Lecture 17: Three-Phase Systems



Autotransformers

Autotransformers

- Only has one winding

 One portion of winding for both primary and secondary
- Standard equations still apply
- Require less copper
 - Cheaper
 - Smaller
- Disadvantage is more hazardous





Three-Phase Systems



- Why generate in three-phase?
 - More efficient generation/transmission/use
 - Three-phase equipment smaller per unit power
 - Easy to create rotating magnetic fields (motors)
 - Smoother power transfer
 - Smoother torque in motors
 - Smoother conversion to DC (e.g. for battery storage)
 - Cost effective transmission
 - Less conductors required
- If we generate/transmit in three-phase, how do we get single-phase?
 - Tap into single leg of three-phase using transformer

Three-Phase Systems



- AC electricity primarily generated and transmitted in the form of three-phase
 - Each phase voltage 120° apart

If load balanced, current 120° apart as well

Typical Color Scheme: Phase 1 Phase 2 Phase 3





- What is a balanced load?
 - All branches of load have equivalent

impedance





Balanced generation with balanced load





Balanced generation with balanced load:

- Combined phase currents sum to zero.



Perfect balance \rightarrow no neutral current \rightarrow decreased copper





- Three-phase motors
 - Windings pretty well equivalent (if non-damaged)
 - Usually well balanced
- Power system distribution
 - Each house only has single-phase distribution
 - But on average (lots of houses), reasonably balanced
- Why balance loads?
 - More efficient
 - Cost effective
 - Better on equipment

And makes analysis a whole lot easier!

We will only consider balanced systems

Three-Phase Configurations



- Wye-Configuration (Star-Configuration)
 - 4-wire distribution
 - Neutral used for any return current due to imbalance



Phase/line relationships

$$E_L = \sqrt{3} E_P$$
$$I_L = I_P$$

Three-Phase Configurations



- Delta-Configuration
 - 3-wire distribution
 - No neutral (any required current return due to imbalance distributed on other legs)



Three-Phase Transformers

delta-delta





delta-wye

MO STATE

wye-delta







- If balanced, can do analysis as single-phase.
 - Use phase variables (voltage, current, impedance, etc)
 - Need to find line variables for some circuits
 - Can easily calculate total three-phase power.
- Can also include transformers
 - For this class we will not consider 3-phase transformers
 - See Ch. 12 if interested.



- Can have wye or delta out of transformer secondary
- Can have wye or delta load

wye secondary - wye load



wye secondary - delta load



delta secondary - wye load



delta secondary - delta load



- What to calculate?
 - Transformer secondary phase voltage, $E_{S,P}$
 - Transformer secondary line voltage, $E_{S,L}$
 - Transformer secondary phase current, $I_{S,P}$
 - Transformer secondary line current, $I_{S,L}$
 - Load phase voltage, $E_{L,P}$
 - Load line voltage, $E_{L,L}$
 - Load phase current, $I_{L,P}$
 - Load line current, $I_{L,L}$
 - Circuit real, reactive, apparent power, P Q S
 - Circuit power factor, PF





- Relevant Equations (we'll consider magnitude only):
 - Ohm's Law: $E_P = I_P Z_P$
 - Real Power: $P = 3 E_P I_P PF$ $P = \sqrt{3} E_L I_L PF$
 - Apparent Power: $S = \sqrt{P^2 + Q^2}$ $S = 3 E_P I_P$ $S = \sqrt{3} E_L I_L$
 - Reactive Power: $Q = \sqrt{S^2 P^2}$

- Power Factor:
$$PF = \frac{P}{S}$$



A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of 8 Ω .

Step 1: Determine transformer phase voltage and line voltage:

Step 2: Determine load phase voltage and line voltage:

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of 8 Ω .

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480V$

Step 2: Determine load phase voltage and line voltage:

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of 8 Ω .

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480V$$
 $E_{L,P} = \frac{1}{\sqrt{3}} E_{S,L} = 277V$

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of 8 Ω .

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480V$$
 $E_{L,P} = \frac{1}{\sqrt{3}} E_{S,L} = 277V$

Step 3: Calculate load phase and line current:

$$I_{L,P} = E_{L,P} / Z_{L,P} = 34.6A$$
 $I_{L,L} = I_{L,P} = 34.6A$



A wye-connected three-phase transformer supplies power to a wye-connected resistive load. The transformer secondary has a phase voltage of 277 V and the resistors of the load have a resistance of 8 Ω .

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480V$$
 $E_{L,P} = \frac{1}{\sqrt{3}} E_{S,L} = 277V$

Step 3: Calculate load phase and line current:

$$I_{L,P} = E_{L,P} / Z_{L,P} = 34.6A$$
 $I_{L,L} = I_{L,P} = 34.6A$

$$I_{S,L} = I_{L,L} = 34.6 \text{A} \quad I_{S,P} = I_{S,L} = 34.6 \text{A}$$



For previous circuit example, determine real, reactive, and apparent power:



For previous circuit example, determine real, reactive, and apparent power:

$$PF = 1$$

 $P = 3 E_{L,P} I_{L,P} PF = 3 * 277V * 34.6A = 28.8kW$
 $S = P = 28.8kVA$
 $Q = 0kVAR$

Resistive circuit so no reactive power!



A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of 8 Ω . The motor operates with a power factor of 0.8.

Step 1: Determine transformer phase voltage and line voltage:

Step 2: Determine load phase voltage and line voltage:

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of 8 Ω . The motor operates with a power factor of 0.8.

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277 V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480 V$

Step 2: Determine load phase voltage and line voltage:

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of 8 Ω . The motor operates with a power factor of 0.8.

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277 V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480 V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480 V$$
 $E_{L,P} = E_{S,L} = 480 V$

Step 3: Calculate load phase and line current:



A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of 8 Ω . The motor operates with a power factor of 0.8.

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277 V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480 V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480 V$$
 $E_{L,P} = E_{S,L} = 480 V$

Step 3: Calculate load phase and line current:

$$I_{L,P} = E_{L,P} / Z_{L,P} = 60A$$
 $I_{L,L} = \sqrt{3} I_{L,P} = 104A$



A wye-connected three-phase transformer supplies power to a delta-connected induction motor. The transformer secondary has a phase voltage of 277 V and motor windings have a total impedance of 8 Ω . The motor operates with a power factor of 0.8.

Step 1: Determine transformer phase voltage and line voltage:

$$E_{S,P} = 277 V$$
 $E_{S,L} = \sqrt{3} E_{S,P} = 480 V$

Step 2: Determine load phase voltage and line voltage:

$$E_{L,L} = E_{S,L} = 480 V$$
 $E_{L,P} = E_{S,L} = 480 V$

Step 3: Calculate load phase and line current:

$$I_{L,P} = E_{L,P} / Z_{L,P} = 60A$$
 $I_{L,L} = \sqrt{3} I_{L,P} = 104A$

$$I_{S,L} = I_{L,L} = 104A$$
 $I_{S,P} = I_{S,L} = 104A$



For previous circuit example, determine real, reactive, and apparent power:



For previous circuit example, determine real, reactive, and apparent power:

$$PF = 0.8$$

$$P = 3 E_{L,P} I_{L,P} PF = 3 * 480V * 60A * 0.8 = 69.1 kW$$

$$S = \frac{P}{PF} = \frac{69,100}{0.8} = 86.4 kVA$$

$$Q = \sqrt{S^2 - P^2} = 51.8 kVAR$$

Upcoming in class

- More 3-phase circuits
 - Delta and Wye connections
- Electrical Distribution
- CHANGE TO SYLLABUS
 - There IS lab next week
 - We will do project later (probably week before Thanksgiving)

