Inductance

Recall from last lecture:

Self-inductance in a conductor causes a change in the magnetic field. This induces a voltage opposed to the applied voltage such that the change in conductor current is retarded (slowed).

Current-voltage relationship

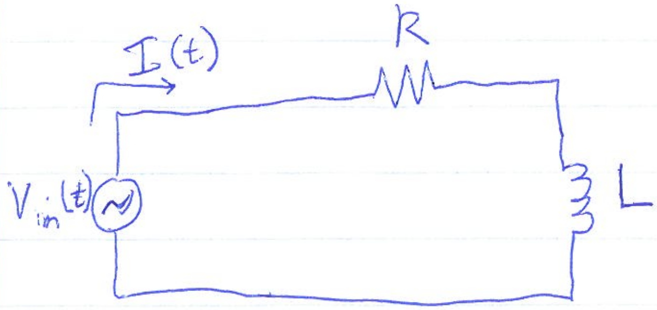
$$V_L = L \frac{di}{dt}$$

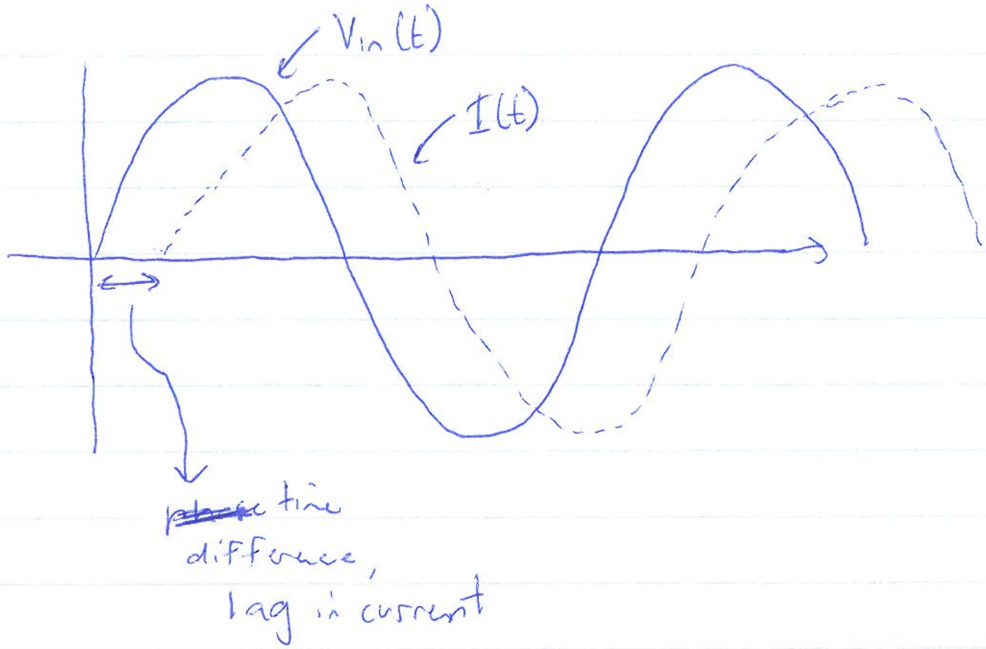
time
↳ change in current

In DC circuits, causes a slowing of the response to a step change in applied voltage

(see plots from previous lecture)

In AC circuits, causes a lag in the current response to the applied voltage





Inductive Reactance

In AC circuits, there is an additional opposition to current flow as compared to DC circuits. This is <u>inductive reactance</u>.

$$X_L = 2\pi F L$$

~~However~~ However, ~~Because~~ this opposition is "out of phase" with the resistance.

