Thermal Noise:

The noise density for electrical noise may be shown to be:

$$P(f) = \frac{2h|f|}{e^{\frac{h|f|}{kT}} - 1} \quad \frac{watts}{Hz}$$

h is Planck's constant: 6.624×10^{-34} W-sec per Hz

k is Boltzman's constant : 1.379×10^{-27} W per degree K

- Due to the thermal motion of electrons in a conduction medium and is present in any circuit with resistance.
- Thermal noise limits the sensitivity of all electronic systems.
- This density may be assumed flat (white) for frequencies we use in electronics.

$$P(f) = kT \quad \frac{Watts}{Hz} \quad for\ 0 < f < 6000GHz$$

$$\left(\frac{hf}{kT} = 1\ at\ 6000GHz \right)$$

The total noise power for white noise is

$$P = \int_0^B P(f)df = kTB \ \ Watts$$

where B is the bandwidth in Hz.

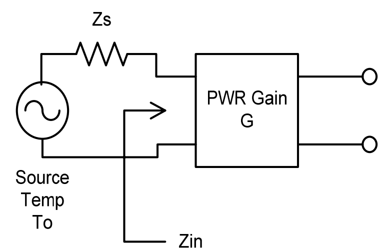
When white noise is passed through a system with a voltage transfer function of H(f), the total noise power is found by:

$$P = \int_0^B P(f)|H(f)|^2 df \quad Watts$$

Noise Power Transfer

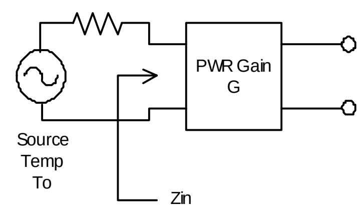
The source impedance will generate a thermal noise due to random electron motion and have a one-sided spectral density of:

$4kT_oZ_s\ \ V^2/Hz$



$Z_{in} = Z^*_s$ Matched Load

The noise power coupled into the device from the source impedance over a bandwidth B is:

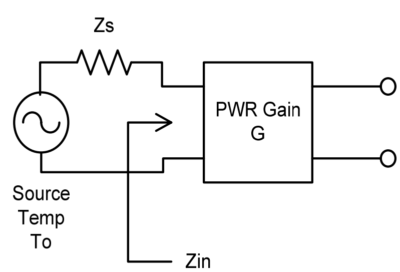


$$4kT_oZ_s\left(\frac{Z_{in}}{Z_{in}+Z_s}\right)^2\frac{1}{Z_{in}}B$$

Which is kT_oB $Z_{in} = Z^*_s$ Matched Load

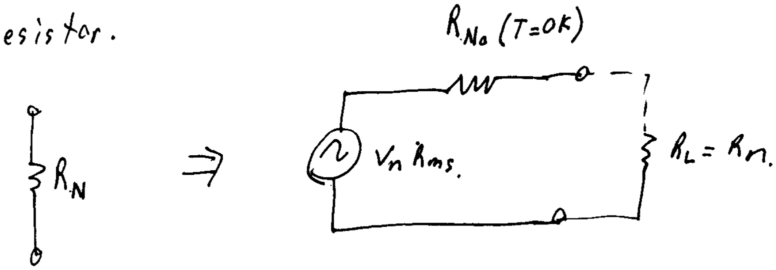
Noise Power Transfer

The source noise power at the output is then:



$$P_n = kT_oBG$$

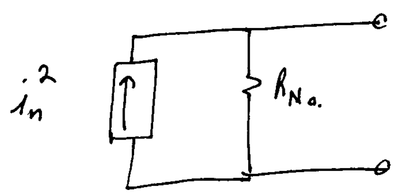
Resistor.



$$\frac{V_n^2}{4R_{N_0}} = P_N \rightarrow \text{power delivered to load under matched conditions.}$$

$$P_N = KTB = \frac{V_n^2}{4R_{N_0}} \qquad V_n^2 = 4KTBR_{N_0}$$

$$\Rightarrow \frac{V_n^2}{B} = 4KTR \quad V^2/Hz.$$



$$\frac{i_n^2}{B} = \frac{4KT}{R} \quad A^2/Hz.$$

---

example. Calculate the noise power (in dBm) and the rms noise voltage at $T = 290K$ for.

$R_n = 1\Omega \qquad B = 1Hz$

$$P_N = KTB = 1.374\times10^{-23}\, J/K \cdot 290 \cdot 1 = \underline{3.985\times10^{-21}\, W}$$

$$P_N\,dBm = 10\log\left(\frac{3.985\times10^{-21}}{10^{-3}}\right) = \underline{-174\,dBm}$$

$$V_N = 2\sqrt{P_N R_N} = 2\sqrt{3.958\times10^{-21}\cdot 1} = 12.6\times10^{-11}\,V = \underline{0.0126\,nV}$$

1.26 pV

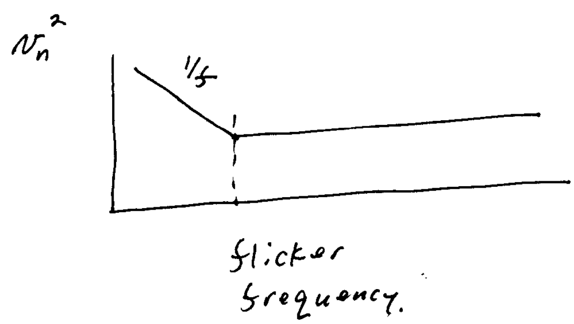
Let $R_n = 2\ M\Omega$ $B = 5\ KHz$.

$$P_N = 3.985 \times 10^{-21} \cdot 5000 = 19.925 \times 10^{-18}\ W \Rightarrow -137\ dBm.$$

or $P_N = -174\ dBm/Hz + 10 \log 5000 = -137\ dBm.$

$$V_N = 2\sqrt{P_n R_n} = 2\sqrt{19.925 \times 10^{-18} \cdot 2 \times 10^{6}} = \underline{12.6\ uV}$$

- noise in Resistors is independent
- reduce Resistive networks to thev or Nort equiv.
- practical Resistors have $\frac{1}{f}$ noise and N_o above KTR



14.2 IMPORTANCE OF NOISE

Noise is passed into a microwave component or system from an external source, or it is generated within the unit itself. Regardless of the manner of entrance of the noise signal, the noise level of a system greatly affects the performance of the system by setting the minimum detectable signal in the presence of noise. Therefore, it is often desirable to reduce the influence of external noise signals and minimize the generation of noise signals within the unit to achieve the best performance.

14.3 NOISE DEFINITION

Because noise considerations have important consequences, we need to define noise first:

DEFINITION-ELECTRICAL NOISE (OR NOISE): *Any unwanted electrical disturbance or spurious signal. These unwanted signals are random in nature and are generated either internally in the electronic components or externally through impinging electromagnetic radiation.*

Because noise signals are totally random and uncorrelated in time, they are best analyzed through statistical methods. Their statistical properties can be briefly summarized as follows:

a. The "mean value" of the noise signal is zero, i.e.,

$$\bar{V}_n = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} V_n(t)dt = 0 \tag{14.1}$$

where $\bar{V}_n$ is the noise mean value, $V_n(t)$ is the instantaneous noise voltage, t_1 is any arbitrary point in time, and T is any arbitrary period of time, ideally a large one approaching infinity.

b. The "mean-square value" of the noise signal is a constant value, i.e.,

$$\overline{V_n^2} = \lim_{T\to\infty} \frac{1}{T} \int_{t_1}^{t_1+T} [V_n(t)]^2 dt = \text{Constant} \tag{14.2}$$

c. The "root-mean-square" (rms) of a noise signal is given by:

$$V_{n,rms} = \sqrt{\overline{V_n^2}} \tag{14.3}$$

or

$$V_{n,rms}^2 = \overline{V_n^2} \tag{14.4}$$

The concept of "root-mean-square value" (rms value) of noise, as given by Equation 14.3, is based on the fact that the "mean-square value," $\overline{V_n^2}$, is proportional to the "noise power." Thus, if we take the square root of Equation 14.2, we obtain the rms value of the noise voltage, which is the "effective value" of the noise voltage.

