The Free Space Wireless Link
(Text Section 8-5)

EELE445
Lecture 39

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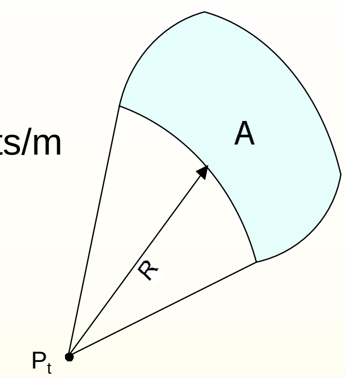
Isotropic Radiator

- Power is radiated equally in all directions
- F= flux density in Watts/m^2
- Also use Field strength, F_s= Volts/m

$$F = \frac{P_t}{4\pi R^2} \frac{Watts}{m^2}$$



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Flux Density and Field Strength

$$F = \frac{P_t}{4\pi R^2} \frac{Watts}{m^2}$$

$$Z_o = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi = 377\Omega$$

$$F_s = \sqrt{F \bullet Z_o} = \sqrt{\frac{P_t 120\pi}{R^2 4\pi}} = \sqrt{\frac{30 P_t}{R^2}} \frac{Volts}{meter}$$

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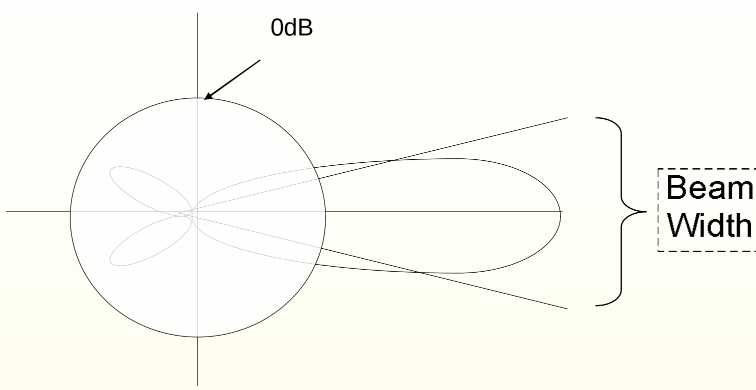
Real Antenna – Not Isotropic
Polar Plot of Normalized F



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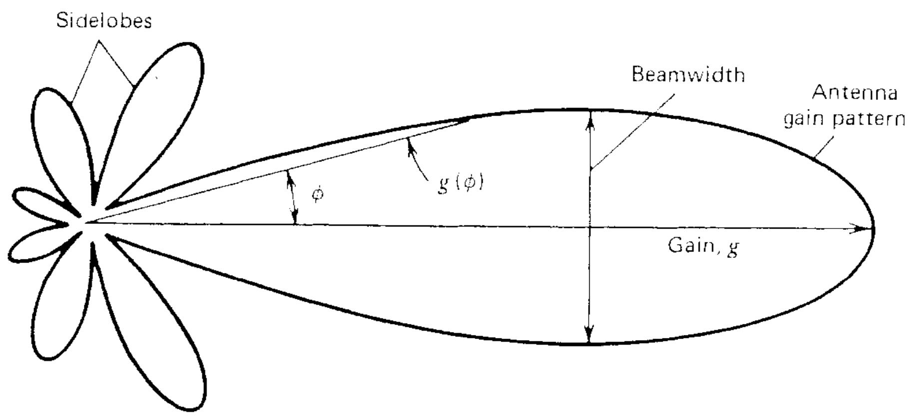
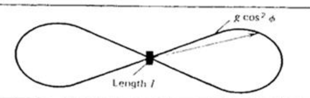
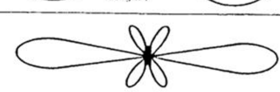
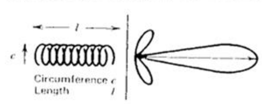
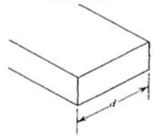
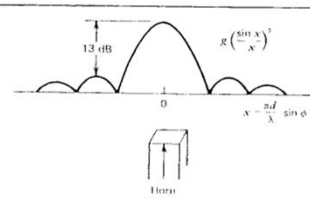
Polar Antenna Beam Pattern