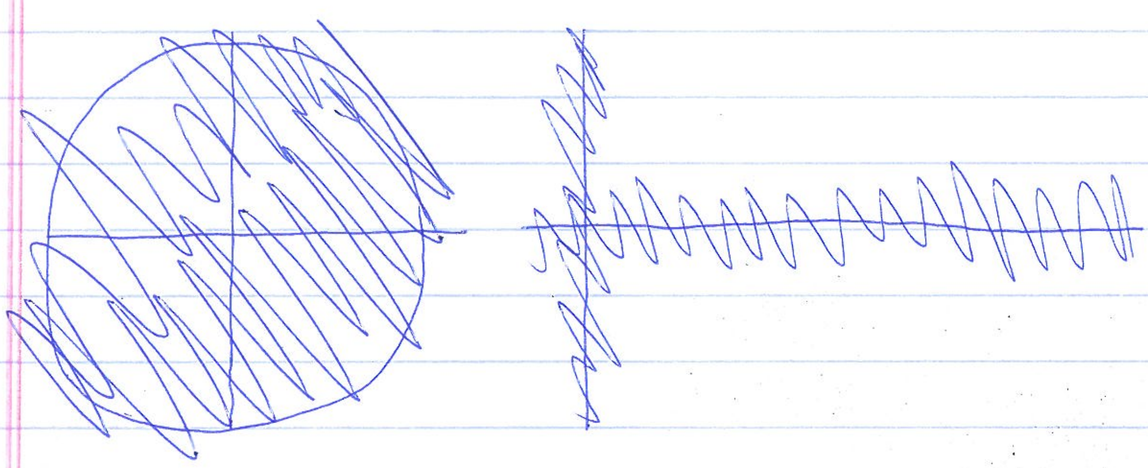
↓

Thus, total impedance (resistive + reactive) is written as

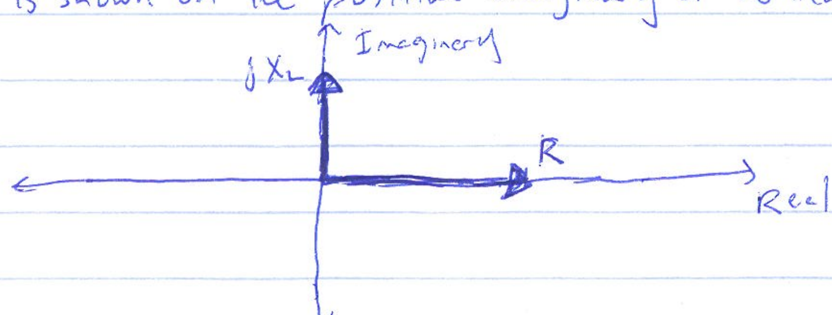
$$Z = R + jXL$$

↳ imaginary . indicates $+90^\circ$ out of phase.

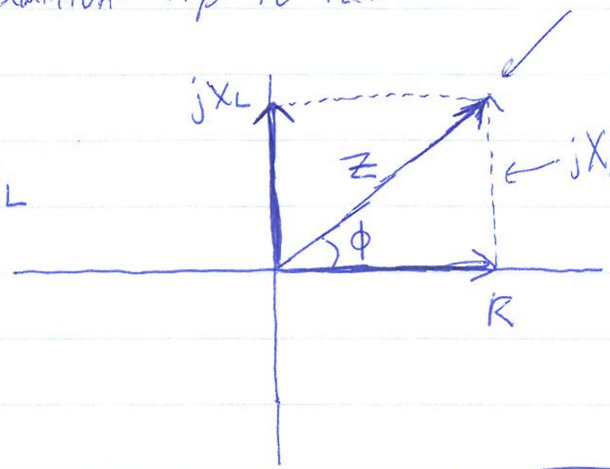
Phasor Diagram - Impedance indicates the relationship of the amplitude and phase between voltage and current.



The total impedance then of an RL circuit can be represented on a phasor diagram. Typically, resistance is shown on the positive real or horizontal axis and inductance is shown on the positive imaginary or vertical axis.



