EELE445 lab 9: FM Modulation and Occupied Bandwidth

Purpose

We consider the bandwidth of a Frequency Modulated (FM) carrier when it is modulated by a 1 KHz sinewave. The modulation index for FM is defined to be:

$$\beta = \Delta F_{pk} / (F_m)$$

Where ΔF_{pk} is the peak frequency <u>deviation</u> of the carrier frequency, and F_m is the frequency of the modulating sinewave.

The amplitude of the spectral components of the modulated signal are determined by Bessel functions.

 $J_0(\beta)$ is the carrier amplitude

 $J_1(\beta)$ is the amplitude of the modulating frequency sidebands, F_c +/- F_m

 $J_2(\beta)$ is the amplitude of the sidebands at F_c +/- $2F_m$

$$s(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos\left[\left(\omega_c + n\omega_m\right)t\right]$$

The occupied bandwidth for 98% of the transmitted power is determined from Carson's Rule:

$$BW = 2(\beta + 1)f_m$$

Reference Sources

Couch Chapter 5-6, Table 5-2,3, figure 5-11, eq 5-61 and Bessel handout for lab 10

Prelab:

- Define narrow band FM.
- Determine the magnitude of the spectral components you will see on a spectrum analyzer for a modulating frequency of f_m= 1 KHz with a peak frequency deviation of ∆f=600 Hz and for a peak deviation of ∆f=2.4 KHz. (You may use the FM sideband calculator on the website or modify Couch E5_60m Matlab file.) The carrier frequency is 12 MHz.
- Understand how to apply Carson's rule to predict the occupied bandwidth of the signal. (Bessel function order for Carson $n_{max} = \beta + 1$)

Method:

Narrowband FM:

- 1. Set the generator for a 12 MHz, 2volt p-p into 50 ohms sinewave output with no modulation. Connect the generator to the spectrum analyzer (SA) through the 3dB attenuator. Verify that the generator output is about 10dBm. Remember that the generator output is: <u>SApower dBm+ attenuator loss of 3dB</u>.
 - Record the actual power for use in the following steps for use as the total s(t) power. Be sure to record the unmodulated carrier power level in dBm as accurately as possible to minimize your measurement error.
- 2. Turn on the FM sinewave modulation with a frequency of 1KHZ and a deviation of 100 Hz.
 - Record the levels in dBm of the carrier, first and second sidebands.
 - What is the measured occupied bandwidth of the measured signal? (approx. 98% of total signal power)
 - How does this compare to AM modulation?
 - What does Carson's rule predict?
- 3. Using the setup from part 1, increase the frequency deviation of the modulation to 600 Hz.
 - Record the levels in dBm of the carrier, first and second sideband pairs.
 - Use Bessel functions to calculate the theoretical power of the spectrum and compare your measured amplitudes with the predicted amplitudes.
 - Does this still qualify as narrowband FM?

J_0 Carrier Null for β =2.4 and J_1 Null for β =3.83:

- 4. Using the setup from part 1, continue to increase the frequency deviation of the 1 KHz modulating signal in 100 Hz steps and watch the amplitude of the carrier.
 - At what deviation does the carrier amplitude disappear?
 - What is the measured modulation index, how does it compare to the theoretical $J_0(2.4)$ zero crossing?
- 5. Set the frequency deviation to 2.4 KHz.
 - Record the levels of the sidebands and compare them to the levels predicted by the Bessel functions.
 - Calculate the percent of total power <u>measured</u> in the spectrum in Carson's rule.
 - How does this compare to the bandwidth predicted by Carson's rule?
 - How does the bandwidth compare to AM?
- 6. Continue to increase the deviation of the modulating signal in 100 Hz steps until the 1KHz sidebands decrease to a minimum power.
 - Sketch the spectrum and record the deviation where the J_1 term has zero amplitude.
 - Identify the approximate occupied bandwidth on your plots and compare this value calculated by Carson's rule.
- 7. Continue to increase the deviation of the modulating signal until the J_2 terms minimize.
 - Sketch the spectrum and record the deviation.
 - Does this β correspond to the Bessel chart J₂ null? Give a reason for the error, if the null does not appear at the correct β .

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Figure 5-10 Bessel functions for n = 0 to n = 6.

