Lecture 12

AC Circuit Analysis

- This will build upon our current understanding of electrical circuit analysis

- Recall, for purely resistive circuits ⇒

  - Could analyze any circuit with three laws:

    - Ohm's Law ⇒ $V = IR$

    - KVL ⇒ Sum of voltages in any circuit loop is 0.

    - KCL ⇒ Sum of currents into a node is 0.

  - To get power quantities, we used $P = IV$.

- In lab last week, we learned that KVL and KCL do not hold for AC circuits if we only consider the magnitude of the currents and voltages. However, if we also consider the <u>phase</u> in addition to magnitude, then they <u>do</u> hold.

  - To do this, we must use phasor diagrams.

AC Series circuits:

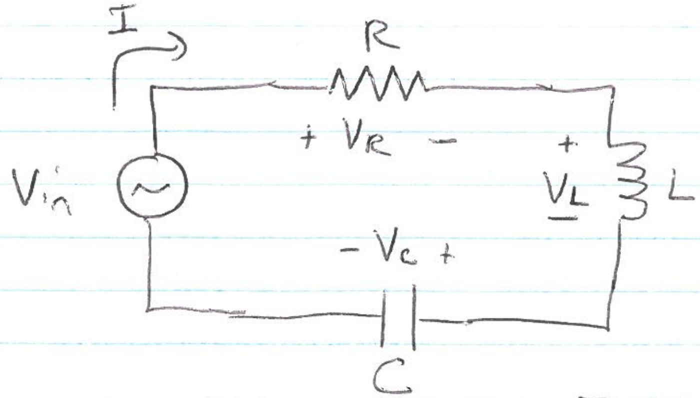
- In series circuits, the following statements are true:

1: The instantaneous current through all parts of the circuit is the same.

2: The applied voltage (source voltage) is equal to the vector sum of the voltage drops through each device.
↳ This is the same as KVL!

3: The combined impedance of an AC series circuit is the vector sum of the individual impedances in the circuit.
↳ This can be derived from KVL.

In previous class, we considered RL and RC circuits. To understand how to fully analyze any series AC circuit, let's consider an RLC series circuit



- From point 1 above, I is the same throughout circuit
- From 2, vector sum of V_R, V_L, V_c equals V_{in}
- From 3, vector sum of R, jX_L, $-jX_c$ equals Z.

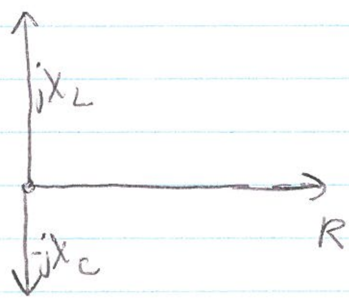
How to find total impedance of circuit:

- Creat vector diagram for impedances:
  - ↳ Just like in RL or RC circuits, R usually shown on + real axis

recall:

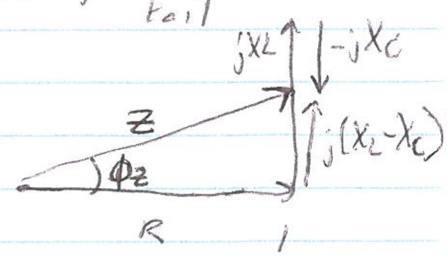
$X_L = 2\pi F L$

$X_C = \dfrac{1}{2\pi F C}$



$\Rightarrow$

adding using tip to tail



So $Z = R + jX_L \underbrace{-j}\, X_C$ ← This is why we use complex number notation!

90° lag ↲ ↳ 90° lead

So $Z = R + j(X_L - X_C)$

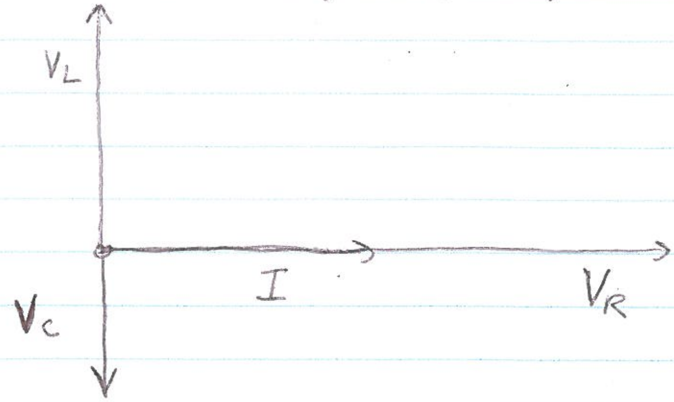
$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$, $\phi_Z = \tan^{-1} \dfrac{X_L - X_C}{R}$

↳ We will come back to this later.

So with impedance of circuit, we can find the current:

$|I| = \dfrac{|V|}{|Z|}$ ← This is equivalent of Ohm's Law.

