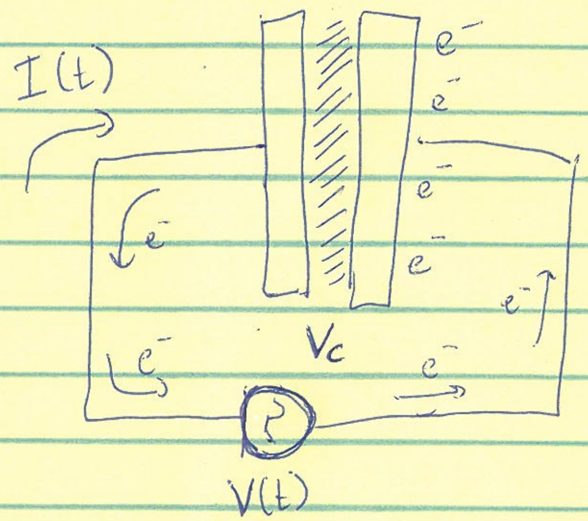
Lecture 11

Finishing capacitance

Why current leads voltage

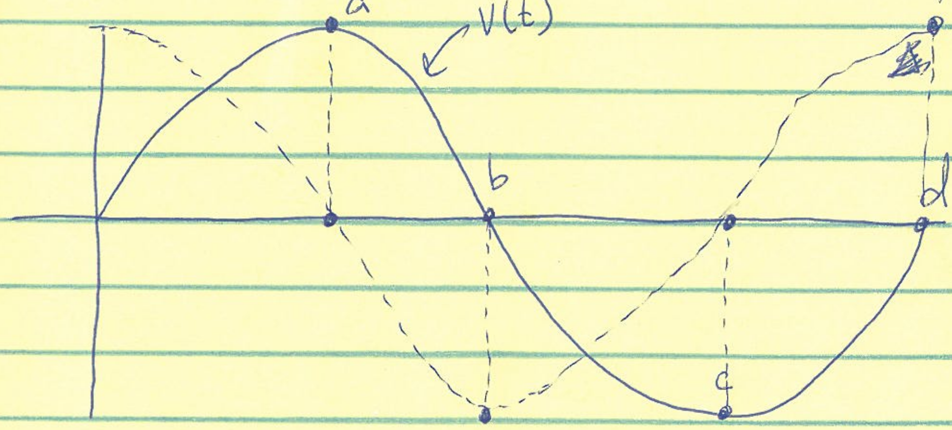


For voltage to build on capacitor, electrons must move.

Think about wave forms and equations

$CV = Q$

C → capacitance, V → voltage, Q → charge



$I = \frac{dQ}{dt}$

→ time rate change in charge is current.

a - Point a, voltage is at a peak and not changing very fast. So, the flow of electrons through the circuit is relatively small. That is current is 0.

b - voltage across capacitor is 0V but changeing quickly.
~~Bec~~ For Voltage to change quickly, the charge on the cap must be changing quickly.

↳ Change in charge over change in time is current.

Because voltage is going from postive to negative, ~~the~~ electrons must be moving from the negatively charged plate to the positive. Thus, current, which is in the opposite direction, is negative.

c - Voltage is ~~p~~ peaking negatively and changing slowly, so current is near zero.

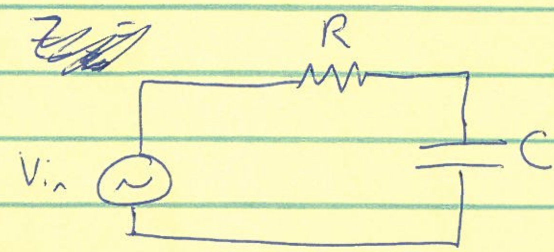
d - Voltage changing quickly so $\frac{\Delta Q}{\Delta t}$ (i.e. current) ~~changing~~ is large, and because voltage is going from negative to positive, current must be positive.

If we connect the dots with a sine wave, we see current must lead voltage by 90°.

