EELE445 Telecommunications Lab 5, Spring 2015
A/D Conversion and Quantization Noise

Purpose

The purpose of the lab is to demonstrate the signal analysis capabilities of Matlab. The oscilloscope will be used as an A/D converter to capture several signals we have examined in previous labs. These signals will then be examined and analyzed in Matlab. Effects of A/D quantization noise will be seen.

Reference Sources

Proakis chapter 7.4.1, equations 7.4.1 to 7.4.3 as attached

Prelab

- **YOU WILL NEED thumb drive**
- What is the quantization noise power P_q (eq 7.4.2) for an 8-bit A/D with an input range of +1V to -1V? (n=8 and x_{max}=1V)
- Does this noise power depend on the sample frequency? Explain
- The power spectral density, PSD, of the noise is $S_q = \frac{P_q}{F_s/2}$ Watts/Hz. This is the quantization noise floor of the A/D. What is the noise floor PSD when F_s is 10Ks/sec?
- What would the Signal to quantization noise ratio SQNR be if the input signal is a 2Vp-p sine wave, x_{max}=1V, and n=8 bits? (SQNR=P_{signal}/P_q)

Capture the following signals on a floppy disk as csv files for analysis in Matlab. You may do the data analysis at your leisure before the next lab.

1. Set the oscilloscope for a sample rate of 100 Ks/sec, 10,000 points, and 1mV/div. Disconnect the probe from the scope. This is the minimum resolution of the scope.
    - Record the mean, Pk-Pk, and RMS voltages using the scope measurements
    - **Capture the trace to a .csv type data file**
    - Write a Matlab file to read the data file and:
        - Plot the data in time
        - Find the mean, Pk-Pk and RMS noise voltages. Compare the results to what you recorded on the oscilloscope.
        - Plot the power spectral density (PSD) in Watts/Hz. Be sure to correct for the resolution bandwidth of the FFT. You may assume R=1
        - Estimate the Δ (voltage resolution of the A/D) by finding the minimum non-zero voltage change between points.
        - The full scale range of the A/D is Δ x 512 (9-bit). How does this compare to the full scale range of 8 mV as seen on the scope screen? Explain

2. Set the oscilloscope for a sample rate of 100 Ks/sec, 10,000 points, and 200mV/div. Disconnect the probe from the scope.
   - **Capture the trace to a data file**
   - Repeat part 1 using this data.

3. Set the oscilloscope for a sample rate of 100 Ks/sec, 10,000 points, and 200mV/div. Now connect a signal generator and apply a 1 KHz sine wave of 1 Vp-p.
   - **Capture the trace to a data file.**
   - Repeat part 1 using this data.
   - Estimate the full scale range of the scope, +/1 X_{max}, under these settings. How does this compare to the displayed +/- 800 mV signal on the scope display.
   - Estimate the dynamic range of the scope in dB, $SNQR=20*\log(X_{max}/V_{noise})$. V_{noise} is the no input rms noise voltage from 2.
   - Compare the level of the noise you expect to see for the sine wave with what you see in the FFT plot. Explain. (Hint: Resolution Bandwidth)

4. Set the oscilloscope for a sample rate of 100 Ks/sec, 10,000 points, and 200mV/div. Now connect a signal generator and apply a 1 KHz sine wave of 2 Vp-p.
   - **Capture the trace to a data file.**
   - Repeat part 1 using this data.
   - Is the sine wave distorted in the Matlab plots? Explain

5. Change the input waveform to a 100 KHz triangle waveform 2.5 Vp-p
   - **Capture the trace to a data file.**
   - Repeat part 1 using this data.
   - Is the triangle wave distorted in the Matlab plots? Use this to find the pk-pk range of the A/D.

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7.4.1 Pulse Code Modulation (PCM)

Pulse code modulation is the simplest and oldest waveform coding scheme. A pulse code modulator consists of three basic sections: a sampler, a quantizer and an encoder. A functional block diagram of a PCM system is shown in Figure 7.7. In PCM, we make the following assumptions:

1. The waveform (signal) is bandlimited with a maximum frequency of W. Therefore, it can be fully reconstructed from samples taken at a rate of $f_s = 2W$ or higher.
2. The signal is of finite amplitude. In other words, there exists a maximum amplitude $x_{\max}$ such that for all t, we have $|x(t)| \leq x_{\max}$.
3. The quantization is done with a large number of quantization levels N, which is a power of 2 ($N = 2^\nu$).

