## Lecture 17: Three-Phase Systems

## Autotransformers

Autotransformers

- Only has one winding
- One portion of winding for both primary and secondary
- Standard equations still apply
- Require less copper
- Cheaper
- Smaller

- Disadvantage is more hazardous


## Three-Phase Systems

- Why generate in three-phase?
- More efficient generation/transmission/use
- Three-phase equipment smaller per unit power
- Easy to create rotating magnetic fields (motors)
- Smoother power transfer
- Smoother torque in motors
- Smoother conversion to DC (e.g. for battery storage)
- Cost effective transmission
- Less conductors required
- If we generate/transmit in three-phase, how do we get single-phase?
- Tap into single leg of three-phase using transformer


## Three-Phase Systems

- AC electricity primarily generated and transmitted in the form of three-phase
- Each phase voltage $120^{\circ}$ apart

If load balanced, current $120^{\circ}$ apart as well

Typical Color Scheme: Phase 1
Phase 2
Phase 3


## Balanced Loads

- What is a balanced load?
- All branches of load have equivalent impedance


All phase impedances equivalent:
$Z=Z_{P I}=Z_{P 2}=Z_{P 3}$

## Balanced Loads

## - Balanced generation with balanced load



## Balanced Loads

- Balanced generation with balanced load:
- Combined phase currents sum to zero.


Perfect balance $\rightarrow$ no neutral
current $\rightarrow$ decreased copper


## Balanced Loads

- Three-phase motors
- Windings pretty well equivalent (if non-damaged)
- Usually well balanced
- Power system distribution
- Each house only has single-phase distribution
- But on average (lots of houses), reasonably balanced
-Why balance loads?
- More efficient
- Cost effective
- Better on equipment

And makes analysis
a whole lot easier!

We will only consider balanced systems

## Three-Phase Configurations

- Wye-Configuration (Star-Configuration)
- 4-wire distribution
- Neutral used for any return current due to imbalance


Phase/line relationships

$$
\begin{aligned}
& E_{L}=\sqrt{3} E_{P} \\
& I_{L}=I_{P}
\end{aligned}
$$

## Three-Phase Configurations

- Delta-Configuration
- 3-wire distribution
- No neutral (any required current return due to imbalance distributed on other legs)


Phase/line relationships

$$
\begin{aligned}
& E_{L}=E_{P} \\
& I_{L}=\sqrt{3} I_{P}
\end{aligned}
$$



## Three-Phase Circuits

- If balanced, can do analysis as single-phase.
- Use phase variables (voltage, current, impedance, etc)
- Need to find line variables for some circuits
- Can easily calculate total three-phase power.
- Can also include transformers
- For this class we will not consider 3-phase transformers
- See Ch. 12 if interested.


## Three-Phase Circuits

- Can have wye or delta out of transformer secondary - Can have wye or delta load
wye secondary - wye load

wye secondary - delta load

delta secondary - wye load

delta secondary - delta load



## Three-Phase Circuits

- What to calculate?
- Transformer secondary phase voltage, $E_{S, P}$
- Transformer secondary line voltage, $E_{S, L}$
- Transformer secondary phase current, $I_{S, P}$
- Transformer secondary line current, $I_{S, L}$
- Load phase voltage, $E_{L, P}$
- Load line voltage, $E_{L, L}$
- Load phase current, $I_{L, P}$
- Load line current, $I_{L, L}$
- Circuit real, reactive, apparent power, $P Q S$
- Circuit power factor, PF


## Three-Phase Circuits

- Relevant Equations (we'll consider magnitude only):
- Ohm's Law: $\quad E_{P}=I_{P} Z_{P}$
- Real Power: $P=3 E_{P} I_{P} P F \quad P=\sqrt{3} E_{L} I_{L} P F$
- Apparent Power: $S=\sqrt{P^{2}+Q^{2}} \quad S=3 E_{P} I_{P} \quad S=\sqrt{3} E_{L} I_{L}$
- Reactive Power: $Q=\sqrt{S^{2}-P^{2}}$
- Power Factor: $\quad P F=\frac{P}{S}$


## Example: Wye-Wye Circuit

A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of $8 \Omega$.

Step 1: Determine transformer phase voltage and line voltage:

Step 2: Determine load phase voltage and line voltage:

Step 3: Calculate load phase and line current:

Step 4: Determine transformer secondary phase and line current:

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E_{S, P}=277 \mathrm{~V} \quad E_{S, L}=\sqrt{3} E_{S, P}=480 \mathrm{~V}
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E_{L, L}=E_{S, L}=480 \mathrm{~V} \quad E_{L, P}=\frac{1}{\sqrt{3}} E_{S, L}=277 \mathrm{~V}
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Step 3: Calculate load phase and line current:

Step 4: Determine transformer secondary phase and line current:

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Step 3: Calculate load phase and line current:

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I_{L, P}=E_{L, P} / Z_{L, P}=34.6 \mathrm{~A} \quad I_{L, L}=I_{L, P}=34.6 \mathrm{~A}
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## Example: Wye-Wye Circuit

For previous circuit example, determine real, reactive, and apparent power:

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For previous circuit example, determine real, reactive, and apparent power:

$$
\begin{aligned}
& P F=1 \\
& P=3 E_{L, P} I_{L, P} P F=3 * 277 \mathrm{~V} * 34.6 \mathrm{~A}=28.8 \mathrm{~kW} \\
& S=P=28.8 \mathrm{kVA} \\
& Q=0 \mathrm{kVAR}
\end{aligned}
$$

Resistive circuit so no reactive power!

## Example: Wye-Delta Circuit

A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of $8 \Omega$. The motor operates with a power factor of 0.8 .

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Step 3: Calculate load phase and line current:

$$
I_{L, P}=E_{L, P} / Z_{L, P}=60 \mathrm{~A} \quad I_{L, L}=\sqrt{3} I_{L, P}=104 \mathrm{~A}
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## Example: Wye-Wye Circuit

For previous circuit example, determine real, reactive, and apparent power:

$$
\begin{aligned}
& P F=0.8 \\
& P=3 E_{L, P} I_{L, P} P F=3 * 480 \mathrm{~V} * 60 \mathrm{~A} * 0.8=69.1 \mathrm{~kW} \\
& S=\frac{P}{P F}=\frac{69,100}{0.8}=86.4 \mathrm{kVA} \\
& Q=\sqrt{S^{2}-P^{2}}=51.8 \mathrm{kVAR}
\end{aligned}
$$

# Upcoming in class 

- More 3-phase circuits
- Delta and Wye connections
- Electrical Distribution
- CHANGE TO SYLLABUS
- There IS lab next week
- We will do project later (probably week before Thanksgiving)