<u>Capacitive Reactance:</u>

$$X_C = \frac{1}{2\pi f C} \quad \text{units in ohm's } (\Omega)$$

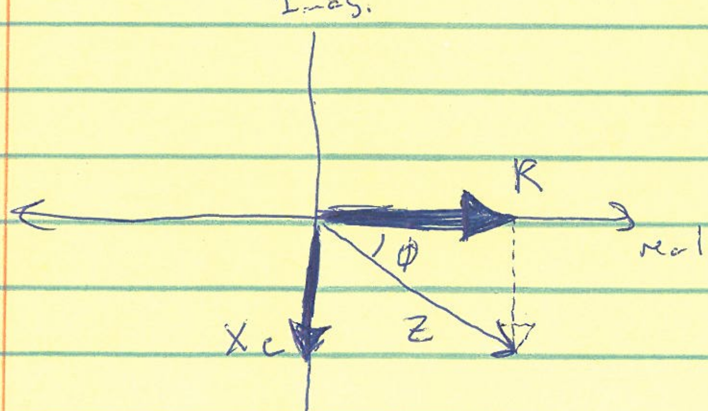
However, similar to inductance, this is "out of phase" with resistance. This time though, ~~90° lead~~ current <u>leads</u> voltage by 90° so in an RC series circuit,



$$Z = R \underbrace{-j}_{} X_c$$

$\hookrightarrow$ $-j$ means $-90°$ out of phase

So if we do our phasor diagram and put R on the positive real axis, then capacitive reactance, X_c will be on the negative imaginary axis:



Then we add the two to get impedance, Z total

Similar to inductance, we can get the magnitude of this impedance as:

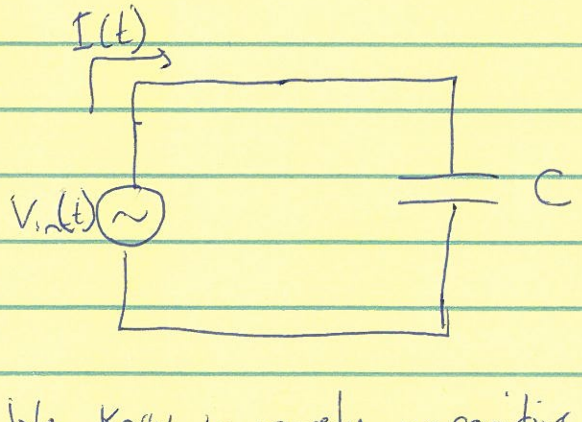
$$Z = \sqrt{R^2 + X_c^2}$$

And the total phase of the impedance as:

$$\phi = -\tan^{-1}\left(\frac{X_c}{R}\right)$$

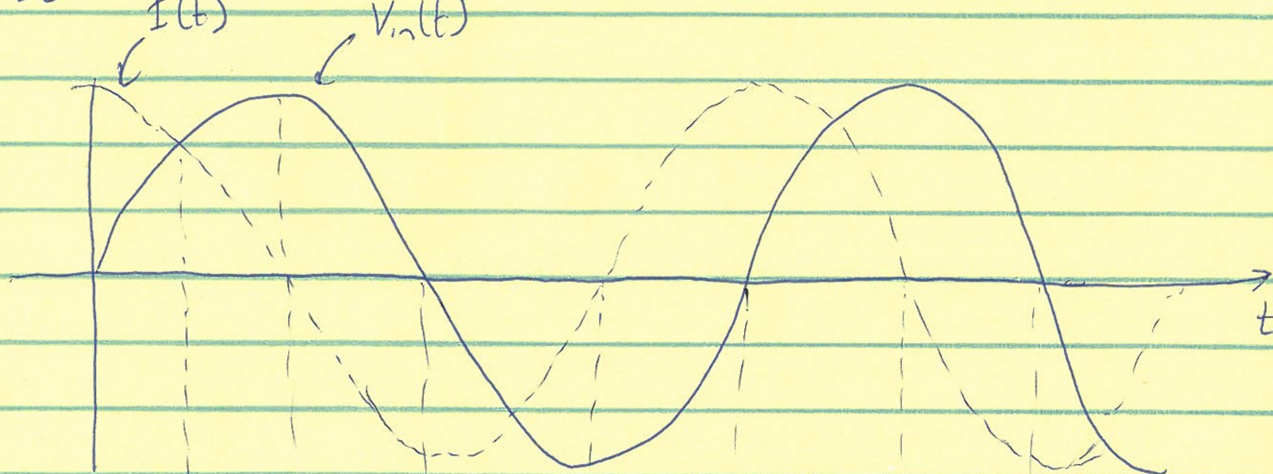
Power in AC Circuits

Use purely capacitive circuit for example:

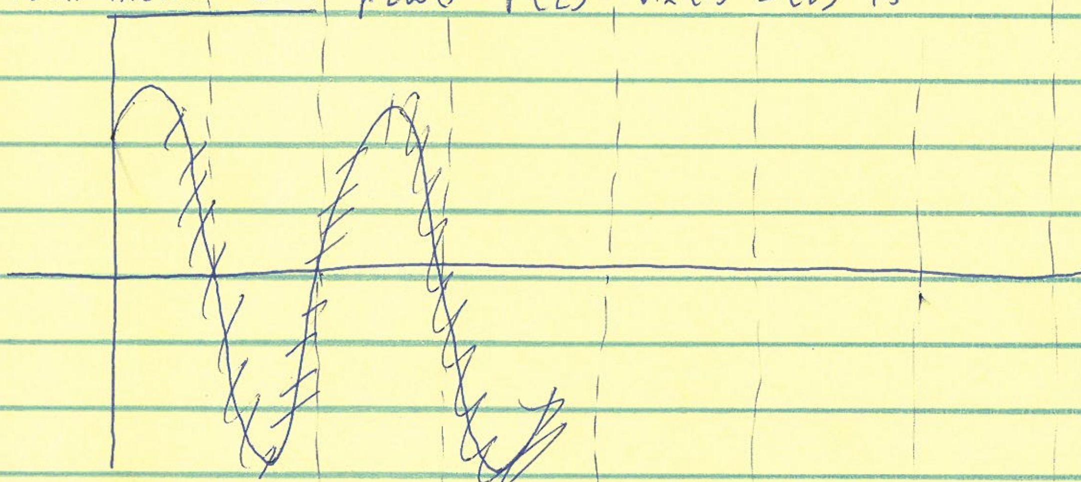


We know in purely capacitive circuit, current leads voltage by $90°$

So



Then instantaneous power $P(t) = V_{in}(t)\, I(t)$ is



$P(t) > 0$ because $I(t), V_{in}(t) > 0$

$P(t) < 0$ because $I(t) < 0, V(t) > 0$

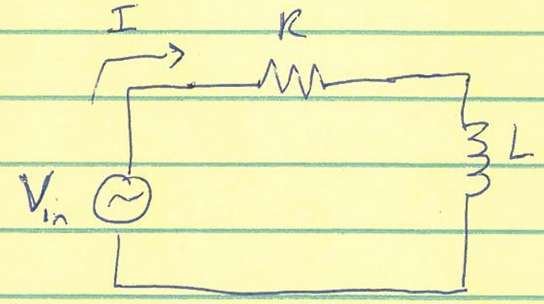
< 0
$V(t) < 0$
$I(t) > 0$.

- Thus net [redacted] power (average power) delivered to circuit is 0!!!

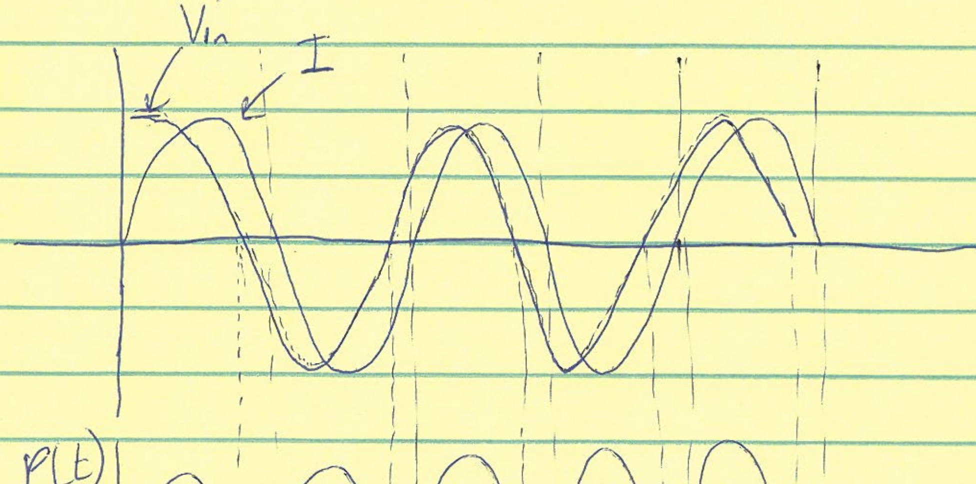
- Similar in purely inductive circuits.