14.4 SOURCES OF NOISE

There are several types of noise that need to be defined:

DEFINITION-THERMAL NOISE (OR JOHNSON NOISE OR NYQUIST NOISE): *The most basic type of noise, which is caused by thermal vibration of bound charges and thermal agitation of electrons in a conductive material. This is common to all passive or active devices.*

DEFINITION-SHOT NOISE (OR SCHOTTKY NOISE): *Caused by random passage of discrete charge carriers (causing a current I, due to motion of electrons or holes) in a solid-state device while crossing a junction or other discontinuities. It is commonly found in a semiconductor device (e.g., in a pn junction of a diode or a transistor) and is proportional to $(I)^{1/2}$.*

DEFINITION-FLICKER NOISE (ALSO CALLED 1/f NOISE): *Small vibrations of a current due to the following factors:*

a. *Random injection or recombination of charge carriers at an interface, such as at a metal and semiconductor interface (in semiconductor devices)*
b. *Random changes in cathode emissions of electric charges such as at a cathode-air interface (in a thermionic tube)*

Flicker noise is a frequency-dependent noise, which distorts the signal by adding more noise to the lower part of the signal band than the upper part. It exists at lower frequencies, almost from DC extending down to approximately 500 kHz to 1 MHz at a rate of −10 dB per decade.

14.5 THERMAL NOISE ANALYSIS

To analyze noise, let's consider the circuit shown in Figure 14.1a where a noisy resistor is connected to the input port of a two-port network. Focusing primarily on thermal noise, we note that the available noise power (i.e., maximum power available under matched conditions) from any arbitrary resistor has been shown by Nyquist to be:

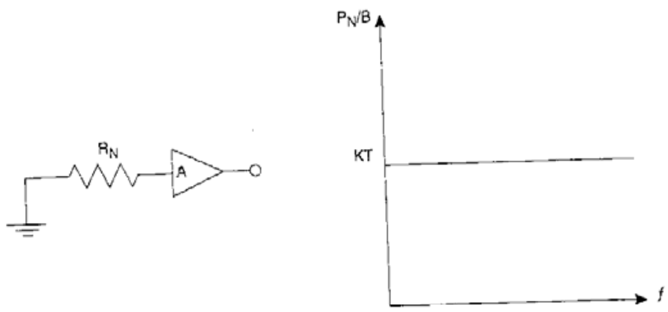
$$P_N = kTB \tag{14.5}$$

where

k = Boltzmann's constant (= 1.374×10^{-23} J/K)
T = The resistor's physical temperature (in Kelvin)
B = The two-port network's bandwidth (i.e., $B = f_H - f_L$)

Because the noise power does not depend on the center frequency of operation but only on the bandwidth, it is called "white noise," as shown in Figure 14.1b.
A few observations about noise power (P_N) are worth considering:

- As bandwidth (B) is reduced, so is the noise power, which means narrower bandwidth amplifiers are less noisy.
- As temperature (T) is reduced, the noise power is also lessened, which means cooler devices or amplifiers generate less noise power.



(a) A noisy resistor (b) Noise spectral power density

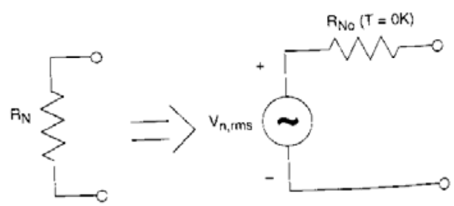
FIGURE 14.1 White noise in an amplifier.

- Increasing bandwidth to infinity causes an infinite noise power (called ultraviolet catastrophe), which is incorrect because Equation 14.5 for noise power is valid only up to approximately 1000 GHz.

14.6 NOISE MODEL OF A NOISY RESISTOR

A noisy resistor (R_N) at a temperature (T) can be modeled by an ideal noiseless resistor (R_{No}) at 0° K in conjunction with a noise voltage source ($V_{n,rms}$), as shown Figure 14.2.

FIGURE 14.2 Model of a noisy resistor.

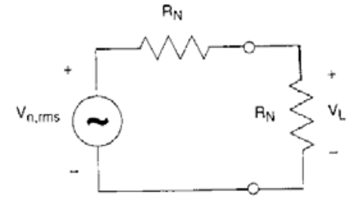


If we assume that the resistor value is independent of temperature then $R_{No} \doteq R_N$. From this model, the available noise power to the load (under matched condition) is given by (see Figure 14.3):

$$P_N = \frac{V^2_{n,rms}}{4R_N} \qquad (14.6)$$

Equation 14.6 provides the noise power available from a noisy resistor, which equals Equation 14.5 for any arbitrary resistor. Thus:

FIGURE 14.3 Available noise power.



$$P_N = kTB \qquad (14.7a)$$

$$V_{n,rms} = 2\sqrt{P_N R_N} = 2\sqrt{kTBR_N} \qquad (14.7b)$$

From Equation 14.7b, we can observe that the noise voltage is proportional to $R_N^{1/2}$. Thus, higher-valued resistors have higher noise voltage even though they provide the same noise power level as the lower-valued resistors.

EXAMPLE 14.1

Calculate the noise power (in dBm) and rms noise voltage at T = 290° K for:

a. $R_N = 1\ \Omega, B = 1$ Hz
b. $R_N = 2\ \text{M}\Omega, B = 5$ kHz

Solution:

a. The noise power is given by:

$$k = 1.374 \times 10^{-23}\ \text{J/° K}$$

$$B = 1\ Hz$$

$$P_N = kTB = kT = 1.374 \times 10^{-23} \times 290 \times 1 = 3.985 \times 10^{-21}\ \text{W}$$

Or, in dBm, we have:

$$P_N(\text{dBm}) = 10\log(3.985 \times 10^{-21}/10^{-3}) = -174\ \text{dBm}$$

This is the power per unit Hz. The corresponding noise voltage for a 1 Ω resistor is given by:

$$V_{n,rms} = 2\sqrt{P_N R_N} = 2\sqrt{3.958 \times 10^{-21} \times 1} = 12.6 \times 10^{-11}\ \text{V} = 12.6 \times 10^{-5}\ \mu\text{V}$$

b. For a 5 kHz bandwidth, we have

$$P_N = kTB = 3.985 \times 10^{-21} \times 5000 = 19.925 \times 10^{-18}\ \text{W} = 19.925 \times 10^{-15}\ \text{mW}$$

Or, in dBm:

$$P_N(\text{dBm}) = -137\ \text{dBm}$$

(alternately, $P_N = 10\log(kT) + 10\log B = -174 + 10\log 5000 = -137$ dBm)

The corresponding noise voltage for a 2 MΩ resistor is given by

$$V_{n,\,rms} = 2\sqrt{P_N R_N} = 2\sqrt{19.925\times10^{-18}\times2\times10^{6}} = 12.6\times10^{-6}\text{ V} = 12.6\ \mu\text{V}$$

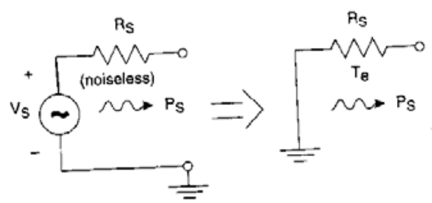
4.7 EQUIVALENT NOISE TEMPERATURE

Any type of noise, in general, has a power spectrum that can be plotted in the frequency domain. If the noise power spectrum is not a strong function of frequency (i.e., it is white noise), then it can be modeled as an equivalent thermal noise source characterized by an "equivalent noise temperature" (T_e).

To define the equivalent noise temperature (T_e), we consider an arbitrary white noise source with an available power (P_S) having a noiseless source resistance (R_S), as shown in Figure 14.4a. This white noise source can be replaced by a noisy resistor with an equivalent noise temperature (T_e) defined by:

$$T_e = \frac{P_S}{kB} \tag{14.8}$$

where B is the bandwidth of the system or the component under consideration.