Some common antenna gains

Antenna type	Pattern	Gain g	Half-power beamwidth
Short dipole $l \ll \lambda$		1.5	90°
Long dipole $l \gg \lambda$ $l = \lambda/2$		1.5 1.64	47° 78°
Helix		$15\left[\frac{cl}{\lambda^2}\right]$	$52°\left[\frac{\lambda}{c\sqrt{l}}\right]$
Square horn dimension d 		$\frac{4\pi d^2}{\lambda^2}$	$\frac{0.88\lambda}{d}$ rad

Some common antenna gains

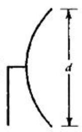
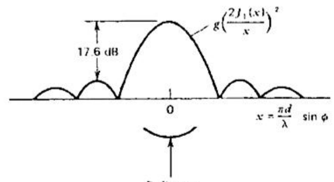
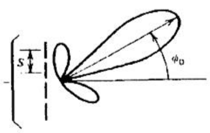
Circular reflector 		$\left[\frac{\pi d}{\lambda}\right]^2$	$\frac{1.02\lambda}{d}$ rad
Phased array, phase difference $= \frac{2\pi s}{\lambda}\cos\theta_0$		$\left[\frac{N\pi s}{1.4\lambda}\right]^2$	$50°\left[\frac{\lambda}{Ns}\right]$ $\theta_0 = 0$

Figure 3.12 (continued)

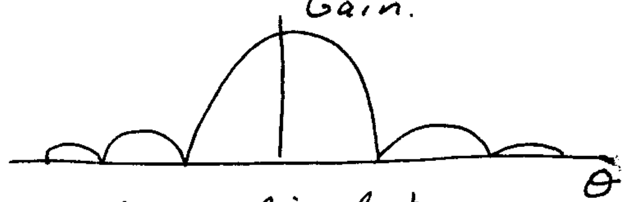
2

From the polar Plot.

$$\frac{F(\theta,\phi)}{F_{isotropic}} = G_t$$ the gain over isotropic

$$G(\theta,\phi) = \frac{P(\theta,\phi)}{\frac{P_0}{4\pi}}$$ steradians.

Linear Plot



for a fixed ϕ

EIRP → Effective Isotropic Radiated Power

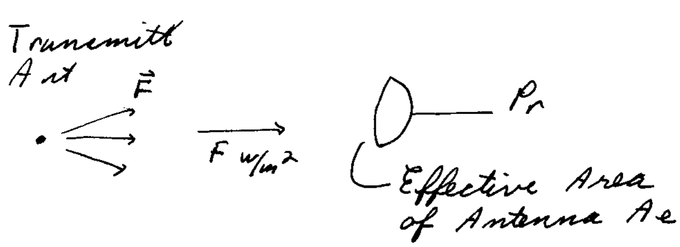
$EIRP \triangleq P_t G_t$ watts/m²

3

Field Strength with directional Antenna:

$$F = \frac{P_t G_t}{4\pi R^2} \quad w/m^2$$

<u>Receive Antenna</u>



P_r = Flux density × Receive Ant Area.

$P_r = F \cdot A_e$

For a parabolic dish

$A = \pi R^2$

due to spill-over $A_e = n_a A$

$n_a \Rightarrow$ aperature Effeciency

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Free Space Path Loss (Friss loss):

Combining :

$$P_r = \frac{P_t G_t A_e}{4\pi R^2} \; watts$$

P_t = transmitt power in watts

G_t = antenna gain in watts/watt

A_e = effective receive antenna area in meters squared

R = distance between transmitt and receive antennas in meters

It can be shown that :

$$G = \frac{4\pi A_e}{\lambda^2}$$ where λ is the wavelength in meters

Think of this as the number of wavelengths the area of the antenna intersects

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Free Space Path Loss (Friss loss):

given A_e and f, we know the gain

$$A_e = \frac{G_r \lambda^2}{4\pi} \qquad P_r = \frac{P_t G_t G_r}{4\pi R^2} \frac{\lambda^2}{4\pi}$$

$$P_r = \frac{P_t G_t G_r}{\left(\frac{4\pi R}{\lambda}\right)^2}$$

free space (Friss) path loss is :

$$L_p \equiv \left(\frac{4\pi R}{\lambda}\right)^2$$

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Free Space Path Loss (Friss loss):

$$P_r = \frac{P_t G_t G_r}{L_p}$$

This is usually written in dB as :

$$P_{rdBm} = P_{tdBm} + G_{tdB} + G_{rdB} - L_{pdB}$$

$$L_{pdB} = 20 Log\left(\frac{4\pi R}{\lambda}\right) = 20 Log\left(\frac{4\pi R f}{c}\right)$$

