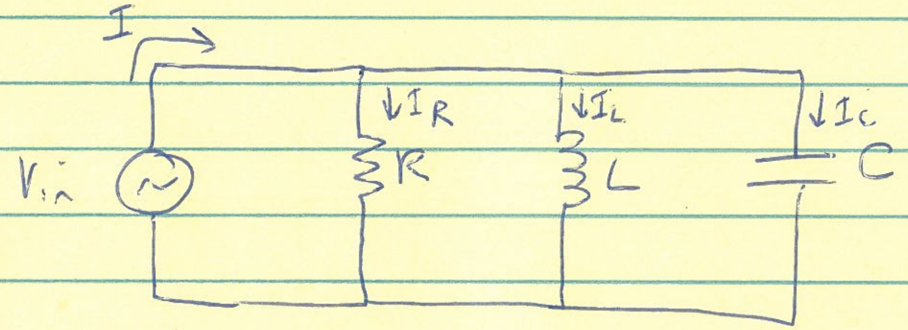
Lecture 13

Continuing on with AC circuits:

Parallel circuits continued:

RLC parallel circuit:



What we might want to know:

- Z
- I
- I_R, V_R
- I_C, V_C
- I_L, V_L
- P
- Q
- S
- P_R
- Q_L
- Q_C

As long as we consider both magnitude and phase, then

$$I = I_R + I_L + I_C$$ From point 1

$$V_{in} = V_R = V_L = V_C$$ From point 2

$$Z = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}}$$ where

$Z_R = R$ (no reactance)

$Z_L = jX_L$

$Z_C = -jX_C$

Phasor diagram

Since input voltage is common to all branches, we can use that as reference. So from Ohm's Law:

$$I_R = \frac{V_{in}}{R} \qquad I Z_L = V_{in} \qquad I Z_C = V_{in}$$

$$I(jX_L) = V_{in} \qquad I(-jX_C) = V_{in}$$

Recall from math class: $\frac{1}{j} = -j$

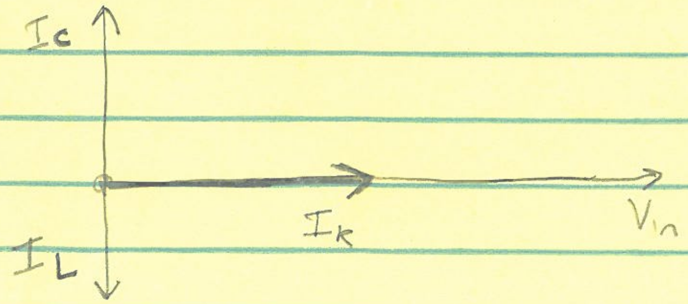
$\frac{1}{-j} = j$

So $I_R = \frac{V_{in}}{R}$ $\quad I_C = \frac{V_{in}}{-jX_C} = j\frac{V_{in}}{X_C}$ $\quad I_L = \frac{V_{in}}{jX_L} = -j\frac{V_{in}}{X_L}$

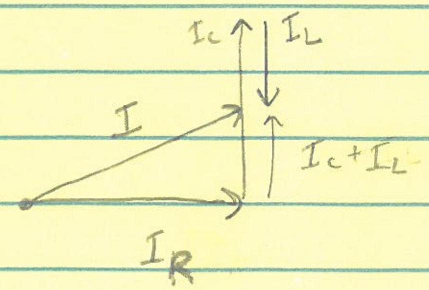
Does this make sense? Yes

Recall for inductor that current lags voltage by 90°, and for capacitor, current leads voltage by 90°.

If we draw these phasors with V_{in} as reference:



IF we add tip to tail:



What about power?