(a) (b)

FIGURE 14.4 (a) An arbitrary white noise source, (b) equivalent circuit.

EXAMPLE 14.2

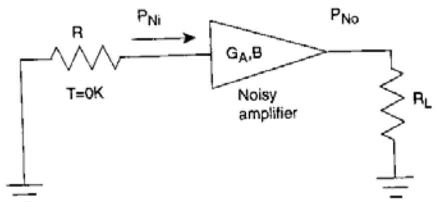
Consider a noisy amplifier with available power gain (G_A) and bandwidth (B) connected to a source and load resistance (R) both at $T = T_S$, as shown in Figure 14.5. Determine the overall noise temperature of the combination and the total output noise power if the amplifier alone creates an output noise power of P_n.

Solution:

To simplify the analysis, let's first assume that the source resistor is at $T = 0°$ K. This means that no noise enters the amplifier, i.e., $P_{Ni} = 0$.

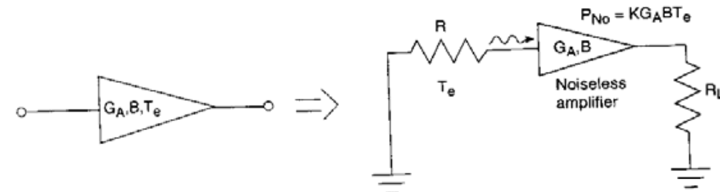
The noisy amplifier can be modeled by a noiseless amplifier with an input resistor at an equivalent noise temperature of:

FIGURE 14.5 A noisy amplifier.



$$T_e = \frac{P_n}{G_A kB} \tag{14.9}$$

T_e is called the equivalent noise temperature of the amplifier "referred to the input," as shown in Figure 14.6.



(a) (b)

FIGURE 14.6 Equivalent models of a noisy amplifier.

Because source resistor (R) is at a physical temperature other than zero ($T = T_S$), then as a result the combined equivalent noise temperature (T'_e) is the addition of the two noise temperatures:

$$T'_e = T_e + T_S \tag{14.10}$$

Assuming the noise power at the input terminals of the amplifier is P_{Ni} ($= kT_SB$), the total output noise power due to the amplified input thermal noise power will be ($G_A P_{Ni}$), which adds to the amplifier's generated noise power (P_n) linearly by using the superposition theorem (see Figure 14.7), i.e.,

$$P_{No,tot} = G_A P_{Ni} + P_n = G_A kB(T_S + T_e)$$

$$P_{No,tot} = G_A kBT'_e \tag{14.11}$$

NOTE: *It is important to note that from Equation 14.11, the "equivalent noise temperature" (T'_e) is defined by "referring" the total output noise power to the input port. Thus, the same noise power is delivered to the load by driving a "noiseless amplifier" with a resistor at an equivalent temperature ($T'_e = T_e + T_S$).*

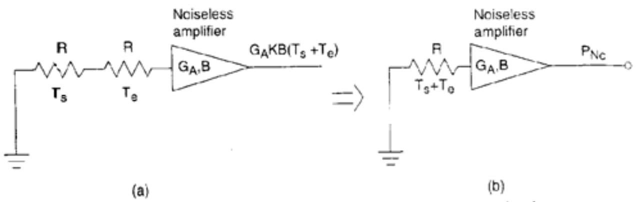


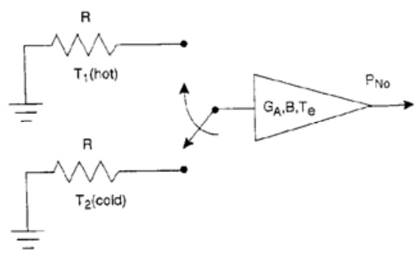
FIGURE 14.7 Total output noise power and its equivalent circuit.

14.7.1 A Measurement Application: Y-Factor Method

The concept of equivalent noise temperature is commonly used in the measurement of noise temperature of an unknown amplifier using the "Y-factor method." In this method, the physical temperature of a matched resistor is changed to two distinct and known values:

- One temperature (T_1) is at boiling water ($T_1 = 100°C$) or at room temperature ($T_1 = 290° K$)
- The second temperature (T_2) is obtained by using either a noise source (hotter source than room temperature) or a load immersed in liquid nitrogen at $T = 77°$ K (a colder source than room temperature), as shown in Figure 14.8.

FIGURE 14.8 Y-factor method



The amplifier's unknown noise temperature (T_e) can be obtained as follows:

$$P_{No,1} = G_A kB(T_1 + T_e) \tag{14.12}$$

$$P_{No,2} = G_A kB(T_2 + T_e) \tag{14.13}$$

Now define:

$$Y \equiv \frac{P_{No,1}}{P_{No,2}}$$

Thus, we can write:

$$Y = \frac{T_1 + T_e}{T_2 + T_e} \tag{14.14}$$

or

$$T_e = \frac{T_1 - YT_2}{Y - 1} \tag{14.15}$$

From a measurement of T_1, T_2 and Y, the unknown amplifier's noise temperature (T_e) can be found.

POINT OF CAUTION: *To obtain an accurate value for Y, the two temperatures ideally must be far apart; otherwise, $Y \approx 1$ and the denominator of Equation 14.15 will create relatively inaccurate results.*

NOTE: *A noise source "hotter" than room temperature, as used in the Y-factor measurement, would be a solid-state noise source (such as an IMPATT diode) or a noise tube. Such active sources, providing a calibrated and specific noise power output in a particular frequency range, are most commonly characterized by their "excess noise ratio" values versus frequency. The term excess noise ratio or ENR is defined as:*

$$ENR(\text{dB}) = 10\log_{10}\left(\frac{P_N - P_o}{P_o}\right) = 10\log_{10}\left(\frac{T_N - T_o}{T_o}\right) \tag{14.16}$$

where P_N and T_N are the noise power and equivalent noise temperature of the active noise generator, and P_o and T_o are the noise power and temperature of a room-temperature passive source (e.g., a matched load), respectively.

14.8 DEFINITIONS OF NOISE FIGURE

As discussed earlier, a noisy amplifier can be characterized by an equivalent noise temperature (T_e). An alternate method to characterize a noisy amplifier is through the concept of noise figure, which we need to define first.

DEFINITION-NOISE FIGURE: *The ratio of the total available noise power at the output, $(P_o)_{tot}$, to the output available noise power $(P_o)_i$ due to thermal noise coming only from the input resistor at the standard room temperature ($T_o = 290° K$).*

To formulate an equation for noise figure (F), let us transfer the noise generated inside the amplifier (P_n) to its input terminals and model it as a "noiseless" amplifier that is connected to a noisy resistor (R) at noise temperature (T_e) in series to another resistor (R) at $T = T_o$, both connected at the input terminals of the "noiseless" amplifier, as shown in Figure 14.9.

From this configuration, we can write:

$$P_n = G_A kT_e B \tag{14.17a}$$

$$(P_o)_i = G_A P_{Ni} = G_A kBT_o \tag{14.17b}$$

$$(P_o)_{tot} = P_{No} = P_n + (P_o)_i \tag{14.18}$$

The following notes are from "Solid State Radio Engineering" by H. Krauss

2-1 Thermal Noise in Resistors and Networks

As the name implies, *thermal noise* is due to the random motion of charge carriers in any conducting medium whose temperature is above absolute zero. The velocity of this motion increases with temperature in such a way that the electrical noise power density produced is proportional to the resistance of the conductor and to its absolute temperature, hence the name *thermal noise*. It is also called *white noise* because it has been shown both theoretically and experimentally to have a uniform spectrum up to frequencies on the order of 10^{13} Hz (just as white light is composed of all colors of the visible spectrum).