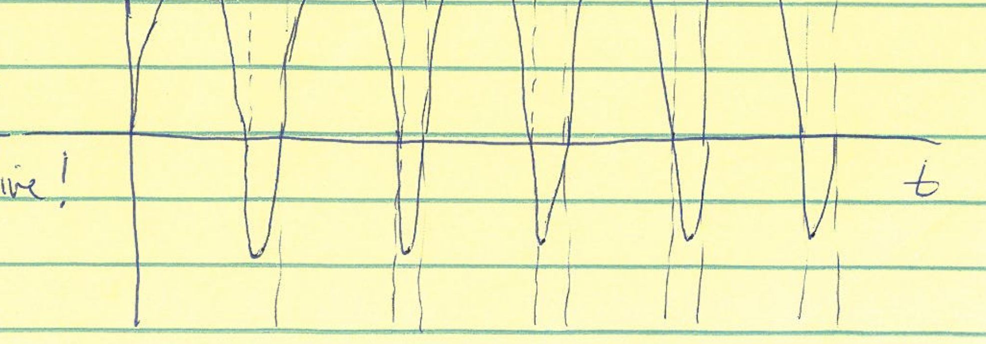
What about RL or RC circuits: Phase angle no longer $\pm 90°$ so some effective power delivered.

RL as example:



$Z = R + jX_L$





Most power positive!

~~This~~ The average of this power is called the true power in your book. This is the power utilized in the circuit,

- The total power required by the circuit from the source through is larger. That is the apparent power!

~~So~~ The ratio of these two is called the power factor.

$$P_F = \frac{P}{P_{app}}$$

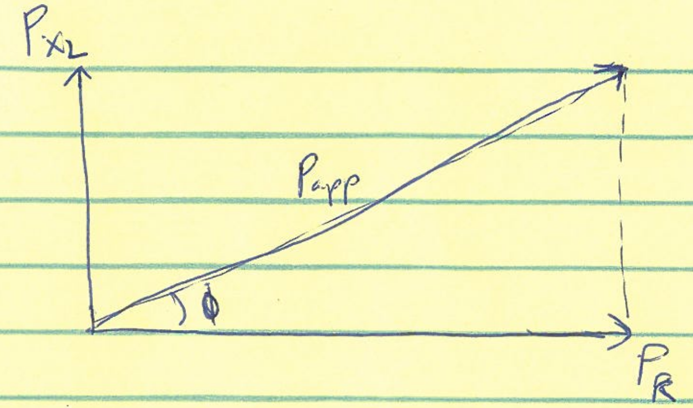
$P_{app} = |V_{in}||I|$ ← So magnitudes of both, Ignore phase angle.

Apparent power unit → VA (volt-amps) to differentiate

The true power ~~is~~ in an RL series circuit is the power dissipated in the ~~a~~ resistor only!

Why? Because voltage in resistor is in phase with the current. while voltage across inductor is 90° out of phase with current.

- Thus power in resistor is real power
- Power in inductor is reactive power (VAR)
- Apparent power is magnitude of vector sum of the two.



So $P_{X_L} = |I|^2 X_L$

$$P = P_R = |I|^2 R$$

$$P_{app} = \sqrt{P_R^2 + P_{X_L}^2}$$

also

$$P_{app} = |V_{in}||I|$$

so real power also given by:

$$P = P_{app} \cos(\phi)$$

$\uparrow$ ϕ is the same as that of the impedance.

$$P = \sqrt{P_{app}^2 - P_{X_L}^2}$$