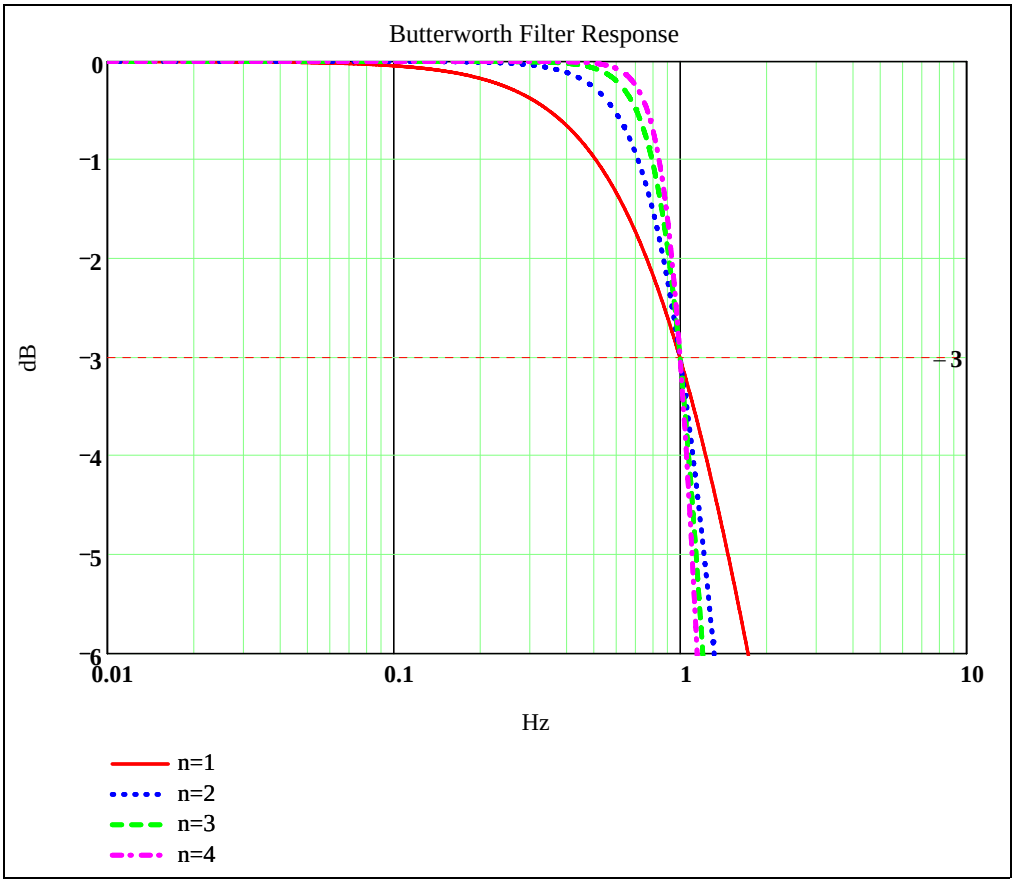
EE445
FILTER RESPONSE

IDEAL FILTER MAGNITUDE RESPONSE FOR BANDWIDTH B:

$$\Pi(f, B) \equiv \text{if}\left[\left(|f| \le B\right), 1, 10^{-10}\right]$$

MAGNITUDE RESPONSE FOR BUTTERWORTH FILTERS NORMALIZED TO 1Hz:

$$H(f, n) := \sqrt{\frac{1}{1 + f^{2 \cdot n}}} \qquad H_{dB}(f, n) := 10 \cdot \log\left(\frac{1}{1 + f^{2 \cdot n}}\right)$$