A metallic resistor may be considered a thermal noise source that can be represented by either of the noise equivalent circuits shown in Fig. 2-1. The *mean-square noise voltage* (V_n^2) and *current* (I_n^2) are given by the following expressions in which R is the resistance, $G = 1/R$ the conductance, T the temperature of the resistor in kelvin units, k Boltzmann's constant (1.38×10^{-23} J/K), and B the bandwidth in hertz in which the noise is observed.

$$V_n^2 = 4kTRB \qquad (2\text{-}1)$$

$$I_n^2 = 4kTGB \qquad (2\text{-}2)$$

(At low frequencies, practical resistors also exhibit excess current noise [4].)

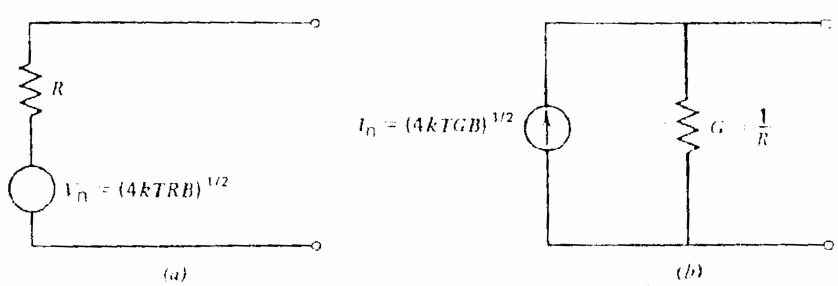
The noise power that is transmitted through a circuit is proportional to the circuit bandwidth. Consequently, the circuit bandwidth should never be greater than that necessary to transmit the desired signal if the maximum output signal-to-noise ratio (SNR) is to be achieved.

Example 2-1.1. Calculate the mean-square noise voltage produced in a 100-kilohm resistor in a bandwidth of 10^6 Hz at room temperature ($T = 20°\text{C} = 293$ K).

$$4kT = 1.62 \times 10^{-20}$$

$$V_n^2 = 1.62 \times 10^{-20} \times 10^5 \times 10^6 = 16.2 \times 10^{-10} \text{ volts}^2$$

Fig. 2-1 Equivalent circuits to represent thermal noise in a resistor.



represented as though it were thermal noise generated in a fictitious resistance equal to the radiation resistance, at a temperature T_A that would account for the noise actually measured. This is called the ***noise temperature*** of the antenna.

Example 2-2.1. Suppose that a 200-ohm antenna exhibits an rms noise voltage of 0.1 μV at its terminals, when measured in a bandwidth $B = 10^4$ Hz. By the use of (2-1),

$$V^2 = 4kT_ARB$$

or

$$T_A = \frac{V^2}{4kRB} = \frac{10^{-14}}{1.38 \times 10^{-23} \times 200 \times 10^4} = 90.6 \text{ K}$$

Thus the noise at the antenna terminals is equivalent to that from a 200-ohm resistor at a temperature of 90.6 K. It will be shown later that other parts of a receiving system can also be characterized by equivalent noise temperatures in order to simplify the computation of signal-to-noise ratio at the output of a receiver.

2-3 Noise in Diodes, Transistors, and FETs

Because resistors and antennas are two-terminal devices, it is easy to describe their noise characteristics in terms of a noise temperature or an equivalent noise resistance. The situation is more complicated for transistors and other multiterminal circuit elements because their internally generated noise depends upon temperature, operating point, and input and output terminations. For noise calculations in transistor circuits, the transistors are conveniently represented as black boxes with specified noise figures and the physical causes of the transistor noise are represented by equivalent noise sources.

Diode Noise

The noise generated in thermionic and junction diodes is called *shot noise.* It arises because the diode current is made up of charge carriers that are emitted randomly from the cathode or emitter region; the number of these carriers fluctuates statistically from instant to instant. Shot noise has essentially a flat spectral distribution and is treated in the same manner as thermal noise.

The shot noise generated by a diode may be represented as coming from a current source with a mean-square noise current equal to where q is the electron charge, 1.6×10^{-19} coulombs, I_{DC} is the diode direct current in amperes, and B is the bandwidth in hertz over which the noise is measured. This model is invalid for diodes operated in the reverse breakdown or avalanche region; here a large-amplitude impulse noise called microplasma noise is generated. Microplasma noise is important in the construction of diode noise generators.

Junction Transistor Noise

Two sources of noise in junction transistors are shot noise in each diode junction and thermal noise in the base spreading resistance (variously called r_x, r_b', or r_{bb}'). As the emitter current divides between the collector and the base, the route taken by each charge carrier is randomly selected and a statistical fluctuation in the collector and base currents results. This is called partition noise. Another noise, called $1/f$ noise, flicker noise, or excess noise, is observed at low frequencies and is the principal source of noise in dc amplifiers [5]. Flicker noise is caused primarily by surface recombination of minority carriers in the emitter-base depletion region [6].

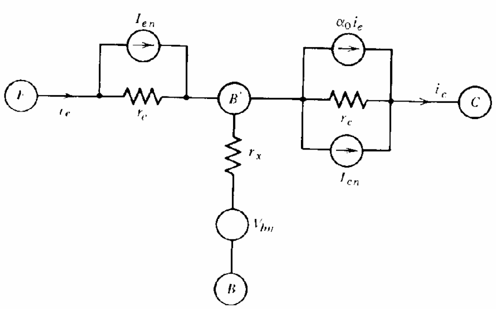
A noise model derived by Van der Ziel for a transistor in the common-base configuration is shown in Fig. 2-4. The noise sources are defined as follows [7]:

$$I_{en}^2 = 2qI_EB \qquad (2\text{-}12)$$

in which I_E is the direct emitter current.

$$V_{bn}^2 = 4kTr_xB \qquad (2\text{-}13)$$

Fig. 2-4 Van der Ziel's noise model for the common-base transistor.



in which r_x is the base-spreading resistance.

$$I_{cn}^2 = 2qB[I_{CO} + I_C(1-\alpha_0)] \qquad (2\text{-}14)$$

in which I_{CO} is the collector reverse saturation current, I_C the direct collector current, and α_0 the low-frequency, common-base, short-circuit current gain. This circuit is valid to frequencies on the order of $f = f_\alpha(1-\alpha_0)^{1/2}$. It does not account for $1/f$ noise.

Figure 2-5 shows a model derived from Fig. 2-4 by converting current sources to voltage sources and making the following simplifying assumptions: $r_x \ll \alpha_0 r_c$, $R_s + r_e \ll \alpha_0 r_c$, and $I_{CO} \ll I_C(1-\alpha_0)$. A source resistance R_s has been added, with its corresponding thermal noise source. The voltage sources in the circuit of Fig. 2-5 are given by (2-13) and by

$$V_{gn}^2 = 4kTR_sB \qquad (2\text{-}15)$$

$$V_{en}^2 = 2kTr_eB, \qquad r_e = \frac{kT}{qI_E} \qquad (2\text{-}16)$$

$$V_{cn}^2 = \frac{2kT\alpha_0 r_c^2(1-\alpha_0)B}{r_e} \qquad (2\text{-}17)$$

For transistors in the common-emitter configuration, a hybrid-pi noise model [8] shown in Fig. 2-6 may be more useful. In this model, V_{bn}^2 is given by (2-13), and

$$I_{on}^2 = 2q|I_C|B \qquad (2\text{-}18)$$

and

$$I_{in}^2 = 2q|I_B'|B \qquad (2\text{-}19)$$

where $I_B' = -I_B + 2I'$, and I' is the total base current that flows when both emitter and collector junctions are reversed-biased (may be significant in germanium transistors at low bias currents). In the frequency range in which

Fig. 2-5 A model derived from Fig. 2-4.

