# Homework 5: Homework 5: AC Circuits 

Assigned: Monday, 10/31/2014
Due: Monday, 11/10/2014

The homework consists of five problems. The last one is an additional problem that combines several topics we've discussed in class and lab. Each problem is worth 10 points total.

1. For the circuit shown in Figure 1, assume the voltage source $\left(V_{i n}\right)$ is a $120 \mathrm{~V}, 60 \mathrm{~Hz}$ source. Assume the resistance is $R=5 \mathrm{k} \Omega$, and the inductance is $L=10 \mathrm{H}$.
(a) Draw a phasor diagram for the impedance (consists of resistance, inductor, and total impedance phasors). Use resistance as your reference.
(b) What is the total impedance of the circuit from the perspective of the power source? Give in terms of a complex number. Then, convert that number to a magnitude $(|Z|)$ and phase ( $\phi_{Z}$ ).
(c) What is the magnitude of the current $(|I|)$ ?
(d) What is the magnitude of the voltages across the resistor and the inductor?
(e) Draw a phasor diagram for the voltage (consists of the resistance voltage, inductor voltage and input voltage phasors). Use resistance voltage as your reference.
(f) What is the real power $(P)$, the reactive power $(Q)$, and the total apparent power ( $P_{a p p}$ ) drawn by the circuit?
(g) Draw a phasor diagram for the power (consists of the real, reactive, and apparent power phasors). Use the real power phasor as your reference.
(h) What is the power factor $(P F)$ of the circuit? Is it leading or lagging?


Figure 1: Practice Circuit 1

Show work:
a) $X_{L}=2 \pi F L=2 \pi 60 \cdot 10=3.77 \mathrm{~K} \Omega$
g)

n)

I-perdace Alasos:

$$
\text { n) } p=\frac{P}{s}=\frac{1.84}{2.3}=0.8
$$

$j_{K_{L}} \uparrow{ }^{\sim}$
$\rightarrow$ PF islagging because in an induetrue aricuit, currat lags woltage ad real
b)

$$
\begin{aligned}
& z=R+j X_{L}=5 i j 3.77 \mathrm{k} \Omega \\
& |z|=\sqrt{5000^{2}+3770^{2}}=6.26 \mathrm{k} \Omega \\
& \phi_{2}=\tan ^{-1} \frac{X_{L}}{k}=37^{\circ}
\end{aligned}
$$

c)

$$
|I|=\frac{\left|V_{\text {in }}\right|}{|z|}=\frac{120}{6.26 \cdot 10^{3}}=19.2 \mathrm{~mA}
$$

d)

$$
\left|V_{k}\right|=|I|\left|Z_{R}\right|=\left|I_{R}\right| R=19.2 \mathrm{~mA} \cdot 51 \mathrm{kar}=95.8 \mathrm{~V}
$$

$\left|V_{L}\right|=1 I \mid X_{L}=19.2 \mathrm{~mA} \cdot 3.77 \mathrm{~K} \Omega=72.2 \mathrm{~V}$
e)

check Vin magnitude:

$$
\begin{aligned}
\left|V_{i n}\right| & =\sqrt{\left|V_{L}\right|^{2}+\left|V_{R}\right|^{2}} \\
& =\sqrt{72.2^{2}+95.8^{2}}
\end{aligned}
$$

$$
\overrightarrow{V_{R}} \quad\left|V_{i n}\right|=119.9 \mathrm{~V} \mathrm{~V}<\text { alls well }
$$

$$
\begin{aligned}
& \text { f) } P=\left|I^{24}\right|^{2} R=1.84 \mathrm{~W} \\
& Q=|I|^{2} X_{L}=1.38 \mathrm{VAR} \\
& S=|I||\mathrm{Vin}|=2.3 \mathrm{VA}
\end{aligned}
$$