with R meters , c meters / sec, *f Hz*

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Free Space Path Loss (Friss loss):

$$L_p = 20Log\left(\frac{4\pi Rf}{c}\right)$$

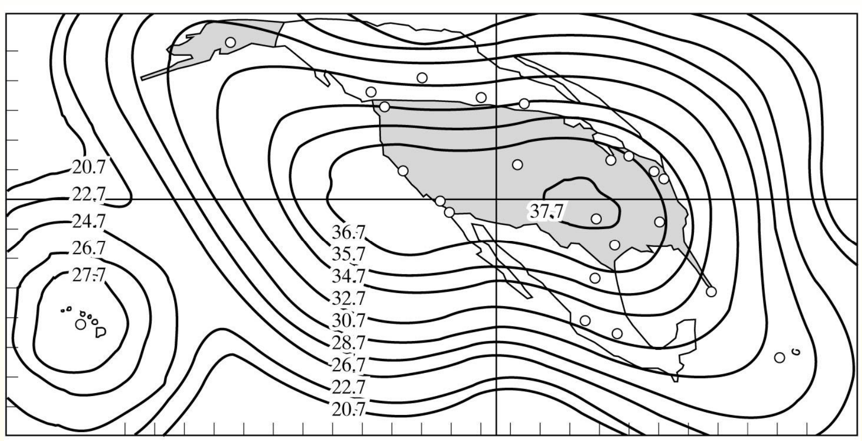
$$= 20Log\left(\frac{4\pi}{c}\right) + 20Log(R) + 20Log(f)$$

For f in GHz, R in Km :

$$L_{pdB} = 92.4 + 20Log(f_{GHz}) + 20Log(R_{Km})$$

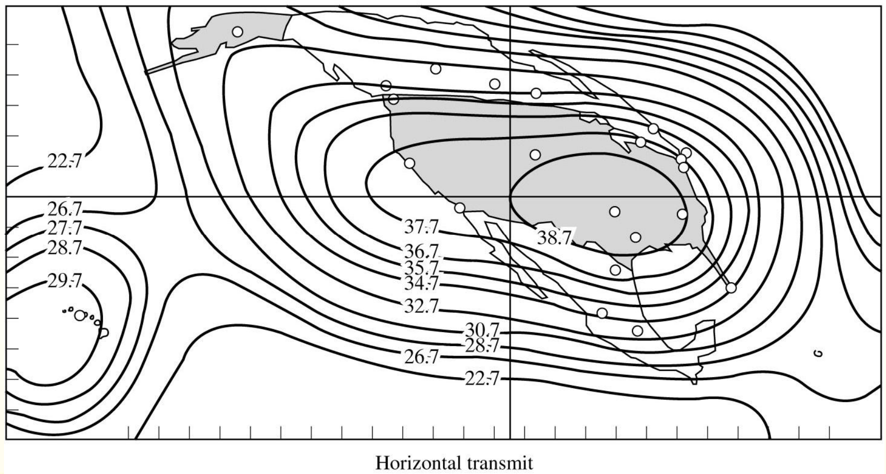
Notice received power : $P_{rdBW} = EIRP_{dBW} + G_{rdB} - L_{pdB}$

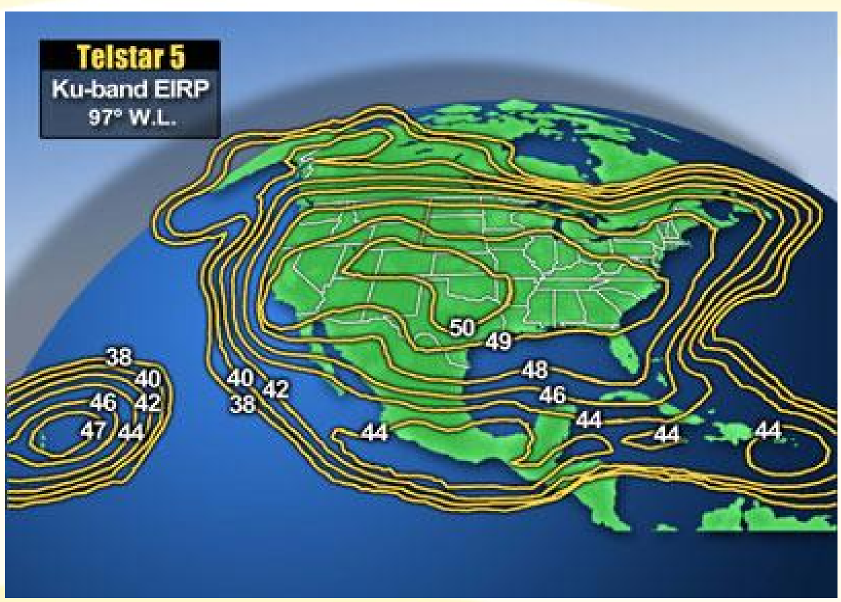
Figure 8–25 *Galaxy I* (134° W longitude) antenna pattern
EIRP footprint contours given in dBw units).
[Courtesy of Hughes Communications, Inc., a wholly owned subsidiary of Hughes Aircraft Company.]