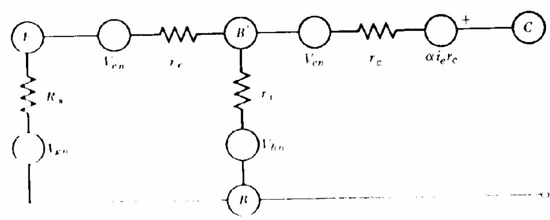
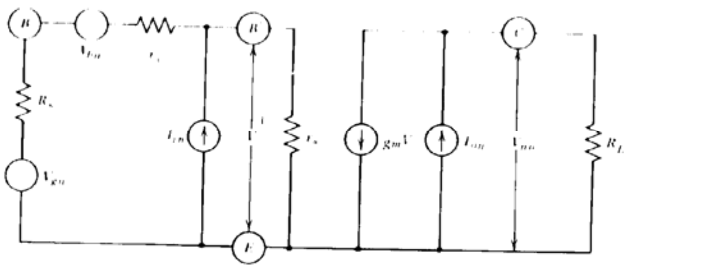


Fig. 2-6 A hybrid-pi noise model for common-emitter operation.



the hybrid-pi model of Fig. 2-6 is valid (i.e., up to about $f_T/2$), the effect of transistor noise can be minimized by the proper choice of R_s. This is shown in Appendix 2-1.

FET Noise

Both JFETs and MOSFETs exhibit noise from several sources as follows [9]:

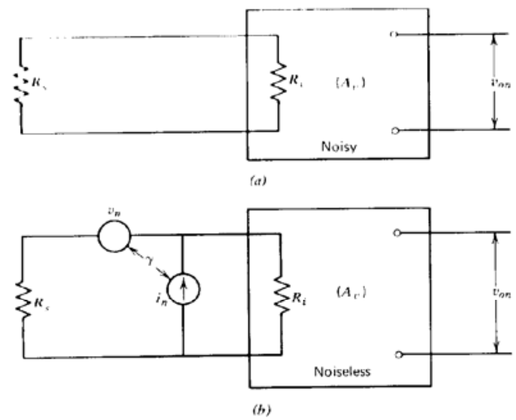
1. Thermal noise generated in the channel resistance.
2. Channel thermal noise coupled to the gate through the gate-channel capacitance.
3. $1/f$ noise that is significant below about 100 Hz in JFETs or 10 kHz in MOSFETs.

In addition, JFETs exhibit shot noise due to the small reverse current in the gate junction.

As an alternative to a noise equivalent circuit that incorporates the separate sources listed above, the noise in an FET is more often characterized by the "two-generator method" [5] illustrated in Fig. 2-7. The noisy FET is represented in Fig. 2-7*a* as an amplifier with voltage gain A_v, input resistance R_i, and rms output noise voltage v_{on}. In Fig. 2-7*b* the FET is replaced by a noise-free amplifier, and the output noise due to the FET is accounted for by the combination of a voltage source v_n and a current source i_n on the input side, with a correlation coefficient γ to represent the mutual dependence between these two sources. The assumption is usually made that v_n and i_n are white-noise sources and that $\gamma = 0$. Since the total v_{on}^2 is proportional to bandwidth B, it is convenient to normalize the values of v_n and i_n to a 1-Hz basis; that is, v_n is given in volts/(hertz)$^{1/2}$ and i_n is given in amperes/(hertz)$^{1/2}$.

With the assumptions that $R_s \ll R_i$, and that $\gamma = 0$, the mean-square output

Fig. 2-7 (*a*) Noisy amplifier with input resistance R_i and voltage gain A_v; and (*b*) two-generator noise equivalent circuit.



noise voltage due to the two sources is

$$V_{on}^2 = A_v^2(v_n^2 + i_n^2 R_s^2) \qquad (2\text{-}20)$$

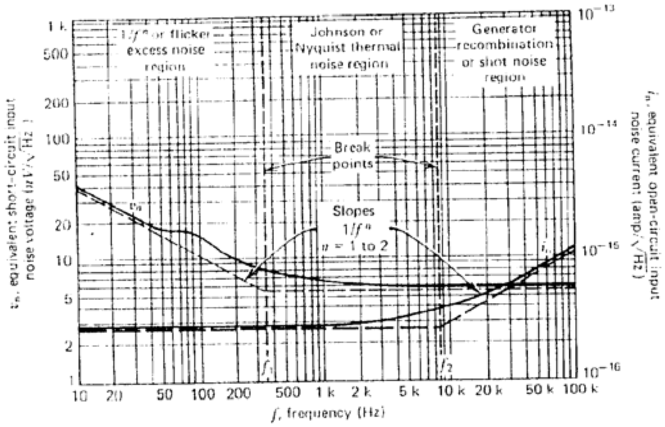
Since v_n and i_n are frequency-dependent, the total output noise in bandwidth B, including the thermal noise contributed by R_s, is then

$$V_{on}^2 = \int_B A_v(f)^2[4kTR_s + v_n(f)^2 + i_n(f)^2R_s^2]\, df \qquad (2\text{-}21)$$

The variation of v_n and i_n with frequency for a typical FET is shown in Fig. 2-8. As is evident from this figure, the effect of the v_n generator predominates at low frequencies, whereas the i_n generator predominates at radio frequencies. The v_n source represents primarily the $1/f$ noise and thermal noise in the channel. The i_n source represents noise due to the gate-channel conductance and the effects of noise transmission through interelectrode capacitances.

The circuit models developed to represent the noise behavior of junction transistors and FETs involve a number of parameters that are rarely available from manufacturers' data sheets. Instead, the devices are represented by the noise figure method employed for amplifier and mixer circuits. This will be described in Section 2-5, but first some definitions are necessary.

Fig. 2-8 Typical variation of v_n and i_n with frequency for an FET. (Courtesy of Siliconix, Inc.)



2-4 Definitions of Noise Terms

Various terms are used to define and compare the relative amounts of noise produced in electrical systems. The following definitions and discussion will provide the basis for understanding the nomenclature on manufacturers' specification sheets and for computing the overall effect of noise in a system.

Signal-to-Noise Ratio (SNR)

In a specified bandwidth, the signal-to-noise ratio is defined as the ratio of signal power to noise power at a port.

$$\text{SNR} = \frac{P_S}{P_N} = \frac{V_s^2}{V_n^2} \tag{2-22}$$

where V_s and V_n are the rms signal and noise voltages, respectively. In decibels,

$$\text{SNR(dB)} = 10 \log_{10} \frac{P_S}{P_N} \tag{2-23}$$

The larger the SNR, the less the signal is "corrupted" by the noise. The lowest permissible value of SNR depends upon the application. Some ap-

proximate minimum values are as follows: 10 dB at the detector input of a AM receiver, 12 dB at the detector input of an FM receiver, and 40 dB at th detector input of a television receiver. Note that as a signal passes through cascade of amplifier stages, the SNR continually decreases because eac stage adds additional noise. In most systems, however, the amplified outpu noise is due primarily to (1) the noise present along with the input signal, an (2) the noise contributed by the first two stages (such as the RF amplifier ar mixer stages in a receiver).

Noise Equivalent Bandwidth

The most common sources of noise (thermal and shot noise) have an esse tially uniform spectral distribution so that the noise transmitted through amplifier is determined by the bandwidth of the amplifier. If the amplifier h a constant gain A_v up to some frequency f_c and zero gain thereafter as sho in Fig. 2-9, the noise bandwidth B (which appeared in a number of t preceding equations) would clearly be equal to f_c. Generally, however, t frequency response is limited by the shunt capacitance or by a tuned circ so that an abrupt cutoff of the frequency response is not achieved. Thu more sophisticated determination of noise bandwidth is required. Conside filter, as shown in Fig. 2-10, that has voltage gain $A_v(f) = V_2/V_1$. Since noise power is related to the mean-square voltage, it is convenient to plot frequency response in terms of $|A_v(f)|^2$, as shown by the solid line in Fig. 2- where $|A_m|^2$ is the maximum value of this curve. If the input to this filte white noise with mean-square voltage v_{1n}^2/Hz, the corresponding mean-squ

Fig. 2-9 Constant gain characteristic, with cutoff at f_c.

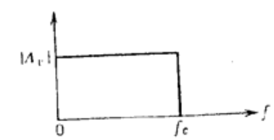


Fig. 2-10 Filter with voltage gain $A_v(f)$, power gain proportional to $A_v(f)^2$.

