EELE445-S15 Homework Assignment 3-10 points

- Grading: -0.5 point per error

HW3

| Set | Problem | Principle |
| :---: | :---: | :--- |
| $\mathbf{3}$ | $2-61$ | Use 2-89 <br> For 2-61: let $\mathrm{A}=1, \mathrm{~b} / \mathrm{T}=.2$, find <br> 1) The total power from the time waveform |
|  | Additional <br> work with | 2) The percent of total power in the fundamental <br> 2 <br> prob 2-61 <br> 3) The percent of total power just the AC components <br> repetitive triangle- (observe the $1 / \mathrm{n}^{2}$ spectrum compared |
| 3 | $2-70$ | to square wave 1/n ) |
| $\mathbf{3}$ | $2-80$ | Energy of a pulse through a filter |
| $\mathbf{3}$ | $2-85$ | comb filter- used in radar and digital communications |

$$
\begin{aligned}
& \text { 2-61 } c_{n}=\frac{1}{T} \int_{T_{0}}^{T_{0}+b} A e^{-j n \omega t} d t \\
&=\left.\frac{-A}{T} \frac{1}{j n \omega} e^{-j n \omega t}\right|_{Y_{0}} ^{Y_{0}+b} \\
&=\frac{-A}{j n \omega T}\left(e^{-j n \omega\left(r_{0}+b\right)}-e^{-j n \omega T_{0}}\right) \\
&=\frac{-A}{j n \omega T} e^{-j n \omega\left(T_{0}+\frac{b}{2}\right)}\left(e^{-j n \omega b} \frac{2}{2}-e^{j n \frac{n b}{2}}\right) \\
&=\frac{2 A}{n \omega T} e^{-j n \omega\left(r_{0}+\frac{b}{2}\right)}\left(e^{\left.j \frac{n \omega b}{2}-e^{-j \frac{n b}{2}}\right)}\right. \\
& \omega=\frac{2 \pi}{T}=\frac{A}{n \pi} e^{-j n \omega\left(r_{0}+\frac{b}{2}\right)} \sin \left(\frac{n \pi b}{T}\right) \\
& C_{n}=\frac{A b}{T} e^{-j n \omega\left(T_{0}+\frac{b}{2}\right)} \frac{\sin \left(\frac{n \pi b}{T}\right)}{n \pi b / T} \\
&
\end{aligned}
$$

2-61 additianal
given $A=1, \frac{b}{T}=0.2$

set $\tau_{0}=0$ for convincuco.

$$
P_{t}=\left\langle v^{2}(t)\right\rangle=\frac{1}{T} \int_{0}^{b} A^{2} d t=\frac{1}{T} \int_{0}^{.2 T} 1^{2} d t=0.2 w a t t=P_{t}
$$

from salution abave: $\left|C_{n}\right|=\frac{A b}{T} \frac{\sin \left(\frac{n \pi b}{T}\right)}{\frac{n \pi b}{T}}$
fundamentel, $n=1$

$$
\begin{aligned}
& \left|C_{1}\right|=0.2 \frac{\sin (0.2 \pi)}{0.2 \pi}=\frac{\sin (0.2 \pi)}{\pi} \\
& P_{1}=2\left|C_{1}\right|^{2}=2\left(\frac{\sin 0.2 \pi}{\pi}\right)^{2}=70 \mathrm{~mW} \\
& \eta_{0} P_{1}=\frac{P_{1}}{P_{t}} \times 100=\frac{0.07}{0.07 \cdot 2} \times 35 \eta_{0}=\eta_{0} p_{1} \\
& \text { M } P_{a c}=\frac{P_{a c}}{P_{t}} \times 100 \\
& P_{a c}=P_{t}-P_{D C}=\left\langle v^{2}(t)\right\rangle-\langle v(t)\rangle^{2} \\
& \rightarrow \frac{A^{2} b}{T} \quad\left(\frac{A b}{T}\right)^{2} \\
& P_{a c}=.2 w-(.2)^{2} w=0.16 w \\
& \eta_{a} P_{a c}=\frac{0.16}{0.20}+100=80 \%_{0}=\eta_{a} P_{a c}
\end{aligned}
$$

$$
\begin{aligned}
& c_{n}=\frac{1}{T_{0}} \int_{-T_{0} / 2}^{T_{0} / 2} u(t) e^{-j n \omega_{0} t} d t \quad T_{0}=2 \\
& =\frac{4}{T_{0}^{2}}\left[\int_{-T_{0} / 2}^{0}\left(t+\frac{T_{0}}{4}\right) e^{-j n \omega_{0} t} d t-\int_{0}^{T_{0} / 2}\left(t-\frac{T_{0}}{4}\right) e^{-j \mu \omega_{0} t} d t\right]
\end{aligned}
$$

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2-70(a) Cont'd
$C_{n}=\left.\frac{-8}{4 n^{2} \pi^{2}}\left[\cos \left(n \omega_{0} t\right)+n \omega_{0} t \sin \left(n \omega_{0} t\right)\right]\right|_{0} ^{T_{0} / 2}$
$\neq c_{n}=\frac{2}{n^{2} \pi^{2}}\left\{1-(-1)^{n}\right\}=\left\{\begin{array}{cl}0, & n=\text { even } \\ \frac{4}{n^{2} \pi^{2}}, & n=\text { odd }\end{array}\right\}$
(b)
b) $\left\langle v^{2}(t)\right\rangle=\frac{1}{T_{0}} \int_{-T_{0} / 2}^{T_{0} / 2} 1(t) d t \frac{2}{1} \frac{2}{T_{0}} \int_{0}^{T_{0} / 2} v^{2}(t) d t$

$$
=\frac{2}{T_{0}} \int_{0}^{T_{0} / 2}\left[\frac{-4}{T_{0}}\left(t-\frac{T_{0}}{4}\right)\right]^{2} d t=\left.\frac{32}{T_{0}^{3}} \frac{\left(t-\frac{T_{0}}{4}\right)^{3}}{3}\right|_{0} ^{T_{0} / 2}
$$

$$
=\frac{2 \cdot 4^{2}}{3 T_{0}^{3}}\left[\frac{2 T_{0}^{3}}{4^{3}}\right]=\frac{1}{3} w_{a} \text { ft }
$$

Computer Solution and comparison of results follows. $M:=5 \mathrm{~N}$

$$
d t:=\frac{T}{N} \quad t_{k}:=k \cdot d t
$$

$$
w_{0}=1 \quad \text { at }=0.063
$$


(a.) Find the complex Fourier series.
$n:=0 \ldots N-1$

$$
\mathrm{f}_{\mathrm{n}}:=\frac{\mathrm{n}}{\mathrm{~T}}
$$

$$
\text { fo : }=\frac{1}{\mathrm{~T}}
$$

From analytical computation,

$$
e_{n}:=\text { if }\left[\bmod (n, 2) \neq 0, \frac{4}{(n \pi)^{2}}, 0\right]
$$



Alternately, computing FS using the FFT via (2-187),

$$
\operatorname{cc}:=\left[\frac{1}{\sqrt{N}}\right] \cdot \operatorname{icfft}(w)
$$


(b)

$$
p:=2 \sum_{n} e_{n}^{2}-c_{0}^{2} \quad p=0.333
$$

(c) and (d) $|V(f)|=\sum_{-\infty}^{\infty}\left|C_{n}\right| \delta\left(f-n f_{\theta}\right), \quad O(f)=\sum_{-\infty}^{\infty}\left|c_{n}\right|^{2} \delta\left(f-n f_{0}\right)$



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$2-85$
Td $:=.1 \quad j:=\sqrt{-1} \quad f:=0,0.02 \ldots 30$
$H(f):=1-\cos (2 \cdot \tau \cdot T d f)+j \sin (2 \pi \cdot T d f)$
(a.)

(b.)
$T:=1$

$$
X(f):=T \cdot \frac{\sin (T T f)}{T T \cdot f}
$$