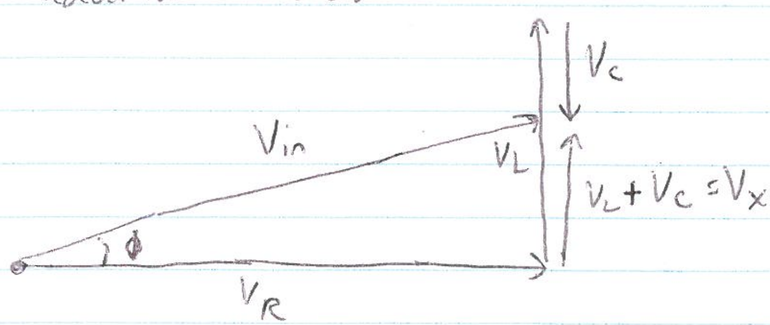
Now, because current is ~~equal~~ same through ~~all~~ each device, let's use that as the reference vector for ~~finding~~ drawing our current and voltage phasor vectors:
(that is ~~to~~ ~~set~~ show I as having an ~~angle~~ phase of $0°$)



Recall:
$V_R = IR$
$V_L = jX_L \cdot I$
$V_C = -jX_C \cdot I$

Note:
This diagram is exactly the same as the impedance diagram except that it is scaled by $|I|$.

tip to tail addition of vectors:



So if we take phase into account, $V_{in} = V_R + V_L + V_C$ is true.
then V_{in}, V_R, V_L, V_C are vectors and

Note ϕ is the same as ϕ_Z

What about power, ⇒ real, reactive, and apparent?

- The real power of a circuit is the product of the current and the portion of the voltage that is in phase with the current.

That is $P = IV\cos\phi$

- The reactive power is the product of the current and the portion of voltage out of phase with the current:

$$Q = IV\sin\phi$$

- The apparent power is the product of the voltage magnitude and current magnitude.

In a series circuit: the real power is the power through the total resistance of the circuit:

$P = |I|^2 R$ ⇒ this is equal to $IV\cos\phi$

$P = |I||V_R|$

In a series circuit: the reactive power is the power required by the vector sum of the reactive components:

$Q = |I|^2 |jX|$ ⇒ in our circuit $X = X_L - X_C$

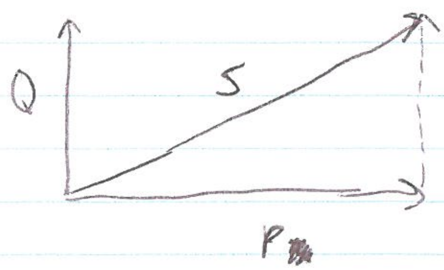
$Q = |I||V_X|$

In any circuit: Apparent power is $S = |V_{in}||I|$

Vector diagram for power:

- Show real power on positive real axis (horizontal)
- Show reactive power on imaginary (vertical) axis
- Apparent power is vector sum of the two

So then ⇒



Power Factor is the ratio of P to S

$$PF = P/S$$

AC Parallel Circuits:

Following statements are true:

1: The source current is equal to the vector sum of the currents through each individual branch.
↳ This is the same as KCL!

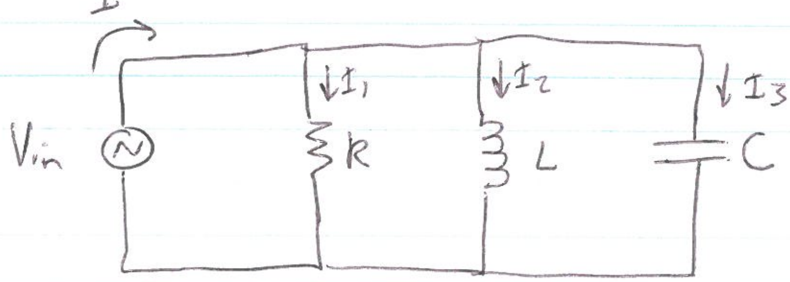
2: At any instant, the voltage across each branch of the circuit is the same.

3: The total admittance of the circuit is equal to the vector sums of the admittance of each circuit branch.
↳ Derived From KCL

→ Do not worry so much about this last one. Basically it is saying that we can find the total circuit impedance in the same way that we found the total resistance in parallel resistive circuits.

Admittance is inverse of impedance

Consider the following circuit:



From point 1: $I_1 + I_2 + I_3 = I$ if we consider phase

2: $V_{in} = I_1 R$

$V_{in} = I_2 jX_L$

$V_{in} = I_3(-jX_C)$

3. ~~Z~~ Total impedance of circuit is

$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$ where $Z_1 = R$

$Z_2 = jX_L$

$Z_3 = -jX_C$

<u>Phasor diagram</u>

Since input voltage is common to all branches, use it as reference:

Recall $\Rightarrow$ $I_1 = \frac{V_{in}}{R}$

$I_2 = \frac{V_{in}}{jX_L}$

$I_3 = \frac{V_{in}}{-jX_C}$