Since resistor current is in phase with applied voltage, power is real:

$$P_R = I_R V_{in} = I_R^2 R = \frac{V_{in}^2}{R}$$

Since inductor current is <u>out of phase</u> with applied voltage power is reactive:

$$Q_L = |I_L||V_{in}| = |I_L|^2 |(j X_L)| = \frac{|V_{in}|^2}{|j X_L|}$$

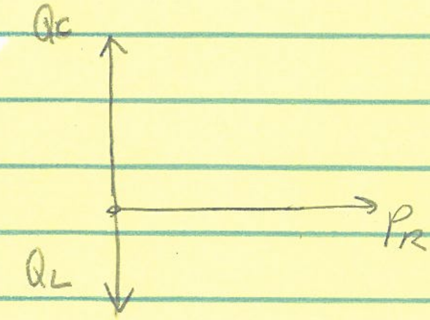
Since current lags voltage reactive power is lagging by 90°

Since capacitor current is <u>out of phase</u> with applied voltage, also reactive power:

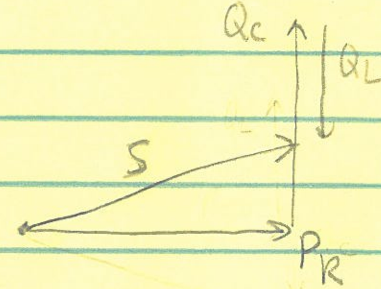
$$Q_C = |I_C||V_{in}| = |I_C|^2 |(-j X_C)| = \frac{|V_{in}|^2}{|-j X_C|}$$

reactive power 90° leading.

Power phasor diagram:



Adding tip to tail:



$$S = \sqrt{P^2 + Q^2} \quad \text{where } Q = Q_C - Q_L$$

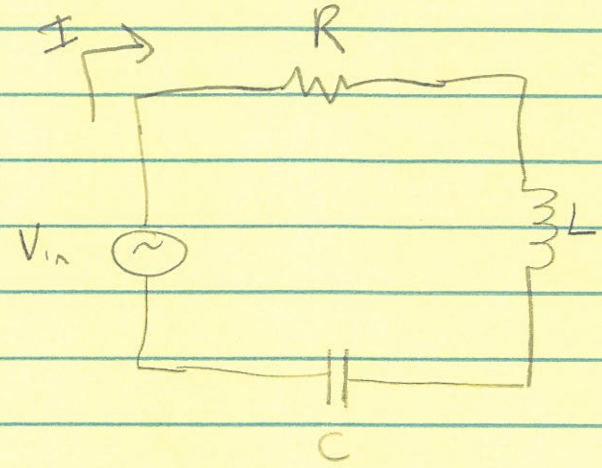
So by having a capacitor in parallel with inductor, we can cancel out the reactive power

Let's do some examples

See example 5 in book

RLC series circuit:

$V_{in} = 120V @ 60Hz$



Let $R = 5\Omega$

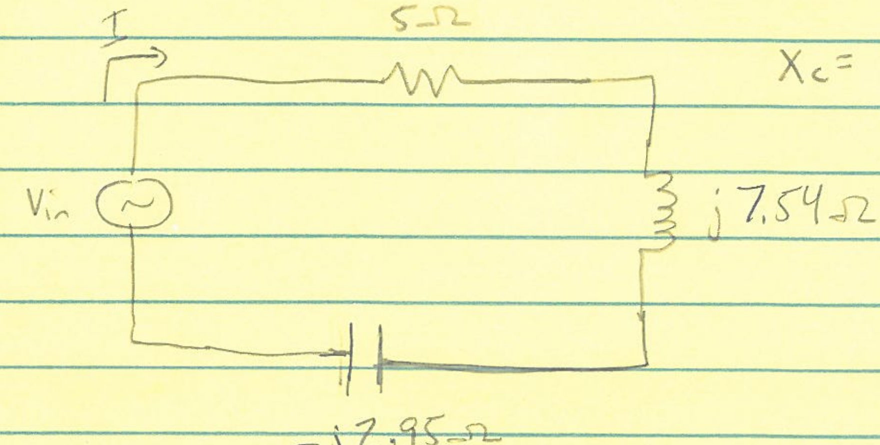
$C = 900\mu F$

$L = 0.02H$

First, rewrite circuit with impedance values!

$X_L = 2\pi FL = 7.54\Omega$

$X_C = \frac{1}{2\pi fC} = 2.95\Omega$



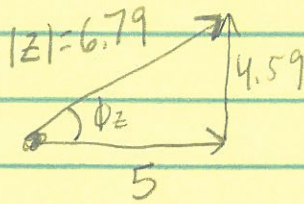
So what if all I cared about was finding I, P, Q, S and PF

That is, I'm not concerned with individual voltages?