Vector addition: Tip to tail:



$Z = R + jX_L$

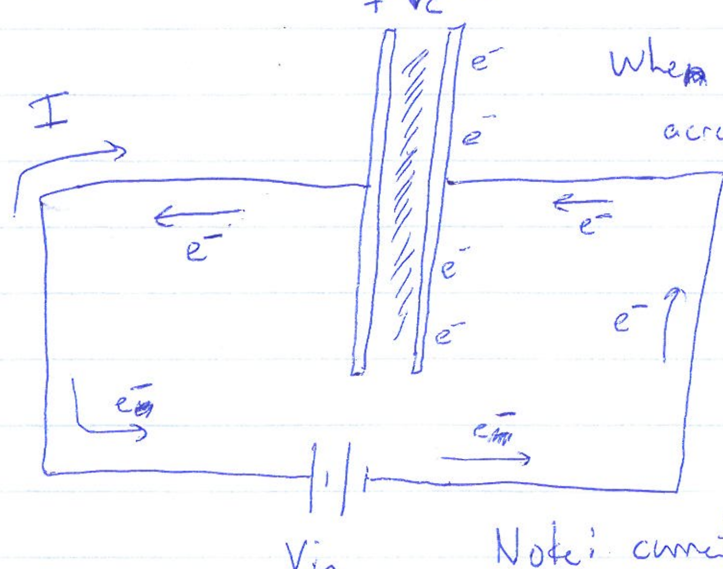
By pythagorean theorem: $Z = \sqrt{R^2 + X_L^2}$
magnitude of impedance

Also phase found as $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Then $|V| = |I||Z|$

Capacitance

Capacitor consists of two conductive plates separated by an insulating material. called the dielectric.



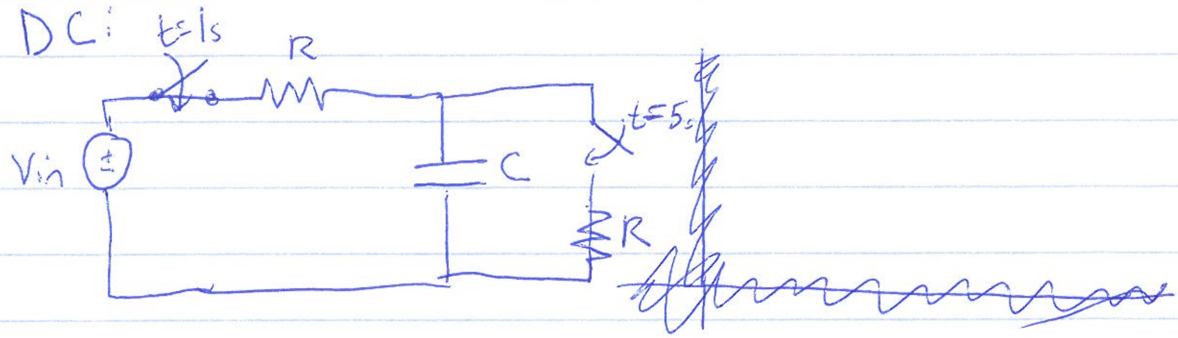
When voltage is applied across it. electrons forced from one plate and accumulate on the other creating a voltage potential across it.