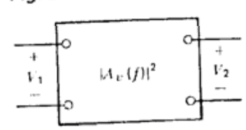
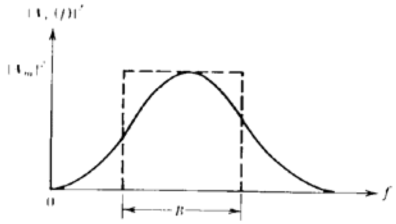


Fig. 2-11 Illustration of noise-equivalent bandwidth B, defined by equal areas under the dashed and solid curves.



output voltage in a 1-hertz interval at frequency f is

$$v_{2n}^2/\text{Hz} = |A_v(f)|^2 v_{1n}^2 \tag{2-24}$$

Addition of all such increments over the frequency band yields

$$\int_0^\infty v_{2n}^2(f)\, df = v_{1n}^2 \int_0^\infty |A_v(f)|^2\, df \tag{2-25}$$

The value of the integral on the right side of (2-25) is the area under the solid-line curve of $|A_v(f)|^2$ in Fig. 2-11. The dashed line shows a rectangular spectrum of the same maximum height $|A_m|^2$ and with bandwidth B. The *noise-equivalent bandwidth* B is the value that gives equal areas under the solid- and dashed-line curves such that

$$A_m^2 B = \int_0^\infty |A_v(f)|^2\, df$$

or

$$B = \frac{\int_0^\infty |A_v(f)|^2\, df}{|A_m|^2} \tag{2-26}$$

This is the value of B that should be used in equations earlier in this chapter, and obviously it must be evaluated for the particular system being analyzed. The integral in (2-26) is not always easy to evaluate. However, in many RF amplifiers, the bandwidth is established by tuned RLC circuits for which the noise bandwidth is[2]

$$B = \frac{\pi f_r}{2Q}, \qquad \text{where } Q = \frac{\omega_r L}{R} \tag{2-27}$$

[2] [7, p. 171].

The noise bandwidth given in (2-27) is $\pi/2$ times the 3 dB bandwidth of the circuit.

Available Noise Power

The *available power* P_a of a source is the maximum power that can be drawn from the source. If the source has internal impedance $Z_s = R + jX$, the maximum power will be delivered to a conjugate-matched load ($Z_L = R - jX$). If the open-circuit voltage of the source is V, the maximum power transfer theorem yields

$$P_a = \frac{V^2}{4R} \tag{2-28}$$

If R is a source of thermal noise, V^2 is given by (2-1), and

$$P_a = kTB \tag{2-29}$$

The available noise power in a bandwidth of 1 Hz is

$$P_{a(1\,\text{Hz})} = kT \tag{2-30}$$

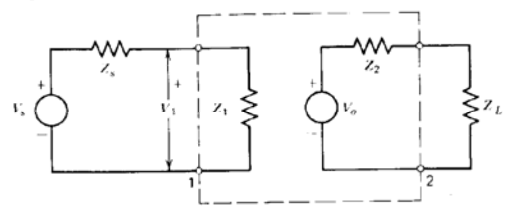
Since P_a is independent of R, the available noise power from all nonzero finite resistances is the same.

Available Power Gain of a Two-Port Network

Figure 2-12 shows a general two-port network that could serve as a model for an amplifier stage, filter, or other network. The source has open-circuit voltage V_s and internal impedance Z_s. The two-port has input impedance Z_1, output impedance Z_2, and open-circuit output voltage V_o. The load is an impedance Z_L. The open-circuit voltage gain of the two-port will be called $H(f)$.

$$H(f) = \frac{V_o}{V_1} \tag{2-31}$$

Fig. 2-12 Model of a general two-port network.



The available signal power from the source, as defined previously, is

$$P_{as} = \frac{|V_s|^2}{4R_s} \qquad (2\text{-}32)$$

Similarly, the available signal power at the output port is

$$P_{ao} = \frac{|V_o|^2}{4R_2} \qquad (2\text{-}33)$$

(Note that R_s and R_2 are the real parts of corresponding impedances.) The *available power gain* G_a of the network is defined as the ratio P_{ao}/P_{as}, that is,

$$G_a = \frac{|V_o|^2 R_s}{|V_s|^2 R_2} \qquad (2\text{-}34)$$

This expression is not very useful because it does not account for impedance mismatches at the input and output ports. However, examination of the network yields the following relations:

$$V_1 = \frac{Z_1 V_s}{Z_1 + Z_s} \qquad (2\text{-}35)$$

and

$$\frac{V_o}{V_s} = \frac{V_o Z_1}{V_1(Z_1 + Z_s)} \qquad (2\text{-}36)$$

Substitution of (2-31), (2-35), and (2-36) into (2-34) gives the following relation for available power gain at frequency f.

$$G_a = \left|\frac{H(f)Z_1}{Z_1 + Z_s}\right|^2 \frac{R_s}{R_2} \qquad (2\text{-}37)$$

The two-port model of Fig. 2-12, and the subsequent development, applies only if the network is unilateral (no reverse transmission). For this case it is worth emphasizing that G_a given by (2-37) is independent of the value of Z_L. Note that this is not the actual gain of the network—the value of G_a would be obtained in practice only with ideal matched conditions ($Z_1 = Z_s^*$ and $Z_L = Z_2^*$) at both ports.[3]

As a final note, the available gain of several cascaded unilateral networks is equal to the product of the G_a values of the individual networks.

Noise Temperature

The noise temperature at any port of a network is defined as follows: A noise source that has an available power P_a in a small frequency interval Δf has an

[3]The symbol Z_s^* denotes "the conjugate of" Z_s.

equivalent noise temperature equal to $T_r = P_a/k\Delta f$. [See (2-29).] If the power spectrum of the source is not flat, P_a and T_r are frequency-depe

Excess Noise Temperature, T_x. Noise generators used for amplifie ting are often calibrated in terms of excess noise temperature,

$$T_x = T - T_0$$

in which T is the noise temperature of the source and $T_0 = 290$ K standard reference temperature.

Effective Input Noise Temperature of a Network. If a thermal source of temperature T is connected to a noiseless network with bandwidth Δf and available gain $G_a(f)$, the available noise power fro source is

$$P_{ns} = kT\Delta f \qquad \text{watts}$$

and the available noise power at the output of the network is

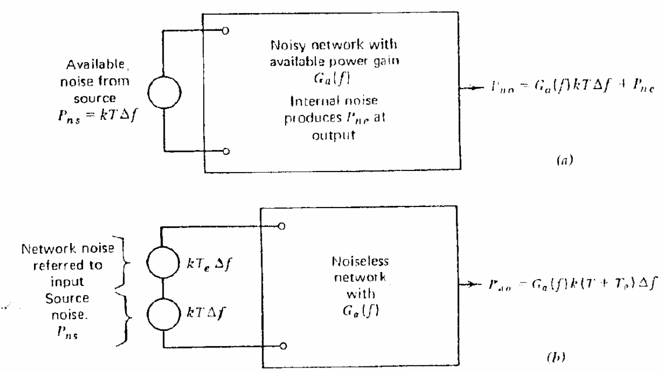
$$P_{no} = G_a(f)kT\Delta f$$

If the network is noisy, it will produce additional noise power, P_{ne}, output. With the same input noise as before, the output noise power

$$P_{no} = G_a(f)kT\Delta f + P_{ne}$$

as shown in Fig. 2–13*a*. Now replace the noisy network with a no

Fig. 2-13 (*a*) Model of a noisy network with output noise power P_{ne} due to i noise sources; and (*b*) equivalent noiseless network with counted for by a source $kT_e\,\Delta f$ at the input.



network having the same available power gain $G_a(f)$, and account for the output noise P_{ne} by means of an extra noise source on the input side as shown in Fig. 2-13b. The temperature T_e of this extra source is adjusted to produce P_{ne} at the output.

$$T_e = \frac{P_{ne}}{G_a(f)k\Delta f} \tag{2-42}$$

and

$$P_{no} = G_a(f)k(T + T_e)\Delta f \tag{2-43}$$

The value T_e is called the *effective input noise temperature* of the network. As will be shown, this way of representing network noise is very useful in the determination of overall signal-to-noise ratios, and so forth, of cascaded amplifiers.