2. For the circuit shown in Figure 2, assume the voltage source $\left(V_{i n}\right)$ is a $120 \mathrm{~V}, 60 \mathrm{~Hz}$ source. Assume the resistance is $R=5 \mathrm{k} \Omega$, and the inductance is $L=10 \mathrm{H}$.
(a) What is the total impedance of the circuit from the perspective of the power source? Give in terms of a complex number. Then, convert that number to a magnitude $(|Z|)$ and phase ( $\phi_{Z}$ ).
(b) Determine the current magnitudes through the resistor and the inductor.
(c) Draw a phasor diagram for the current (consists of the resistance, inductor, and total circuit current phasors). Use resistance current as your reference.
(d) What is the magnitude of the total circuit current $(|I|)$ ?
(e) What is the real power $(P)$, the reactive power $(Q)$, and the total apparent power ( $P_{\text {app }}$ ) drawn by the circuit?
(f) Draw a phasor diagram for the power (consists of the real, reactive, and apparent power phasors). Use the real power phasor as your reference.
(g) What is the power factor $(P F)$ of the circuit? Is it leading or lagging?


Figure 2: Practice Circuit 2
a) Show work: $X_{L}=3.77 \mathrm{~K} \Omega$ (same calc as prob 1)
$Z_{R}=R=5 K \Omega$ (resistor inpedana is just resistane)

$$
z_{L}=j X_{L}=j 3,77 \mathrm{~K} \Omega
$$

$$
z_{\text {total }}=\frac{1}{\frac{1}{z_{R}}+\frac{1}{z_{L}}}=\frac{1}{\frac{1}{5000}+\frac{1}{j 3770}}=1.8+j 2.4 \mathrm{k} \Omega
$$

b) $\left|I_{R}\right|=\frac{V_{i n}}{R}=\frac{120}{5000}=24 \mathrm{~mA}$ calculatur.

$$
\left|I_{L}\right|=\frac{r_{1 n}}{x_{L}}=32 \mathrm{~mA}
$$

$$
|z|=3.01 \mathrm{k} \Omega
$$

$$
\begin{aligned}
& 1 z=3.01 \mathrm{~K} \Omega \\
& \phi_{z}=\tan ^{-1}\left(\frac{z_{\text {inas }}}{z_{\text {ral }}}\right)=\tan ^{-1} \frac{2.4}{1.8}
\end{aligned}
$$

$$
\phi_{z}=53^{\circ}
$$


d) $| \pm|=\frac{V_{\text {in }}}{\left|z_{r_{0}}\right|}=\frac{120}{3.01 .10^{3}}=39.9 \mathrm{~mA}$

$$
\begin{aligned}
& \text { e) } P=\left|I_{R}\right|^{2} R=2.88 \mathrm{~W} \\
& Q=\left|I_{L}\right|^{2} X_{L}=3.86 \mathrm{VAR} \\
& S=\left|V_{\text {in }}\right||I|=4.79 \mathrm{VA} .
\end{aligned}
$$


3. For the circuit shown in Figure 3, assume the voltage source $\left(V_{i n}\right)$ is a $277 \mathrm{~V}, 60 \mathrm{~Hz}$ source. Assume the resistance is $R=1 \mathrm{k} \Omega$, and the capacitance is $C=4 \mu \mathrm{~F}$.
(a) Draw a phasor diagram for the impedance (consists of resistance, capacitor, and total impedance phasors). Use resistance as your reference.
(b) What is the total impedance of the circuit from the perspective of the power source? Give in terms of a complex number. Then, convert that number to a magnitude $(|Z|)$ and phase $\left(\phi_{Z}\right)$.
(c) What is the magnitude of the current $(|I|)$ ?
(d) What is the magnitude of the voltages across the resistor and the capacitor?
(e) Draw a phasor diagram for the voltage (consists of the resistance voltage, capacitor voltage and input voltage phasors). Use resistance voltage as your reference.
(f) What is the real power $(P)$, the reactive power $(Q)$, and the total apparent power ( $P_{\text {app }}$ ) drawn by the circuit?
(g) Draw a phasor diagram for the power (consists of the real, reactive, and apparent power phasors). Use the real power phasor as your reference.
(h) What is the power factor ( $P F$ ) of the circuit? Is it leading or lagging?