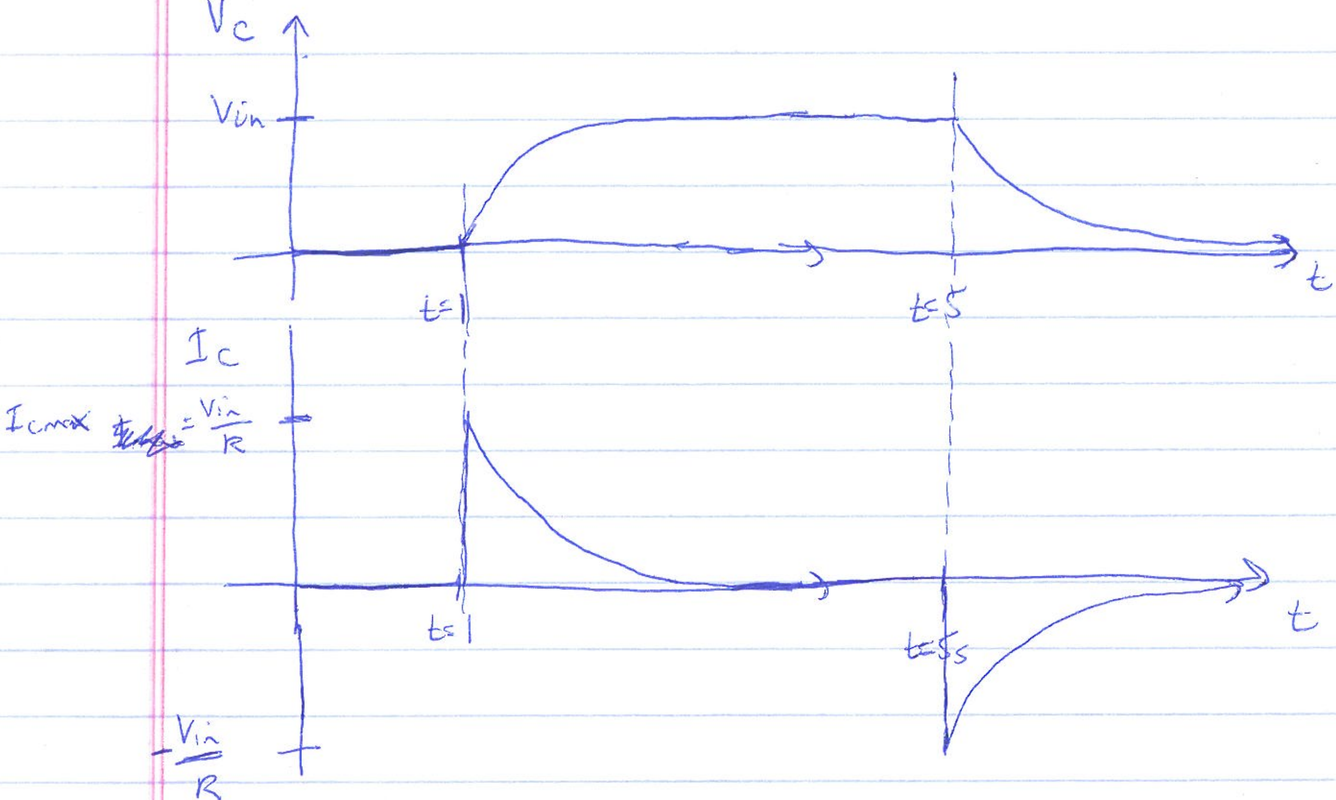
This is stored charge!

Note: current does not flow through dielectric

Effects on current flow:

DC:





Capacitor units is the Farad (F)

Time constant => time it takes for ~~current~~ voltage to reach it's maximum value

__But:__ Maximum value never "truly" reached, only assymptotically

$\tau_c = RC$ units in seconds

↳ time it takes to reach 63.2% of maximum.

Voltage / current relationship:

$V_c(t) = \int \frac{1}{C} I(t)\, dt$ or equivalently $I(t) = C \frac{dV_c(t)}{dt}$

<u>Size of capacitor:</u>

$$C = \frac{Q}{V}$$

Q ← charge in coulombs

V ↳ voltage applied

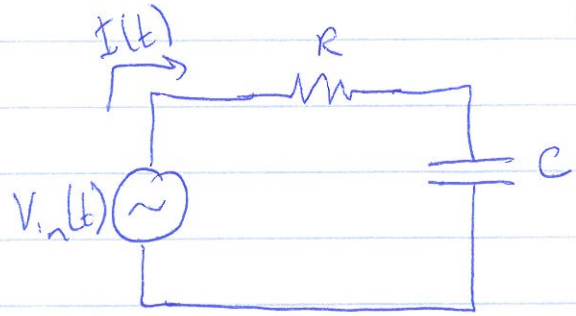
C ↑ capacity in Farads.