2-5 The Noise Figure

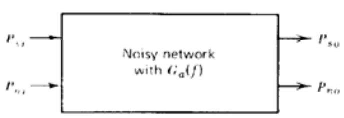
At this point the reader may well be confused by the variety of ways in which the relative noisiness of a device or system can be expressed. Fortunately, a single index called the *noise figure* can be used to compare noise performance. The reader should, however, watch out for several variations of the term with subtle differences that can be confusing.

Noise Figure (Noise Factor)

The noise figure (NF) of a two-port network gives a measure of the degradation of the SNR between the input and output ports. Figure 2-14 shows a noisy network with input signal and noise powers P_{si} and P_{ni}, respectively, and corresponding output signal and noise powers P_{so} and P_{no}. The noise figure is defined over a specified bandwidth as

$$\text{NF} = \frac{\text{input SNR}}{\text{output SNR}} = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}} = \frac{P_{no}}{G_a P_{ni}} = 1 + \frac{P_{ne}}{G_a P_{ni}} \tag{2-44}$$

Fig. 2-14 Signal and noise powers at the input and output of a two-port network.



The value of NF is often expressed in decibels through the relation

$$\text{NF}_{dB} = 10 \log_{10} \text{NF} \tag{2-45}$$

For a noise-free network, the input and output SNRs will be equal and NF = 1 or $\text{NF}_{dB} = 0$. Practical circuits always have larger noise figures than this.

Equation (2-44) provides a conceptual definition of the noise figure, but it requires further qualification to make it more precise. The ratio P_{si}/P_{so} is equal to $1/G_a(f)$, where $G_a(f)$ is the frequency-dependent available power gain of the network. The variation of $G_a(f)$ with frequency must be taken into account. Furthermore, the input power from the signal source is a function of temperature. *In order to obtain a standard value for NF, the source temperature must be assumed to be* 290 K. These considerations have led to the following definitions.

Spot Noise Figure

At a selected input frequency, the spot noise figure is the ratio of (1) the total available noise power *per unit bandwidth* at the output port to (2) the portion thereof produced at the input frequency by the input termination, whose noise temperature is 290 K. If the network provides a conjugate match at the input port, the available power from the standard-temperature source in a 1-Hz bandwidth is equal to kT_0 from (2-30). Hence, the spot noise figure is given by

$$\text{NF} = \frac{P_{no}}{G_a(f)kT_0} \tag{2-46}$$

In practice, the value of P_{no} is measured over a small bandwidth Δf that is more than 1 Hz due to the practical limitation on filter bandwidth. The equation for NF then becomes

$$\text{NF} = \frac{P_{no}}{G_a(f)kT_0\Delta f} \tag{2-47}$$

Average Noise Figure

Over wide bandwidths in which $G_a(f)$ varies appreciably, the average noise figure $\overline{\text{NF}}$ is given by

$$\overline{\text{NF}} = \frac{P_{no}}{kT_0 \int_0^\infty G_a(f)\, df} = \frac{P_{no}}{kT_0 G_{\max} B} \tag{2-48}$$

in which P_{no} is the total noise power delivered to the output termination in noise bandwidth B, and $G_{\max}$ is the maximum value of $|G_a(f)|$.

Overall NF of Cascaded Networks

Of primary interest in the evaluation of noise performance of multistage amplifiers (such as radio receivers) is the overall noise figure of the system. In general, the overall noise figure is evaluated for a bandwidth B that is the bandwidth of the overall system, rather than that of individual stages. (In a radio receiver, for example, the RF amplifier stage has a broad passband, but the IF stages are narrow-band and they determine the amount of noise that reaches the detector.) For convenience, the symbol NF will be used here (rather than $\overline{\text{NF}}$) to denote a value determined from (2-48).

From (2-43), if the source temperature is assumed to be T_0 (required for the standard definition of NF), the output noise power of a single stage in a small frequency band is

$$P_{no} = G_a(f)k(T_0 + T_e)\,\Delta f \tag{2-49}$$

and from (2-47),

$$P_{no} = \text{NF}\; G_a(f)kT_0\,\Delta f \tag{2-50}$$

Solution of the two preceding equations for NF gives

$$\text{NF} = \frac{T_0 + T_e}{T_0} \tag{2-51}$$

or

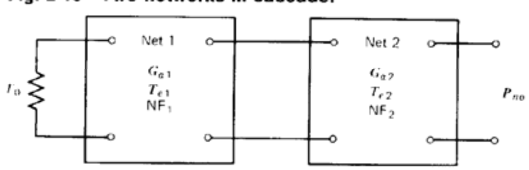
$$T_e = T_0(\text{NF} - 1) \tag{2-52}$$

Thus the noise figure referred to a standard-temperature source can be expressed in terms of the equivalent noise temperature of the network, and vice versa.

Figure 2–15 shows two cascaded networks with their respective available gains, effective noise temperatures, and noise figures. A noise source at standard temperature is input to the system. Small bandwidth Δf is assumed for all parts of the system. The available output noise power is found by the use of (2-49) and some regrouping of terms.

$$P_{no} = \underbrace{G_{a1}G_{a2}kT_0\,\Delta f}_{\substack{\text{due to} \\ \text{source } T_0}} + \underbrace{G_{a1}G_{a2}kT_{e1}\,\Delta f}_{\substack{\text{due to noise} \\ \text{in first net}}} + \underbrace{G_{a2}kT_{e2}\,\Delta f}_{\substack{\text{due to noise} \\ \text{in second net}}} \tag{2-53}$$

Fig. 2-15 Two networks in cascade.



A comparison of this relationship with (2-42) and (2-43) leads to an expression for the effective input temperature of the two networks in cascade.

$$T_{e1,2} = \frac{kG_{a2}(G_{a1}T_{e1} + T_{e2})\,\Delta f}{G_{a1}G_{a2}k\,\Delta f} = T_{e1} + \frac{T_{e2}}{G_{a1}} \tag{2-54}$$

Here $T_{e1,2}$ is the effective input temperature that accounts for all of the output noise introduced by the noisy networks. (Note that the input source temperature does not appear in the expression.) For n networks in cascade, the corresponding expression is

$$T_{e1,n} = T_{e1} + \frac{T_{e2}}{G_{a1}} + \cdots + \frac{T_{en}}{G_{a1}G_{a2}\ldots G_{a(n-1)}} \tag{2-55}$$

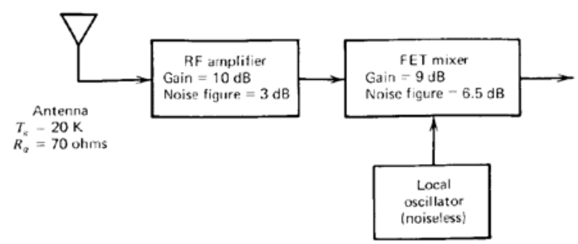
Introduction of the network noise figures by the use of (2-52) in (2-55) leads to the following expression for the overall noise figure.

$$\text{NF}_{1,n} = \text{NF}_1 + \frac{\text{NF}_2 - 1}{G_{a1}} + \cdots + \frac{\text{NF}_n - 1}{G_{a1}G_{a2}\ldots G_{a(n-1)}} \tag{2-56}$$

It is apparent from this relation that the NF of the first stage in a system has the predominant effect on the overall NF, unless G_{a1} is small or NF_2 is large. Therefore, the system designer should always try to minimize the noise produced in the first stage or two by the choice of low-noise transistors and the selection of operating conditions that minimize noise.

Example 2-5.1. The calculation of effective temperatures and noise figures for the receiver front end shown in Fig. 2-16 will serve to illustrate the foregoing discussion. The antenna has a (radiation) resistance of 70 ohms and an effective temperature of 20 K due primarily to external radiation. Noise figures and gains for the RF amplifier and mixer are given in decibels, and must be converted to actual values for use in the computation. The noise

Fig. 2-16 Block diagram of a receiver front end. See Example 2-5.1.



contributed by the receiver local oscillator is assumed to be negligible (which is not always the case).