Vertical transmit

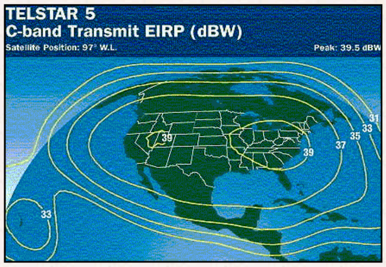
Figure 8–25 *Galaxy I* (134° W longitude) antenna pattern EIRP footprint contours given in dBw units).
[Courtesy of Hughes Communications, Inc., a wholly owned subsidiary of Hughes Aircraft Company.]

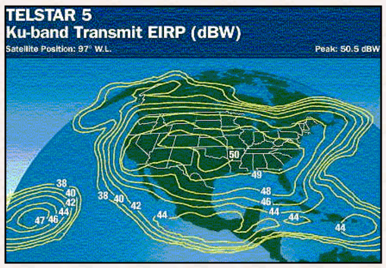




(to Western Hemisphere, Eastern Hemisphere, Footprints by Dish Size)

TELSTAR 5 (w. hemi or size list)





Telstar 5 (at 97.0W)

Dish sizes are minimum for 'Top Grade' reception
(25deg LNB for C-Band, 0.8dB LNB for Ku-Band)

C-Band (Full Transponder Analog)			Ku-Band (3/4 FEC Multiplex, Full Transponder Analog)		
EIRP (dBW)	DTH Size (m)	SMATV Size (m)	EIRP (dBW)	Multiplex Channels (cm)	Analog Channels (cm)
39.0	1.9	2.5	50.0	55	84
38.0	2.1	2.9	49.0	60	94
37.0	2.3	3.1	48.0	65	105
36.0	2.6	3.5	47.0	72	115
35.0	2.8	3.8	46.0	77	125
34.0	3.1	4.2	44.0	95	155
33.0	3.5	4.8	42.0	115	185
32.0	3.9	5.4	40.0	138	230
31.0	4.3	6.1	38.0	175	300

NOTE: SMATV is satellite master antenna television.

C band	4 to 8 GHz
X band	8 to 12 GHz
K_u band	12 to 18 GHz

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Figure 8–8 Communications satellite in geosynchronous orbit.

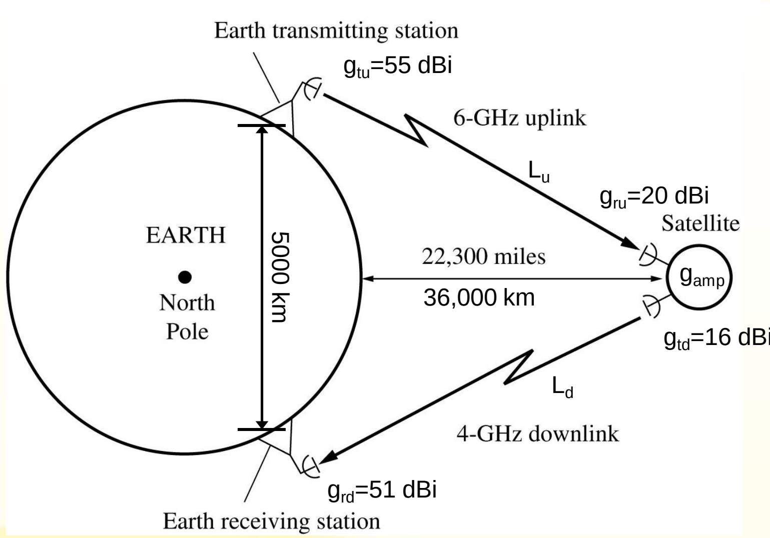

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Figure 8–9 Simplified block diagram of a communications satellite transponder.