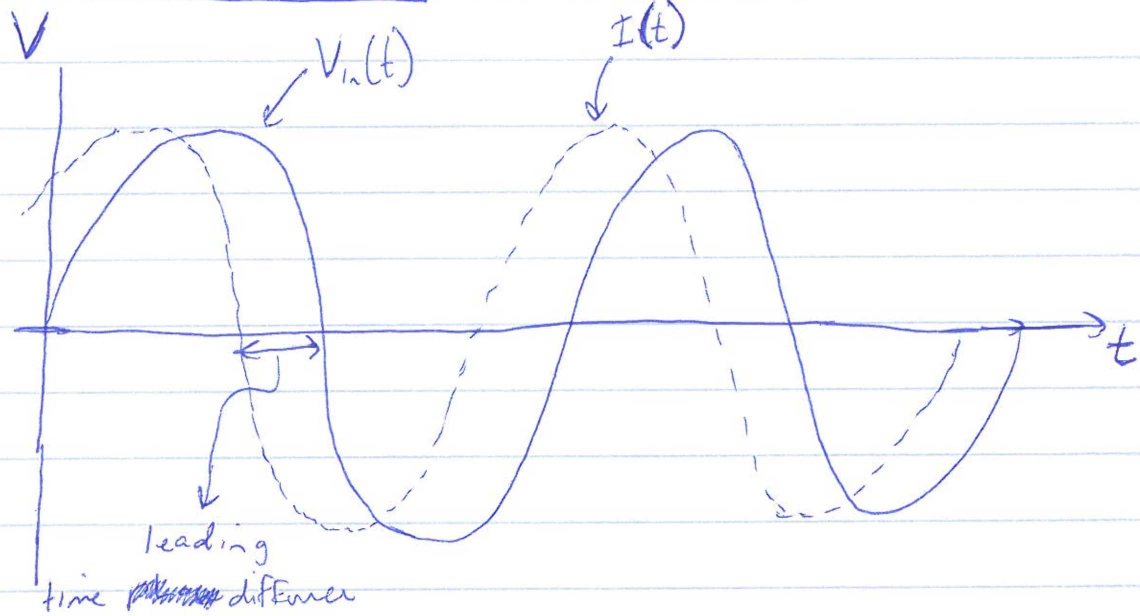
~~Capacitive reactance~~

~~$X_C = \frac{1}{2\pi f C}$~~

↓

Effect on AC circuit current: causes current to <u>lead</u> voltage





<u>Capacitive Reactance</u>

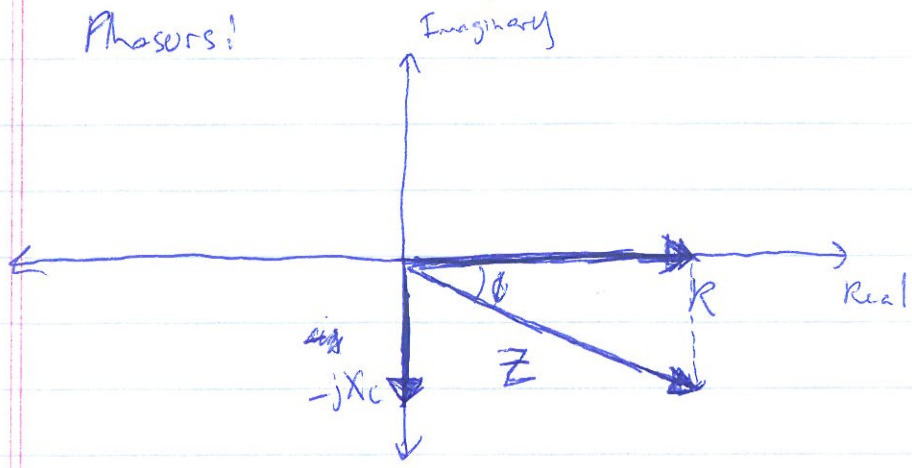
$$X_C = \frac{1}{2\pi f C}$$ units in Ohms (Ω)

however, similar to inductive reactance, this is "out of phase" with resistance. This time though, 90° <u>leading</u> so

$$Z = R - jX_C$$

↳ j is 90° out of phase

Phasors!



$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

So again $|V| = |I||Z|$