Figure 3: Practice Circuit 3

Show work: $X_{c}=\frac{1}{2 \pi F C}=\frac{1}{2 \pi \cdot 60 \cdot 4 \cdot 10^{-6}}=663 \Omega$
a)
b)


$$
\begin{aligned}
& z=R-j X_{c}=1000-j 663 \Omega \\
& |z|=\sqrt{1000^{2}+663^{2}}=1.2 \mathrm{k} \Omega \\
& \phi_{z}=\tan ^{-1}\left(\frac{-x_{c}}{R}\right)=-33.5^{\circ}
\end{aligned}
$$

c) $|I|=\frac{\left|V_{\text {in }}\right|}{|z|}=\frac{277}{1,2 \cdot 10^{3}}=231 \mathrm{~mA}$
d) $\left|V_{R}\right|=\mid \pm 1 \cdot R=231 \mathrm{~V}$

$$
\left|V_{c}\right|=|I| \cdot X_{c}=153 \mathrm{~V}
$$

e)
chects

f)

$$
=\sqrt{231^{2}+153^{2}}
$$

$$
=277 . \mathrm{W} \text { so } V E \text { guad. }
$$

$$
\begin{aligned}
& P=|I|^{2} R=53.3 \mathrm{~W} \\
& Q=|I|^{2} X_{C}=35.3 \mathrm{VAR} \\
& S=|I|\left|V_{\text {in }}\right|=63.95 \mathrm{VA}
\end{aligned}
$$

g)
h) $P F=\frac{P}{s}=0.83$

leeding becarse in copacitione circiit, red power leads the appont power.
4. For the circuit shown in Figure 4, assume the voltage source $\left(V_{i n}\right)$ is a $277 \mathrm{~V}, 60 \mathrm{~Hz}$ source. Assume the resistance is $R=1 \mathrm{k} \Omega$, and the capacitance is $C=4 \mu \mathrm{~F}$.
(a) What is the total impedance of the circuit from the perspective of the power source? Give in terms of a complex number. Then, convert that number to a magnitude $(|Z|)$ and phase $\left(\phi_{Z}\right)$.
(b) Determine the current magnitudes through the resistor and the capacitor.
(c) Draw a phasor diagram for the current (consists of the resistance, capacitor, and total circuit current phasors). Use resistance current as your reference.
(d) What is the magnitude of the total circuit current $(|I|)$ ?
(e) What is the real power $(P)$, the reactive power $(Q)$, and the total apparent power ( $P_{\text {app }}$ ) drawn by the circuit?
(f) Draw a phasor diagram for the power (consists of the real, reactive, and apparent power phasors). Use the real power phasor as your reference.
(g) What is the power factor $(P F)$ of the circuit? Is it leading or lagging?


Figure 4: Practice Circuit 4

$$
\left\{\begin{aligned}
|z| & =552.8 \Omega \\
\phi_{z} & =\tan ^{-1}\left(\frac{-461}{305}\right)
\end{aligned}\right.
$$

a)

Show work: $\quad X_{c}=663 \Omega \quad z_{c}=-j X_{c}, \quad z_{R}=R \quad \phi_{z}=-56,5^{\circ}$

$$
z=\frac{1}{\frac{1}{z_{k}}+\frac{1}{z_{c}}}=\frac{1}{\frac{1}{1000}+\frac{1}{-j 663}}=305-461 j \Omega
$$

b)

$$
\begin{aligned}
& \left|I_{n}\right|=\frac{\left|V_{i n}\right|}{R}=\frac{277}{1000}=277 \mathrm{~mA} \\
& \left|I_{c}\right|=\frac{\left|V_{i n}\right|}{X_{c}}=\frac{277}{663}=418 \mathrm{~mA}
\end{aligned}
$$

昷 ${ }^{\circ}$ )

