.

Solution:

SFDR = 0.67 (10 − (−114) − (−20) − 5)
= 92.67 dB

4.22 IMAGES

In the mixing process it was shown that when two input signals comprised of an RF signal and a local oscillator signal are applied to a mixer, intermediate frequency signals are produced at the mixer's output. More specifically:

$$F_{if} = \left| NF_r \pm MF_{lo} \right| \qquad (4\text{-}58)$$

where

F_{if} is the intermediate frequency
F_r is the receive frequency
F_{lo} is the local oscillator frequency
M and N are intergers

The primary or desired outputs result when $M = N = 1$

Then equation (4-58) results in:

$$F_{if} = \left| F_r \pm F_{lo} \right| \qquad (4\text{-}59)$$

Since F_{if} is a constant, then for a given LO frequency there exist two values of F_r which satisfy the relationship.

EXAMPLE 4-13:

Let F_{lo} = 160.7 MHz
F_{if} = 10.7 MHz
then $F_r = \left| F_{if} \pm F_{lo} \right|$
or $\left| 10.7 \pm 160.7 \right|$ = 150 and 171.4 MHz.

Thus, the mixer is equally responsive to two frequencies, both of which are twice the IF apart. Of these, one is the desired response and the other is called the image frequency. The receiver must reject the image term to provide satisfactory performance. This is one of the reasons mixers are always preceded by preselection filtering of some form. The selectivity requirements of the preselector are governed by the frequency separation between the image frequency and the desired frequency. The amount of image frequency rejection is strictly a function of the attenuation, provided by the preselector filter alone (unless an image rejection mixer is used).

To avoid extreme selectivity requirements from the preselector, the ratio of F_r to F_{if} should not exceed 10 or 20 to 1 for a first conversion in a down conversion superheterodyne receiver.

In up-conversion systems, the high value of intermediate frequency effectively removes the image frequency out of the preselector bandwidth. Here, simple fixed tuned filters often suffice as preselector filters, eliminating tracking and tuning problems. Fig. (4-27) is an example illustrating the relative image frequency behavior between up and down conversion.

4.23 HIGH ORDER IMAGES

When a receiver design utilizes more than one conversion, the image problem becomes more complex. For every mixer in a receiver there will exist an image frequency. A double conversion receiver will have primary and secondary