Then start with Z and find rest:

$Z = Z_R + Z_L + Z_C$, $Z_R = R$ $Z_L = jX_L$ $Z_C = -jX_C$

then $Z = 5 + j7.54 - j2.95$



$Z = 5 + j4.59$

$|Z| = 6.79$, $\phi_Z = \tan^{-1} \frac{4.59}{5} = 43°$

So $|I| = \frac{|V_{in}|}{|Z|} = \frac{120}{6.79} = 17.7A$

$S = |V_{in}||I| = 120 \cdot 17.7 = 2.1 kVA$

$PF = \cos \phi = 0.74$

$PF = \frac{P}{S} \Rightarrow P = S \cdot PF = 1.56 kW$

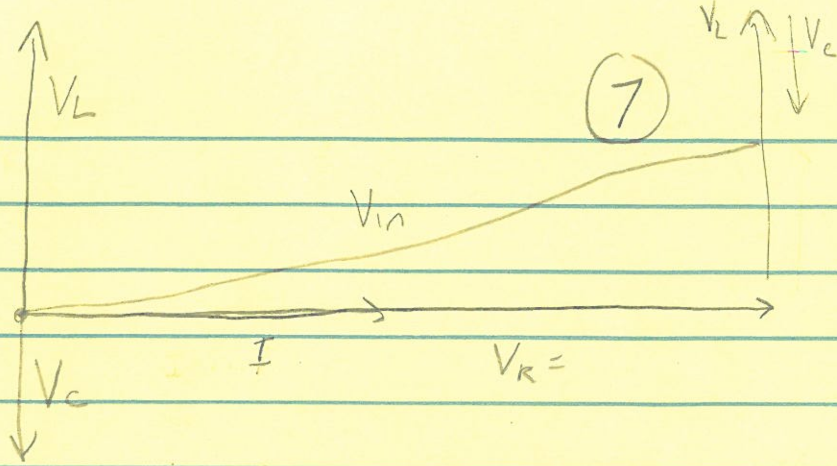
$S = \sqrt{P^2 + Q^2} \Rightarrow Q = \sqrt{S^2 - P^2} = 1.43 kVAR$

And I'm done.

Now if I want voltages:

Use I as reference and

↓



$V_R = I R$
$V_R = 88.5\ V$

$|V_L| = |I||jX_L|$
$= 133\ V$

$|V_C| = |I||-jX_C| = 52.2\ V$

Double check to make sure numbers are correct?
should be:
$|V_{in}| = 120$

$$|V_{in}| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{88.5^2 + 80.8^2} = 119.8 \checkmark$$

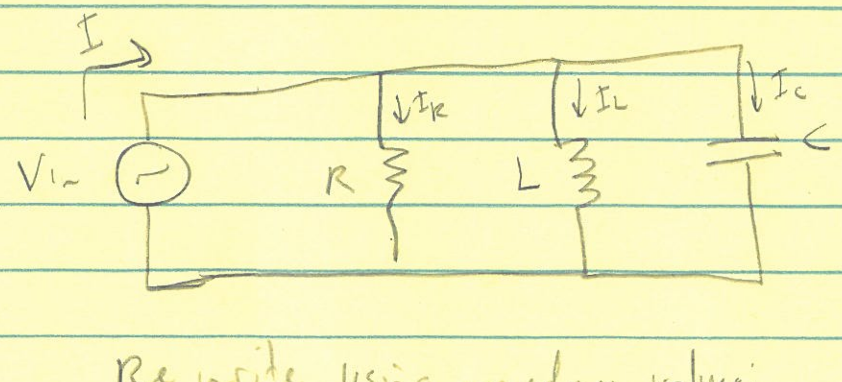
Where we left off.

RLC Parallel Circuit

Example 14 in book:
Let

$V_{in} = 120$ @ 60Hz

$R = 30\Omega$
$L = 0.531 H$
$C = 66.3 \mu F$



Re write using impedance values: