## EELE445 Lab 3: White noise, $|\mathrm{H}(\mathrm{f})|$, and a x3 Frequency Multiplier

## Purpose

The purpose of the lab is to become acquainted with PSD, white noise and filters in the time domain and the frequency domain. White noise and swept sine waves are used examine the to show the transfer function $|\mathrm{H}(\mathrm{f})|$ of a LPF (lowpass filter), and a BPF (bandpass filter). The Q of the BPF is considered. The noise out of the BPF is considered. The a square wave is compared going into the BPF and out of the filter. The filter output is a sine wave at three times the frequency of the square wave. The amplitudes of undesired signals are measured.

## Components Required:

- Small bread-board to build the circuit on( or just use clip leads directly)
- 2ea 220pF capacitors
- 1 ea 1 nF
- 10uH inductor


## Pre-lab

- Find the frequencies for a RC lowpass filter with $\mathrm{R}=50$ ohm and $\mathrm{C}-220 \mathrm{pF}$ where the magnitude response is $-3 \mathrm{~dB},-6 \mathrm{~dB},-10 \mathrm{~dB},-20 \mathrm{~dB},-40 \mathrm{~dB}$.
- Calculate the center frequency, $Q$ and 3 dB bandwidth across a parallel RLC with $\mathrm{R}=25, \mathrm{C}=1.29 \mathrm{nF}$, and $\mathrm{L}=220 \mathrm{nH}$. What is the equivalent noise bandwidth, $\mathrm{B}_{\mathrm{n}}$ of the filter for your calculation? $\left(\mathrm{B}_{\mathrm{n}}=\mathrm{B}_{30 \mathrm{~B}}{ }^{*} \pi / 2\right)$
- Remember for a Bandpass filter: $Q=\frac{f_{0}}{B}=\frac{R}{X_{L}} X_{L}=\omega_{0} L \omega_{0}=\frac{1}{\sqrt{R C}}$ and B is the 3dB bandwidth.
- Single pole Butterworth Bandpass frequency Response:

$$
(|H(f)|)^{2}=\frac{1}{1+Q \cdot\left(\frac{f}{f_{0}}+\frac{f_{o}}{f}\right)^{2}}
$$

- Be able to calculate the distortion of a sinewave with harmonics. (notes attached)


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## METHOD:

## Generator Noise in Time and Frequency Domains

1) Set the generator for a noise (random) signal output with $2 \mathrm{Vp}-\mathrm{p}$ into a 50 ohm load. Verify your amplitude setting using a sine wave. Set the oscilloscope for sample rate of $250 \mathrm{Ms} / \mathrm{sec}$, input $\mathrm{R}=50$, and the bandwidth to 150 MHz .
a) Measure the p-p, rms, and DC magnitude of the noise waveform using the oscilloscope. Be sure the noise is not clipping. Hint(use the run stop button) Why is it not reading 2vp-p? Calculate the Vp-p to Vrms ratio. What do we use this ratio for?
b) Using the Spectrum Analyzer (SA) to estimate the 3dB bandwidth of the noise. Ask for help setting up the SA when you get to this step.
c) The SA shows the power calculated for all frequencies within the resolution bandwidth (RBW) for the settings you have, $\mathrm{P}(\mathrm{f})=\mathrm{dBm} /$ RBWHz. From the RBW it calculates $P(f)=d B m / H z$ assuming $P(f)$ is constant amplitude across the RBW frequency range. Record the SA resolution bandwidth (RBW), power $\mathrm{P}(\mathrm{f})=(\mathrm{dBm} / \mathrm{RBW})$, and the power/ $\mathrm{Hz} \mathrm{P}(\mathrm{f})=\mathrm{dBm} / \mathrm{Hz}$ at 10 MHz .
d) At what frequency does $\mathrm{P}(\mathrm{f}) \mathrm{dBm} / \mathrm{Hz}=\mathrm{P}(10 \mathrm{MHz}) \mathrm{dBm} / \mathrm{Hz}-3 \mathrm{~dB}$ ? This is the 3 dB noise bandwidth of the function generator.
e) What power in dBm you would measure if the resolution bandwidth was increased by a factor of 10 ?

## RC low pass filter

2) Make a RC low pass filter using a 220pF capacitor by placing it in parallel with the output of the generator. (Remember the generator output is 50 ohms).
a) Measure the 3 dB bandwidth of the filter using the oscilloscope (hi-Z input) and a variable frequency sinewave. Compare this to the theoretical 3dB bandwidth.
b) Set the generator back to $2 \mathrm{Vp}-\mathrm{p}$ noise and look at the signal on the SA. Notice that the 3dB bandwidth of the noise spectrum has been reduced to the 3db bandwidth of the low pass filter.
c) Re-measure the Vp-p and Vrms at the filter output using the settings from part 1. Why did the measured values change? (Explain by showing what happens to a signal going through a low pass filter- LPF using $\mathrm{N}_{0}$ and $\mathrm{H}(\mathrm{f})$ ).
d) Reduce the 3dB bandwidth of the filter by using 440pF of capacitance and remeasure Vp-p and Vrms. Compare these voltages with the voltages measured in 2c. Calculate what you expect the values would be for a bandwidth reduction of $1 / 2$. $P$ watts $=\left(N_{0}\right.$ $\mathrm{W} / \mathrm{Hz})^{*}\left(\mathrm{~B}_{\mathrm{n}}\right)$

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## Coupled LC Bandpass Filter

3) Build the Coupled Bandpass filter shown in reference section and set the scope bandwidth to 20 MHz bandwidth.
a) Use the scope to measure the center frequency and bandwidth of the filter using a swept sinewave. Record the $-3 \mathrm{~dB},-6 \mathrm{~dB},-10 \mathrm{~dB},-20 \mathrm{~dB}$, and -40 dB frequencies.
b) Set the oscilloscope to 2us/div with 20 MHz bandwidth, the generator output for noise with maximum amplitude. The noise voltage at the output of the filter is much lower since the bandwidth of the filter is small, trigger in the center of the screen, and set the trigger level as high as possible to get a trace. Setting the trigger for single trace mode helps. Note how the signal looks like a sinewave of the center frequency of the filter with amplitude that varies at a much slower rate. This is similar to an AM modulated waveform (Couch fig 5.1). Ask for help from the TA for an explanation of the waveform and setting the trigger correctly.
c) What is the approximate frequency of the sine wave? The is related to the center frequency of the filter. Estimate the highest frequency in the envelope of the sinewave. This is related to the BW of the filter.

## X3 Frequency Multiplier and Filtered Signals

4) Change the generator to a square wave with a frequency equal to the center frequency of the filter and connect a probe to the input of the filter to display both the input and output waveforms on the oscilloscope.
a) The output of the filter should be a sinewave at the center frequency of the filter. Record the amplitude of the output sinewave and compare it to what you would expect from the square wave input. Explain any error. (hint, the filter allows some of the other harmonics through)
b) Plot the Spectrum of the input and output signals. Consider the first 3 spectral components of the output in your for your plot and calculate the distortion of the output sinewave.
c) Change the generator frequency to $1 / 3$ the center frequency of the filter. The output of the filter will be a sinewave at 3 times the generator frequency with distortion of $f / 3$, $5^{*} f / 3,7^{*} f / 3 \ldots$. This principal of frequency multiplication is used in communication circuitry in transmitters and in receivers.

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## REFERENCE SOURCES

For Gaussian noise:

- The mean $m$, (DC voltage) is zero $\langle\mathrm{v}(\mathrm{t})>$
- The variance $\sigma^{2}$ is the ac noise power $\left\langle v(t)^{2}\right\rangle-\langle v(t)\rangle^{2}$
- The standard deviation $\sigma$, is the rms noise voltage
- The peak to peak value of the noise voltage is $+/-\mathrm{k} \sigma$ where k is a measure of how "Gaussian" the noise is. $k$ is typically 2.5 to 3 . Have you heard of $6 \sigma$ limits before?

RMS Harmonic Distortion in a sinewave:
$D \%=\sqrt{\frac{P_{\text {distortion }}}{P_{\text {desired } \text { sin wave }}}}=\sqrt{\frac{\sum_{n=2}^{k} V_{n}^{2}}{V_{1}^{2}} \bullet 100} \quad \begin{aligned} & \mathrm{V}_{1} \text { is the desired sinewave amplitude } \\ & \begin{array}{l}\mathrm{V}_{\mathrm{n}} \text { are the amplitudes of all the harmonics other } \\ \text { than the desired sinewave }\end{array}\end{aligned}$

EQUIVELENT NOISE BANDWITH FOR BUTTERWORTH FILTERS

$$
\begin{gathered}
\mathbf{B}=\frac{\text { Noise_Power_From_Real_Filter_With_3db_Bandwidth_of_1 }}{\text { Noise_Power_From_Ideal_Filter_With_Bandwidth_of_1 }} \\
\mathbf{n}:=\mathbf{1} . .6 \text { filter orders from } 1 \text { to } 6
\end{gathered}
$$

$$
B_{n}:=\frac{\int_{0}^{\infty} \frac{1}{1+(f)^{2 \cdot n}} d f}{\infty^{\infty}}
$$

$$
\int_{0}^{\infty} \frac{1}{1+(f)^{2}} d f=\frac{1}{2} \cdot \pi
$$

$$
\int_{0}^{\infty} \Pi(f, 1)^{2} d f
$$

First order filter with a 3 dB bandwidth of 1 passes $57 \%$ more noise power when compared with an ideal filter with a bandwidth of 1. A 3rd order filter only passes $4.7 \%$ more noise power when compared with an ideal filter.

| $\mathbf{n}=$ | $\mathbf{B}_{\mathbf{n}}=$ |
| :--- | :--- |
| 1 | 1.571 |
| 2 | 1.111 |
| 3 | 1.047 |
| 4 | 1.026 |
| 5 | 1.017 |
| 6 | 1.012 |

$$
\frac{\pi}{2}=1.571
$$

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Coupled Bandpass Filter

| PARAMETERS: <br> Cvar $=220 \mathrm{p}$ |  |  |
| :---: | :---: | :---: |
| Generator |  | Scope or SA |

Frequency response dB

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## 1) Generator Noise in Time and Frequency Domains

a) Measure the Vp-p, Vrms, and VDC magnitude of the noise waveform using the oscilloscope. Be sure the noise is not clipping. Hint(use the run stop button)
i) Why is it not reading $2 \mathrm{vp}-\mathrm{p}$ ? Calculate the Vp-p to Vrms ratio. What do we use this ratio for?
b) Measured Spectrum Analyzer (SA) 3dB bandwidth of the noise: Sketch the noise spectrum from 1 MHz to 100 MHz using the data you have. (assume one pole lowpass shape for the noise density.) Identify points for which you have data. Provide information for the axis from your SA settings.

Power dBm RBW:

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c) Record:
i) the SA resolution bandwidth (RBW)
ii) power $\mathrm{P}(\mathrm{f})=\mathrm{dBm} / \mathrm{RBW}$
iii) power/Hz $\mathrm{P}(\mathrm{f})=\mathrm{dBm} / \mathrm{Hz}$ at 10 MHz
d) (Sorry, this was a duplicate of 1b)
e) Calculate the power you would measure for Cii ) if the resolution bandwidth was increased by a factor of 10 ?

## 2) RC low pass filter

a) Measure the 3 dB bandwidth of the filter using the oscilloscope (hi-Z input) and a variable frequency sinewave. Compare this to the theoretical 3dB bandwidth.
b) Set the generator back to $2 \mathrm{Vp}-\mathrm{p}$ noise and look at the signal on the SA. Notice that the 3 dB bandwidth of the noise spectrum has been reduced by the 3db bandwidth of the low pass filter.

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c) Re-measure the Vp -p and Vrms at the filter output using the settings from part 1a. Why did the measured values change? (Explain by showing what happens to a signal going through a low pass filter- LPF using $\mathrm{N}_{0}$ and $\mathrm{H}(\mathrm{f})$ )
d) Reduce the 3 dB bandwidth of the filter by $1 / 2$ by using 440 pF of capacitance and remeasure Vp-p and Vrms. Compare these voltages with the voltages measured in 2c by using your voltage values measured in 2 c to calculate what you expect the values to be for a bandwidth reduction of $1 / 2$. Remember P watts $=\mathrm{N}_{0}{ }^{*} \mathrm{~B}_{\mathrm{n}}$ reducing $\mathrm{B}_{\mathrm{n}}$ to $\mathrm{B}_{\mathrm{n}} / 2$ should reduce the rms voltage by $\frac{1}{\sqrt{2}}$. How closely do the measured voltages compare to the ideal voltage? Calculate the error.

## 3) Coupled LC Bandpass Filter

a) Record the $-3 \mathrm{~dB},-6 \mathrm{~dB},-10 \mathrm{~dB},-20 \mathrm{~dB}$, and -40 dB frequencies. Make a table and a sketch of the filter response identifying your measured value values.

| Level dB | Lower Frequency | Upper Frequency | Notes: |
| :---: | :---: | :---: | :---: |
| $\mathbf{0 ~ d B}$ |  |  |  |
| $-\mathbf{3 ~ d B}$ |  |  |  |
| $-\mathbf{- 6 ~ d B}$ |  |  |  |
| $\mathbf{- 1 0 ~ d B}$ |  |  |  |
| $-\mathbf{- 2 0 ~ d B}$ |  |  |  |
| $-\mathbf{- 4 0 ~ d B}$ |  |  |  |
| 3 of 6 |  |  |  |

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Frequency

## Filter Frequency Response

b) Use what you found to answer c)
c) What is the approximate frequency of the sine wave? This is related to the center frequency of the filter. Estimate the highest frequency in the envelope of the sinewave. This is related to the BW of the filter.

## 4) X3 Frequency Multiplier and Filtered Signals

a) The output of the filter should be a sinewave at the center frequency of the filter.

Record the amplitude of the output sinewave and compare it to what you would expect from the square wave input. Explain any error. (hint, the filter allows some of the other harmonics through)

Theoretical amplitude of fundamental 2 Vp -p square wave from previous labs:

Measured amplitude from oscilloscope:

Calculated error: Explanation of error:

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b) Plot the Spectrum of the input and output signals. Consider the first 3 spectral components of the output plot and calculate the distortion of the output sinewave. You may use the following table for your data.

| $\mathbf{n}$ | Input dBV | Output dBV | Ouput $\mathbf{V}_{\text {rms }}$ | notes |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ (fundamental) |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

Note that the amplitude difference in dB of the input and output levels is equal to the insertion loss of the filter
$\underset{\text { Amplitude }}{\perp}$

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c) Change the generator frequency to $1 / 3$ the center frequency of the filter. The output of the filter will be a sinewave at 3 times the generator frequency with distortion of $f / 3$, $5^{\star} f / 3,7^{\star} f / 3 \ldots$ This principal of frequency multiplication is used in communication circuitry in transmitters and in receivers.

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