The waveform entering the sampler is a bandlimited waveform with the bandwidth W. Usually, there exists a filter with bandwidth W prior to the sampler to prevent any components beyond W from entering the sampler. This filter is called the presampling filter. The sampling is done at a rate higher than the Nyquist rate; this allows for some guardband. The sampled values then enter a scalar quantizer. The quantizer is either a uniform quantizer, which results in a uniform PCM system, or a nonuniform quantizer. The choice of the quantizer is based on the characteristics of the source output. The output of the quantizer is then encoded into a binary sequence of length ν, where $N = 2^\nu$ is the number of quantization levels.

Uniform PCM. In uniform PCM, we assume that the quantizer is a uniform quantizer. Since the range of the input samples is $[-x_{\max}, +x_{\max}]$ and the number of quantization levels is N, the length of each quantization region is given by

$$\Delta = \frac{2x_{\max}}{N} = \frac{x_{\max}}{2^{\nu-1}}. \qquad (7.4.1)$$

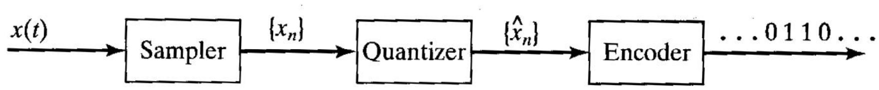

Figure 7.7 Block diagram of a PCM system.

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The quantized values in uniform PCM are chosen to be the midpoints of the quantization regions; therefore, the error $\tilde{x} = x - Q\,(x)$ is a random variable taking values in the interval $(-\frac{\Delta}{2}, +\frac{\Delta}{2}]$. In ordinary PCM applications, the number of levels (N) is usually high and the range of variations of the input signal (amplitude variations $x_{\max}$) is small. This means that the length of each quantization region (Δ) is small. Under these assumptions, in each quantization region, the error $\tilde{X} = X - Q\,(X)$ can be approximated by a uniformly distributed random variable on $(-\frac{\Delta}{2}, +\frac{\Delta}{2}]$. In other words,

$$f(\tilde{x}) = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} \le \tilde{x} \le \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases}.$$

The distortion introduced by quantization (quantization noise) is therefore

$$\mathrm{P_q}= \quad E[\tilde{X}^2] = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta}\tilde{x}^2 d\tilde{x} = \frac{\Delta^2}{12} = \frac{x_{\max}^2}{3N^2} = \frac{x_{\max}^2}{3 \times 4^{\nu}}, \tag{7.4.2}$$

where ν is the number of bits/source sample and we have employed Equation (7.4.1). The signal-to-quantization noise ratio then becomes

$$\mathrm{SQNR} = \frac{P_X}{\overline{\tilde{X}^2}} = \frac{3 \times N^2 P_X}{x_{\max}^2} = \frac{3 \times 4^{\nu} P_X}{x_{\max}^2}, \tag{7.4.3}$$

where P_X is the power in each sample. In the case where $X(t)$ is a wide-sense stationary process, P_X can be found using any of the following relations:

$$\begin{aligned} P_X &= R_X(\tau)_{|_{\tau=0}} \\ &= \int_{-\infty}^{\infty} S_X(f)\,df \\ &= \int_{-\infty}^{\infty} x^2 f_X(x)\,dx. \end{aligned}$$

Note that since $x_{\max}$ is the maximum possible value for X, we always have $P_X = E[X^2] \le x_{\max}^2$. This means that $\frac{P_X}{x_{\max}^2} < 1$ (usually $\frac{P_X}{x_{\max}^2} \ll 1$); hence, $3N^2 = 3 \times 4^{\nu}$ is an upperbound to the SQNR in uniform PCM. This also means that SQNR in uniform PCM deteriorates as the dynamic range of the source increases because an increase in the dynamic range of the source results in a decrease in $\frac{P_X}{x_{\max}^2}$.

Expressing SQNR in decibels, we obtain

$$\mathrm{SQNR}\big|_{\mathrm{dB}} \approx 10\log_{10}\frac{P_X}{x_{\max}^2} + 6\nu + 4.8. \tag{7.4.4}$$

We can see that each extra bit (increase in ν by one) increases the SQNR by 6 dB. This is a very useful strategy for estimating how many extra bits are required to achieve a desired SQNR.

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Example 7.4.1

What is the resulting SQNR for a signal uniformly distributed on $[-1, 1]$, when uniform PCM with 256 levels is employed?