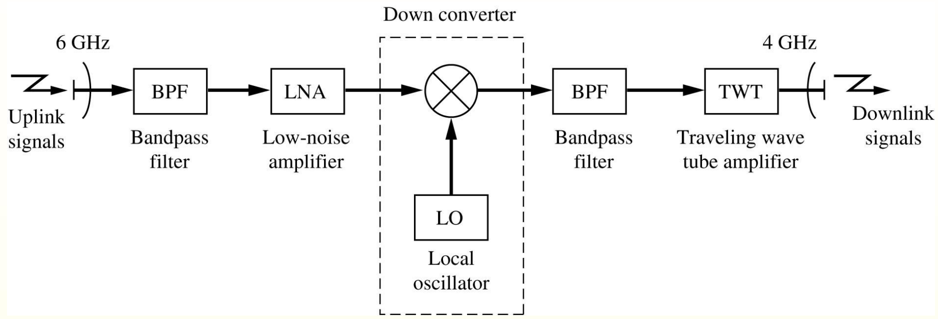
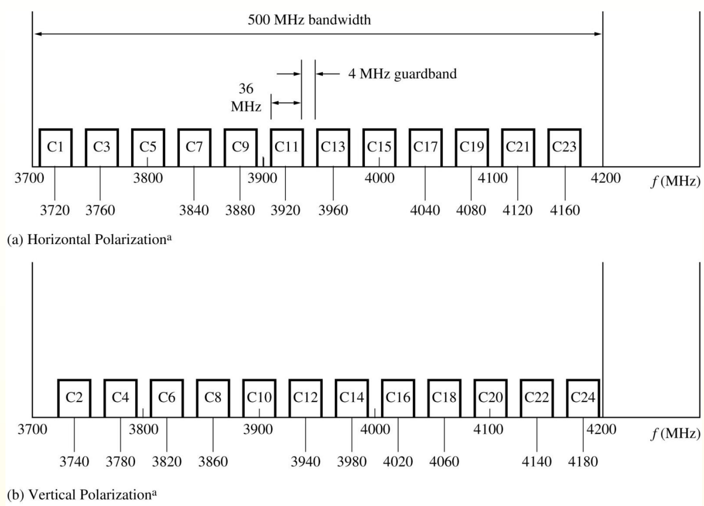


Figure 8–10 64-GHz satellite transponder frequency plan for the downlink channels. (For the uplink frequency plan, add 2225 MHz to the numbers given above.)



[a]These are the polarizations used for the Galaxy Satellites. Some of the other satellites use opposite polarization assignments.

Link Budget
and
Wireless Link Example

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Satellite Relay
The "Bent Pipe"

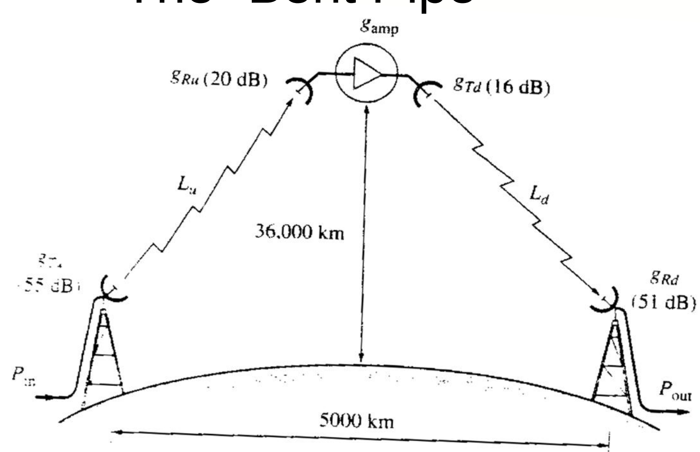

Figure 3.3–5 Satellite relay system.

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Figure 8–8 Communications satellite in geosynchronous orbit.