For the RF amplifier, $NF_1 = 2$, and $G_{a1} = 10$. For the mixer, $NF_2 = 4.47$ and $G_{a2} = 7.94$. By the use of (2-52) the effective noise temperatures of the two units are found to be $T_{e1} = 290$ K and $T_{e2} = 1006$ K. Then from (2-54) the effective input temperature of the receiver (excluding the antenna) is found as $T_r = T_{e1} + T_{e2}/G_{a1} = 290 + 1006/10 = 391$ K.

The overall noise figure can be calculated from either (2-51) or (2-56). From (2-51), $NF = T_r/T_0 + 1 = 391/290 + 1 = 2.35$. Or, from (2-56), $NF = NF_1 + (NF_2 - 1)/G_{a1} = 2 + 3.47/10 = 2.35$ or 3.7 dB.

Actual Noise Figure

By definition, the standard noise figure is the ratio of the input SNR to the output SNR with the input source at the standard 290 K noise temperature. In this example and in many other practical cases, however, the input noise temperature is not 290 K and the standard noise temperature does not describe accurately the SNR degradation from input to output of a system. A true measure of the SNR degradation is the *actual noise figure* defined by (2-57). (For further discussion of this point, see Mumford and Scheibe [10].)

$$NF_{act} = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}} = \frac{P_{no}}{G_a P_{ni}} = \frac{T_s + T_e}{T_s} \qquad (2\text{-}57)$$

in which $T_s \neq T_0$. This value is related to the "standard" noise figure evaluated for $T_s = T_0$ by the relation

$$NF_{act} = 1 + (NF - 1)\left(\frac{T_0}{T_s}\right) \qquad (2\text{-}58)$$

In the preceding example, the value of NF was found to be 2.35 or 3.7 dB. However, since the antenna temperature was 20 K, the use of (2-58) to calculate the actual noise figure yields $NF_{act} = 1 + (2.34 - 1)(290/20) = 20.43$ or 13.1 dB.

2-6 Amplifier Noise Considerations

For calculations of NF or T_e of a system comprising one or more stages, knowledge is required of (1) the noise delivered from the signal source, (2) the noise-equivalent bandwidth B, (3) the thermal noise generated in various resistances in the circuit, and (4) the noise generated within the solid-state devices. Noise produced within diodes and transistors can be predicted by the use of equivalent circuits such as in Figs. 2-4, 2-5, and 2-6. However, because

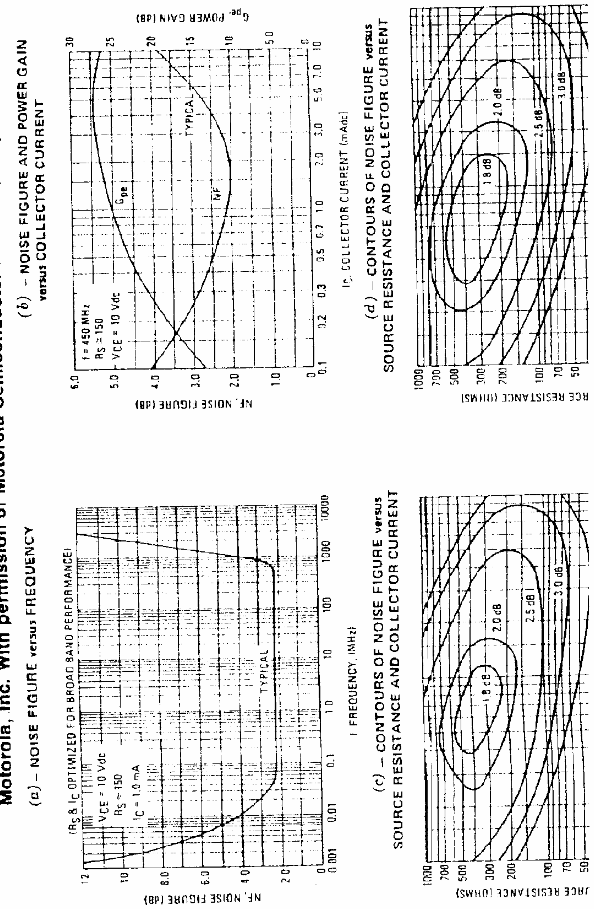


Fig. 2-17 Noise data on the 2N4957 transistor. (From Motorola Data Sheet DS 5227 R2, © 1974, Motorola, Inc. With permission of Motorola Semiconductor Products, Inc.)

the noise produced within a transistor is a function of the Q point, frequency, and transistor parameters, there are so many variables to be taken into account that such predictions are best made by the use of computer analysis programs.

Semiconductor manufacturers generally give data-sheet information on the measured spot noise figure or average noise figure for a variety of operating conditions. For example, Fig. 2-17 shows some data relating to the noise performance of the 2N4957 transistor. The data sheet for the transistor also gives NF = 2.6 dB (typical) and 3.0 dB (max) at $I_C = 2$ mA, $V_{CE} = 10$ V, $f =$ 450 MHz, as measured with the circuit of Fig. 1 in Appendix 4-4. The spot noise figure varies with the frequency, collector current, and source resistance as shown in Fig. 2-17; these variations must be considered in amplifier design when low-noise performance is important.

Since the noise figure variation with frequency is almost flat over the useful operating range of the transistor, primary attention may be given to the selection of a source resistance that minimizes the noise figure for a given collector current. The amplifier can be designed with suitable matching networks so that the transistor "sees" this source resistance at the operating frequency. Unfortunately, the value of the source resistance that yields maximum gain is not the same as the value for the minimum noise figure. Thus an amplifier designed for the minimum noise figure will have less than the maximum gain possible with the transistors chosen. Usually, the gain is reduced from its maximum value by only a few dB when minimum noise figure conditions are met, and this can be compensated for in following high-gain stages.

In the chapters that follow it will be seen that noise considerations play an important part in the design of RF amplifier and mixer stages in receivers. Even in oscillators, noise must be considered if high spectral purity is required.

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PROBLEMS

2-1.1. Two resistances R_1 and R_2 at temperature T are connected in s

(a) Show that the mean-square open-circuit noise voltage that appears terminals is given by $V_n^2 = 4kT(R_1 + R_2)B$.

(b) Let $R_1 = 50$ kΩ, $R_2 = 20$ kΩ, $T = 400$ K, and assume that noise vol can be measured with a true-rms voltmeter that has an effective bandwidth of 5 MHz and infinite input impedance. Calculate the rms voltage measured (1) across R_1, (2) across R_2, and (3) across R_1 and series.

(c) Repeat part (b) if the voltmeter has an input resistance (noiseles 100 kΩ throughout the frequency band.

2-1.2. Given two conductances G_1 and G_2, both at temperature T connected in parallel, show that the equivalent mean-square current sou given by $I_n^2 = 4kT(G_1 + G_2)B$.

2-1.3. Show that Fig. 2-1*b* is the Norton equivalent circuit for th Fig. 2-1*a*.

2-1.4. (a) Redraw Fig. 2-2*a* with the source V_s omitted but incluc voltage source to represent the thermal noise produced in R_2 and current sc to represent thermal noise in R_1 and R_3.

(b) By the use of Thévenin's and Norton's theorems, derive the equivalent Thévenin circuit to the left of R_i. Show that the Thé resistance is $R_T = R_2 + R_1 \| R_3$, and that the mean-square open-circuit voltage is $V_{Tn}^2 = 4kTBR_T$.

2-1.5. In the circuit of Fig. 2-2*a*, let $R_1 = R_3 = 2$ kΩ, $R_2 = R_i = 1$ kΩ 293 K, and $B = 10^4$ Hz. Assume that all resistors are noiseless except R_3. C late the rms noise voltage across R_i due to the thermal noise generated

2-1.6. In Fig. P2-1.6 resistors R_1 and R_2 are at temperatures T_1 an respectively.

(a) Redraw the circuit, representing the thermal noise in each resistor current source.

(b) Derive a general expression for the mean-square noise voltage V_n^2.