Solution We have $P_X = \int_{-1}^{1} \frac{1}{2}x^2 dx = \frac{1}{3}$. Therefore, using $x_{\max} = 1$ and $\nu = \log 256 = 8$, we have

$$\text{SQNR} = 3 \times 4^{\nu} \times P_X = 4^{\nu} = 65536 \approx 48.16 \text{ dB}. \qquad \blacksquare$$

The issue of bandwidth requirements of pulse transmission systems, of which PCM is an example, is dealt with in detail in Chapter 9. In this chapter, we briefly discuss some results concerning the bandwidth requirements of a PCM system. If a signal has a bandwidth of W, then the minimum number of samples for perfect reconstruction of the signal is given by the sampling theorem, and it is equal to $2W$ samples/sec. If some guardband is required, then the number of samples per second is f_s, which is more than $2W$. For each sample, ν bits are used; therefore, a total of νf_s bits/sec are required for transmission of the PCM signal. In the case of sampling at the Nyquist rate, this is equal to $2\nu W$ bits/sec. The minimum bandwidth requirement for binary transmission of R bits/sec (or, more precisely, R pulses/sec) is $\frac{R}{2}$. (See Chapter 9.)[2] Therefore, the minimum bandwidth requirement of a PCM system is

$$\text{BW}_{\text{req}} = \frac{\nu f_s}{2}, \tag{7.4.5}$$

which, in the case of sampling at the Nyquist rate, gives the absolute minimum bandwidth requirement as

$$\text{BW}_{\text{req}} = \nu W. \tag{7.4.6}$$

This means that a PCM system expands the bandwidth of the original signal by a factor of at least ν.

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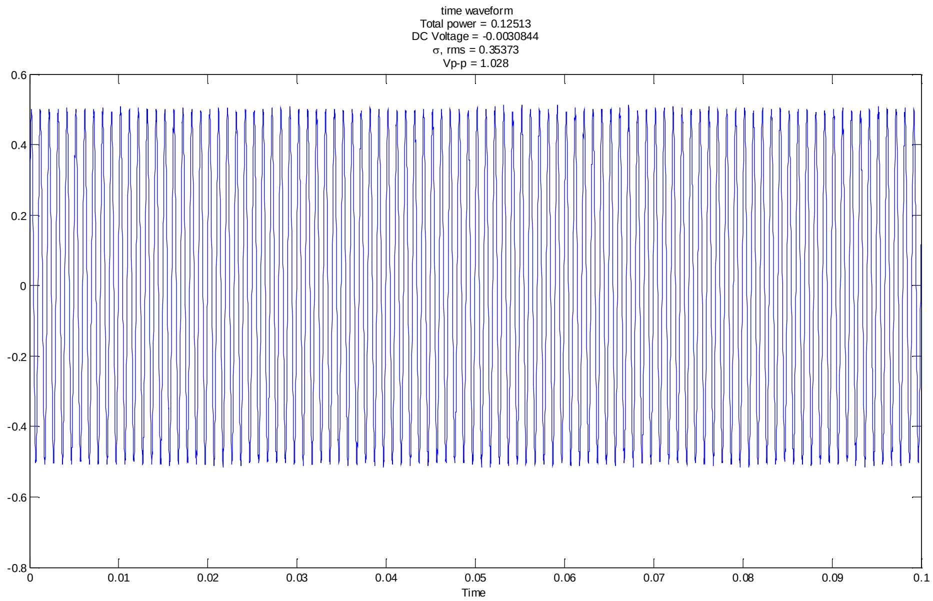
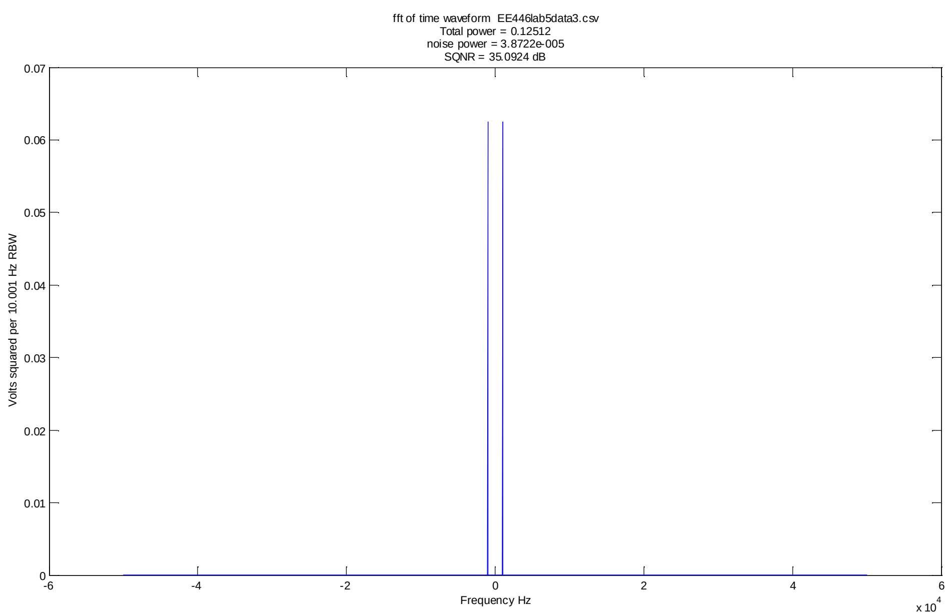
Sample data analysis from Matlab program:

Screen shots from EELE44512lab5.m Matlab file. You should see something similar.





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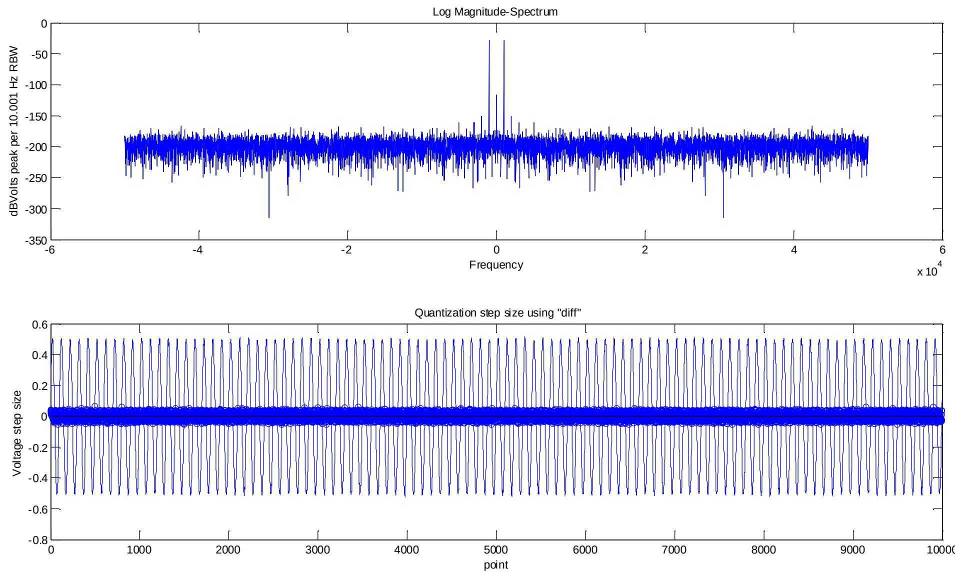

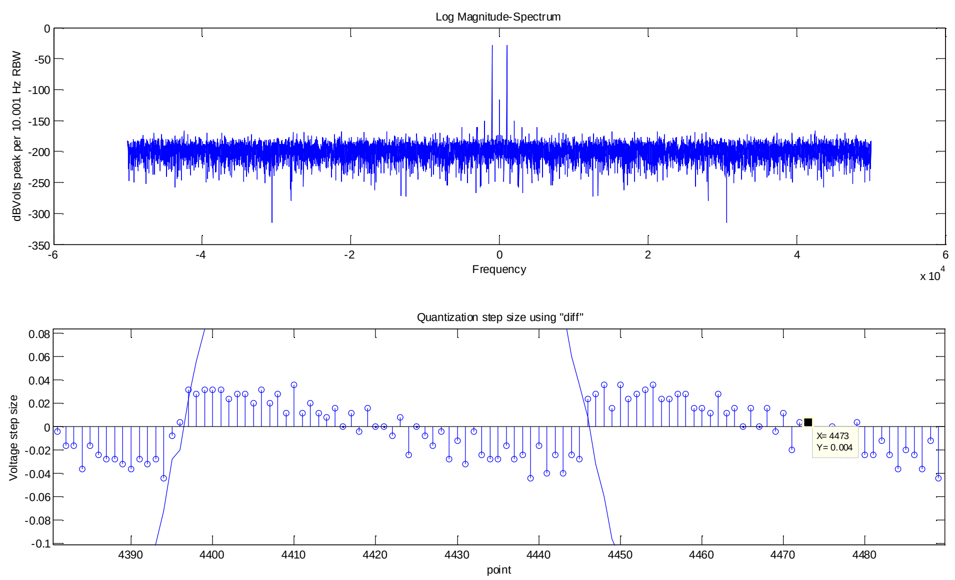

Zoom was used on the above graph to see the individual quantization steps. 4mV in this case.