F)

d) $|I|=\frac{V_{\text {in }}}{|z|}=\frac{277}{552.8}=501 \mathrm{~mA}$
e)

$$
\begin{aligned}
P=\left|I_{R}\right|\left|V_{\text {in }}\right| & =277 \mathrm{~mA} \cdot 277 \mathrm{~V} \\
& =76.7 \mathrm{~W}
\end{aligned}
$$

$$
Q=\left|I_{c}\right| V_{i n} \mid=0.418 \cdot 277
$$

$$
=115.8 \mathrm{VAR}
$$

$$
\begin{aligned}
& S=|I||\operatorname{Vin}|=0,501 \cdot 277 \\
&=138.8 \mathrm{VA}
\end{aligned}
$$

g) $P F=\frac{p}{s}=0.55$
$\rightarrow$ |eadins
5. Application Problem: An AC motor used in a manufacturing plant produces 200 $\mathrm{N}-\mathrm{m}$ of torque at a rotational speed of $100 \mathrm{rad} / \mathrm{s}(955 \mathrm{RPM})$. This motor takes as input a 480 V ac power source, and has a rated efficiency of $75 \%$, and a lagging power factor of 0.8 .
(a) What is the mechanical power produced by the motor? Answer in kW .
(b) What is the amount of real electrical power drawn by the motor? Note: When calculating the efficiency of an electric motor, the input power is the real power.
(c) What is the total apparent power delivered from the power source? What is the amount of reactive power required by the motor?
(d) Draw a phasor diagram for the real, reactive, and total apparent power delivered to the motor.
(e) If you wanted to bring this motor's power factor into unity, what size capacitor would you need to add in parallel with the motor? Note: See Lab 7.
(f) Without the capacitor added, what is the magnitude of the current delivered to the motor?
(g) If the ambient temperature of the manufacturing plant is $125^{\circ} \mathrm{F}$, and the cables used to connect the motor to the power source are THHN insulated wire, what size of cables should be used? Note: See Table 8-1 in Appendix B of your book.

$$
\begin{aligned}
& \text { Power: } \\
& P_{m}=\tau \omega \text { (torque in } \mathrm{N}-\mathrm{m} \text { "rotations speed in } \mathrm{rad} / \mathrm{s} \text { ) }
\end{aligned}
$$

$$
=200 \cdot 100=20 \mathrm{~kW}
$$

b) (rat electric power)

$$
\text { c) } P F=\frac{P}{s} \Rightarrow S=\frac{P}{P F}=\frac{26.7}{0.8}=33.3 \mathrm{kVA}
$$

$$
Q=\sqrt{S^{2}-p^{2}}=\sqrt{33.3^{2}-26.7^{2}}=20 \mathrm{kVAR}
$$



Show work:
d)

e) Need to off set $Q$ so place capacitor in parallel. so that reactive power from cap, $Q_{c}$, is equal to reactor power drawn by load. Thy will hove uppriste signs hecasse reaction load is lagging while

$$
\begin{aligned}
& \text { Thy will hove uppurite sis ns reckoned. } \\
& Q_{c}=Q, Q_{c}=\frac{T}{\left.\frac{\mid V i n}{}\right|^{2}} \\
& X_{c}
\end{aligned} X_{c}=\frac{1}{2 \pi F C} \Rightarrow C=\frac{1}{2 \pi F \frac{\left|w_{c}\right|^{2}}{Q}}
$$

Assme $F=6(6) \mathrm{Hz}_{z}$ sincethat is stadod in U.S.:

$$
c=\frac{1}{2 \pi 60 \cdot \frac{480^{2}}{20000}}=2.3 \cdot 10^{-4} \mathrm{~F}=230 \mu \mathrm{~F}
$$

F) 厘布 $\left.=s=\left|\operatorname{Vin}_{\text {in }}\right||t| \leq\right\rangle$

$$
\begin{aligned}
& \left.s=\left|V_{\text {in }}\right||I| s\right\rangle \\
& |I|=\frac{s}{\left|V_{\text {in }}\right|}=\frac{33000}{480}=68.75 \mathrm{~A}
\end{aligned}
$$

g) Use table: Did not specify if copper or aluminum so will accept correct answers fur either.

$\rightarrow$| So for copper: \#4 AW G |
| :---: |
| is rated at $0.76 \cdot 95 A=72.24$ | is copper. \#4 AW G

(a) Ambient tempi li ${ }^{\circ}{ }^{11}$

THIN ration down by Fader 0.76 aluminvmi \#ZAWG rated at $0.76 \cdot 100 \mathrm{~A}=76 \mathrm{~A}$