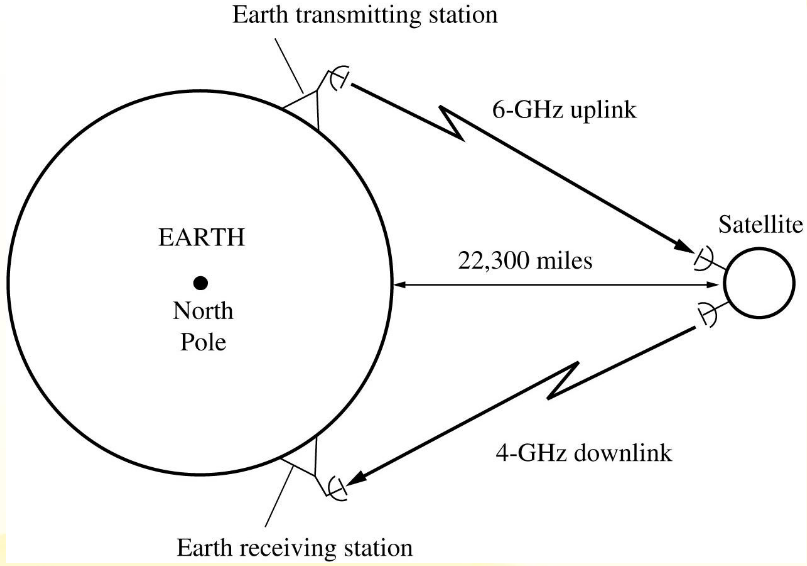
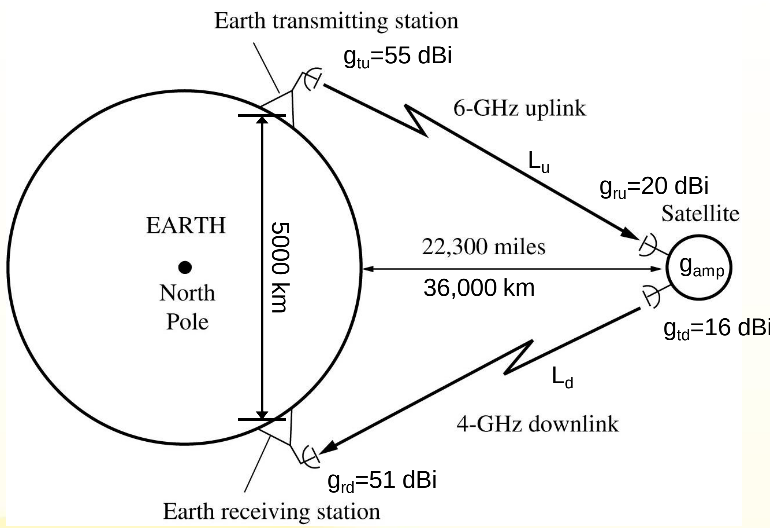


Figure 8–8 Communications satellite in geosynchronous orbit.



Earth Bulge

Earth Bulge

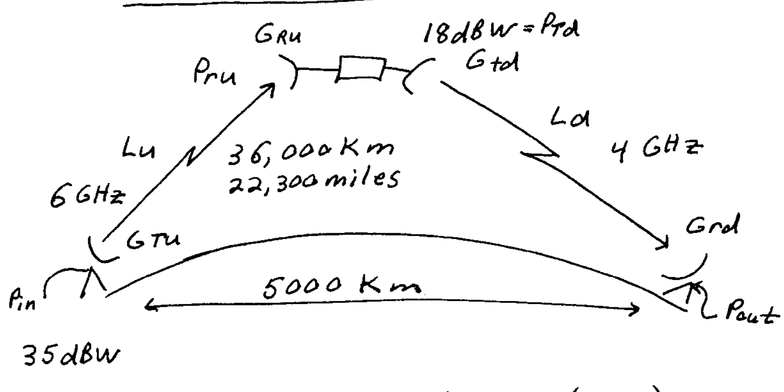
When planning for paths longer than seven miles, the curvature of the earth might become a factor in path planning and require that the antenna be located higher off the ground. The additional antenna height needed can be calculated using the following formula:

$$H = \frac{D^2}{8}$$

where
H = Height of earth bulge (in feet)
D = Distance between antennas (in miles)

$$H = \frac{3100^2}{8} = 1{,}201{,}250 \text{ feet !}$$

Example: Satellite Link



Uplink $L_u = 92.4 + 20\log(6) + 20\log(36{,}000)$

$L_u = 199.1 \text{ dB}$

Downlink $L_d = 92.4 + 20\log(4) + 20\log(36{,}000)$

$L_d = 195.6 \text{ dB}$

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Link Budgets:

uplink: $P_{ru} = P_t + G_{Tu} - L_p + G_{Ru}$

$P_{ru} = 35 dBw + 55 dB - 199.1 dB + 20 dB = -89 dBw$
or $-59 dBm$

note: $EIRP_u = 35 dBw + 55 dB = 90 dBw$
$= P_t + G_{Tu}$

Down Link: $P_{td} + G_{Td} - L_D + G_{rd} = P_{out}$

$P_{out} = 18 dBw + 16 dB - 195.6 dB + 51 dB = -110.6 dBw$
$= 8.7 \times 10^{-12}$ Watts

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A Total Link Analysis includes many Factors.