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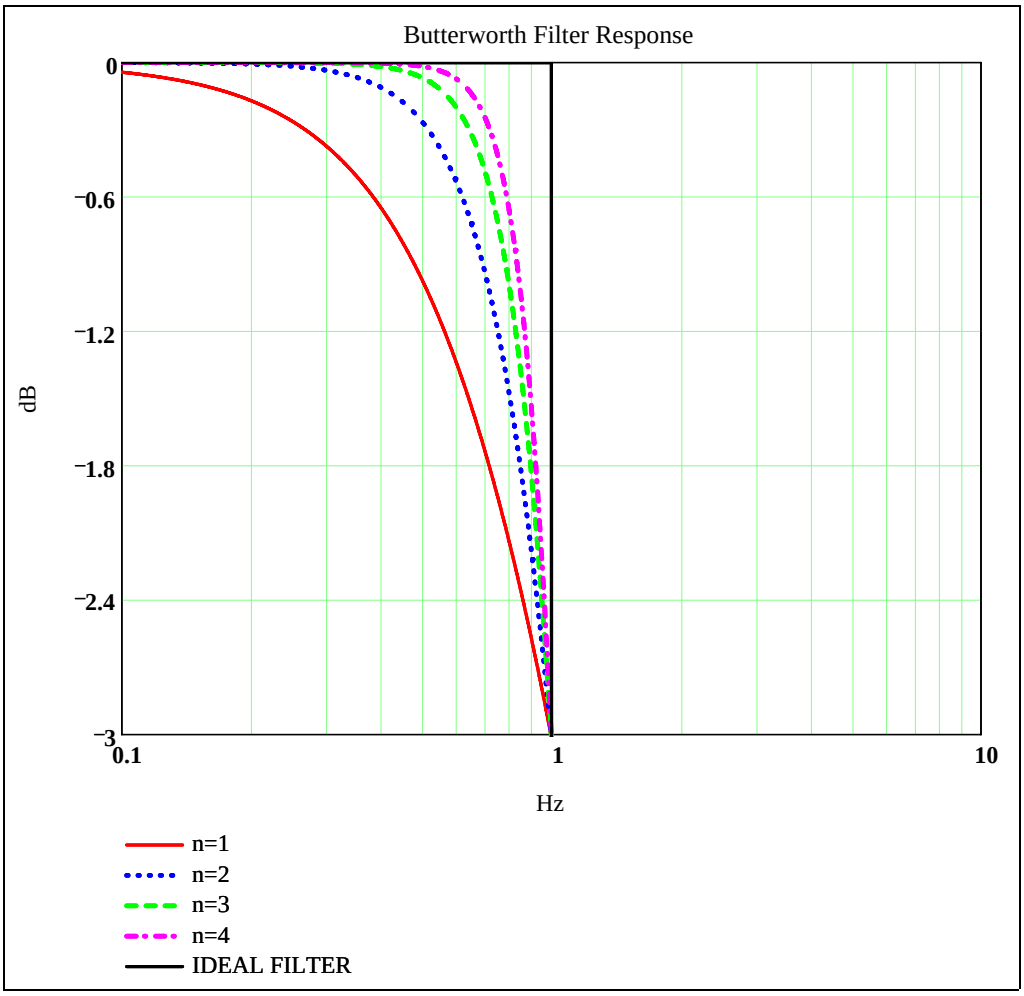

EQUIVELENT NOISE BANDWITH FOR BUTTERWORTH FILTERS

$$B_n = \frac{\text{Noise\_Power\_From\_Real\_Filter\_With\_3db\_Bandwidth\_of\_1}}{\text{Noise\_Power\_From\_Ideal\_Filter\_With\_Bandwidth\_of\_1}}$$

$n := 1 .. 6$ filter orders from 1 to 6

note that: $\int_0^\infty \frac{1}{1+f^2}\,df = \frac{\pi}{2}$

$$B_n := \frac{\int_0^\infty \frac{1}{1+f^{2\cdot n}}\,df}{\int_0^\infty \Pi(f,1)^2\,df}$$

First order filter with a 3dB bandwidth of 1 passes 57% more noise power when compared with an ideal filter with a bandwidth of 1. A 3rd order filter only passes 4.7% more noise power when compared with an ideal filter.

$$M = \begin{pmatrix} 0 & 0 \\ 1 & 1.571 \\ 2 & 1.111 \\ 3 & 1.047 \\ 4 & 1.026 \\ 5 & 1.017 \\ 6 & 1.012 \end{pmatrix}$$

$M_{n,0} := n$
$M_{n,1} := B_n$

$\frac{\pi}{2} = 1.571$