TABLE 5-3 ZEROS OF BESSEL FUNCTIONS: VALUES FOR β WHEN $J_n(\beta) = 0$

		Order of Bessel Function, n					
	0	1	2	3	4	5	6
β for 1st zero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β for 2nd zero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β for 3rd zero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β for 4th zero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β for 5th zero	14.93	16.47	17.96	19.41	20.83	22.21	23.59
β for 6th zero	18.07	19.61	21.12	22.58	24.02	25.43	26.82
β for 7th zero	21.21	22.76	24.27	25.75	27.20	28.63	30.03
β for 8th zero	24.35	25.90	27.42	28.91	30.37	31.81	33.23

Note: underlined values in table 5-2 correspond to Carson's Bandwidth Rule

TABLE 5-2 FOUR-PLACE VALUES OF THE BESSEL FUNCTIONS $J_n(\dot{\beta})$

β: n	0.5	l	2	3	4	5	6	7	8	9	10
0	0.9385	0.7652	0.2239	-0.2601	-0.3971	-0.1776	0.1506	0.3001	0.1717	-0.09033	-0.2459
I	<u>0.2423</u>	0.4401	0.5767	0.3391	-0.06604	-0.3276	0.2767	0.004683	0.2346	0.2453	0.04347
2	0.03060	0.1149	0.3528	0.4861	0.3641	0.04657	-0.2429	-0.3014	-0.1130	0.1448	0.2546
3	0.002564	0.01956	0.1289	0.3091	0.4302	0.3648	0.1148	-0.1676	0.2911	0.1809	0.05838
4		0.002477	0.03400	<u>0.1320</u>	0.2811	0.3912	0.3576	0.1578	-0.1054	-0.2655	-0.2196
5			0.007040	0.04303	0.1321	0.2611	0.3621	0.3479	0.1858	-0.05504	-0.2341
6			0.001202	0.01139	0.04909	<u>0.1310</u>	0.2458	0.3392	0.3376	0.2043	-0.01446
7				0.002547	0.01518	0.05338	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.004029	0.01841	0.05653	0.1280	0.2235	0.3051	0.3179
9						0.005520	0.02117	0.05892	0.1263	0.2149	0.2919
10						0.001468	0.006964	0.02354	0.06077	0.1247	0.2075
11							0.002048	0.008335	0.02560	0.06222	0.1231
12								0.002656	0.009624	0.02739	0.06337
13									0.003275	0.01083	0.02897
14									0.001019	0.003895	0.01196
15										0.001286	0.004508
16											0.001567

EELE445 - LAB 9 Student Report

Section	Name
Date:	Lab Partner:

Narrowband FM:

- 1. Remember that the generator output is: <u>SApower dBm+ attenuator loss of 3dB</u>.
 - Record the actual power for use in the following steps for use as the total s(t) power. Be sure to record the unmodulated carrier power level in dBm as accurately as possible to minimize your measurement error.
- 2. $f_m=1$ KHZ and a deviation of $\Delta f=100$ Hz.
 - Record the levels in dBm of the carrier, first and second sideband pairs.
 - What is the measured occupied bandwidth of the measured signal? (approx. 98% of total signal power)
 - How does this bandwidth compare to AM modulation?
 - What does Carson's rule predict for the bandwidth?
- 3. $f_m=1$ KHZ and a deviation of $\Delta f=600$ Hz.
 - Record the levels in dBm of the carrier, first and second sideband pairs.
 - Use Bessel functions to calculate the theoretical power of the spectrum and compare your measured amplitudes with the predicted amplitudes.
 - Does this still qualify as narrowband FM?

J₀ Carrier Null for β =2.4 and J₁ first sideband Null for β =3.83:

- 4. Using the setup from part 1, continue to increase the frequency deviation of the 1 KHz modulating signal in 100 Hz steps and watch the amplitude of the carrier.
 - At what deviation is the carrier amplitude a minimum?
 - What is the measured modulation index? Compare this to the theoretical carrier null that occurs at $J_0(2.4)=0$
- 5. Set the frequency deviation to $\Delta f=2.4$ KHz.
 - Record the levels of the sidebands and compare them to the levels predicted by the Bessel functions.
 - Calculate the percent of total power <u>measured</u> in the spectrum in Carson's rule.
 - How does this compare to the theoretical power predicted by Carson's rule?
 - How does the bandwidth compare to AM?
- 6. Continue to increase the deviation in 100 Hz steps until the J_1 terms minimize.
 - Sketch the spectrum and record the deviation where the J_1 terms are minimize.
 - Identify the approximate occupied bandwidth on your plots and compare this value calculated by Carson's rule.
- 7. Continue to increase the deviation of the modulating signal until the J_2 terms minimize.
 - Sketch the spectrum and record the deviation.
 - Does this β correspond to the Bessel chart J₂ null? Give a reason for the error, if the null does not appear at the correct β .