Stage	Loss/Gain dB
P_t	
feedline loss	
Connector loss	
Antenna Gain	
Path Loss	
Polarization Loss	
Atmospheric Absorbsion Loss	
Rain, Clouds, Obstruction Loss	
Receive Antenna.	

Total Receive Power.

A full Link Budget will also include a noise analysis

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Additional Link Path Loss Factors:
Fading and Atmospheric Absorption

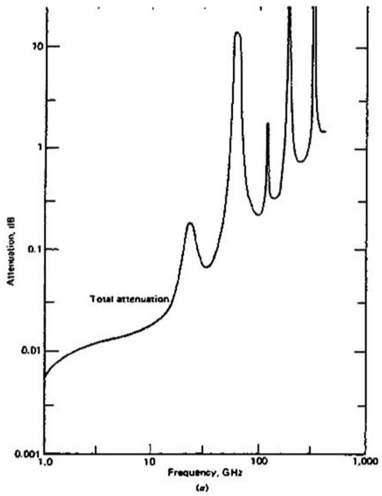


Figure 3.21. Atmospheric attenuation due to absorption: (a) horizontal path at sea le

Additional Link Path Loss Factors:
Fading and Atmospheric Absorption

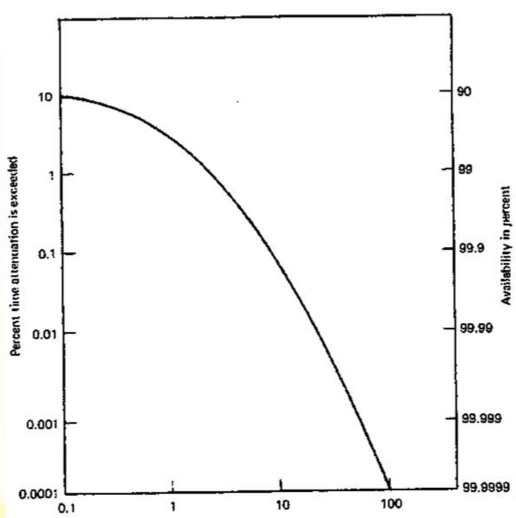


Figure 3.23. Percent time abscissa is exceeded (f = 10 GHz).

Additional Link Path Loss Factors:
Antenna Pointing Error

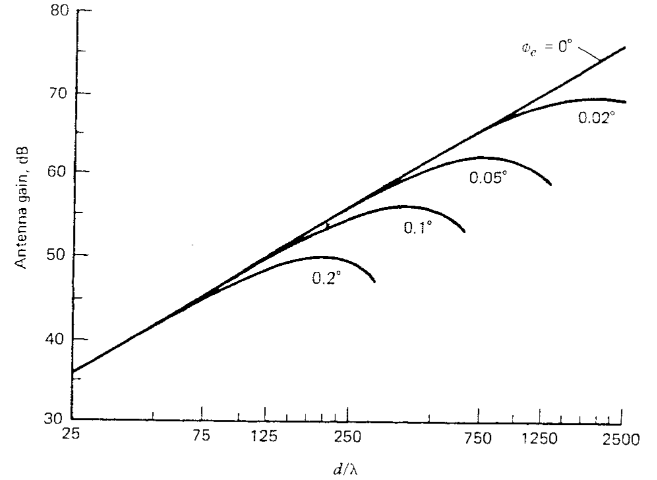
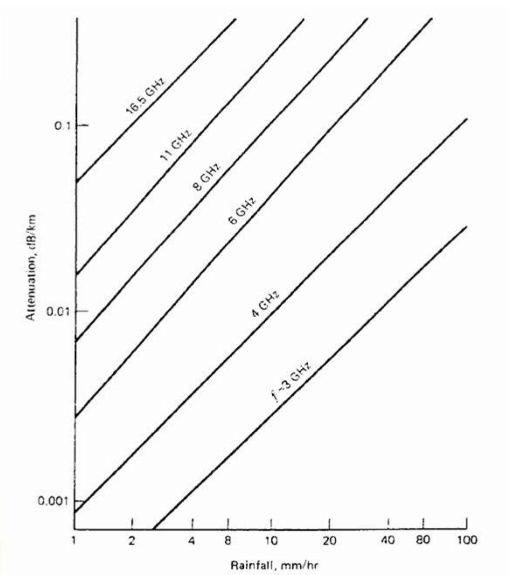
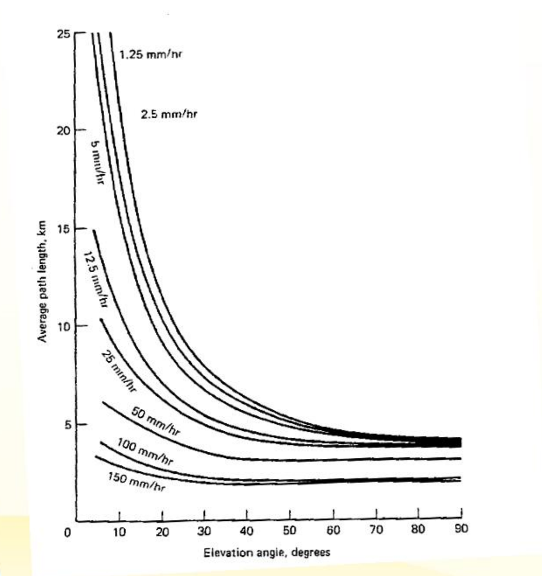


Figure 3.17. Antenna gain versus d/λ. The pointing error is denoted by ϕ_e.

Additional Link Path Loss Factors: Rainfall



Additional Link Path Loss Factor: Elevation Angle



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Plane Earth Loss-

•Free space path loss only applies when the wave does not interact with the ground.
•Examples: mountain top links and satellite links
•The loss is higher with ground interaction or when the wave passes through a material.

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Plane Earth Loss- LOS

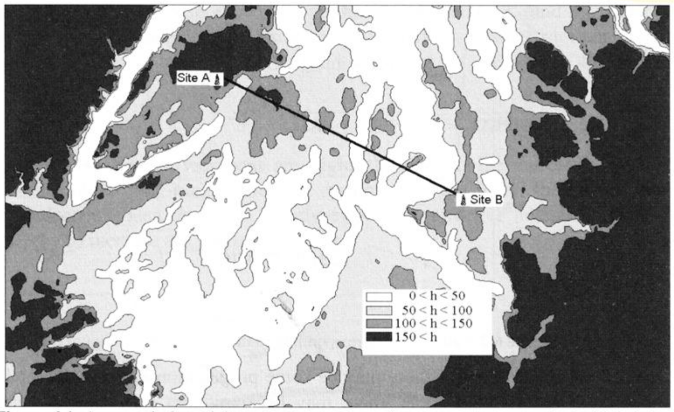

Figure 6.1: A great circle path between two antenna sites; contours are marked with heights in metres above mean sea level. Data is for the Seattle area using the USGS 3 and 1 arcsecond digital terrain models translated into Vertical Mapper format

- When the wave interacts with the ground or some other obstruction we no longer have free space propagation

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Plane Earth Loss with line of site-LOS

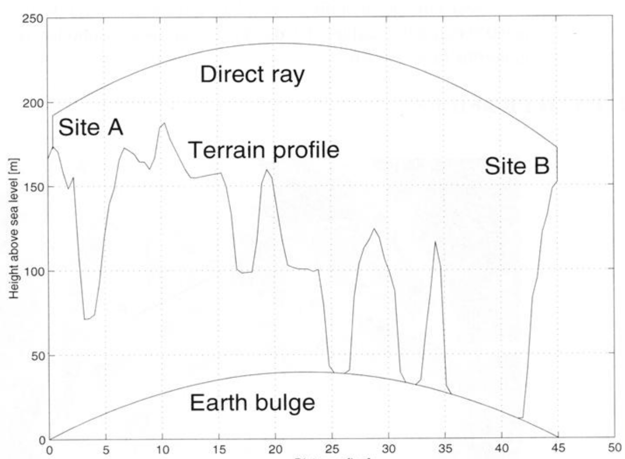

Figure 6.2: Path profile corresponding to the path shown in Figure 6.1

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Fresnel Diffraction (non LOS)

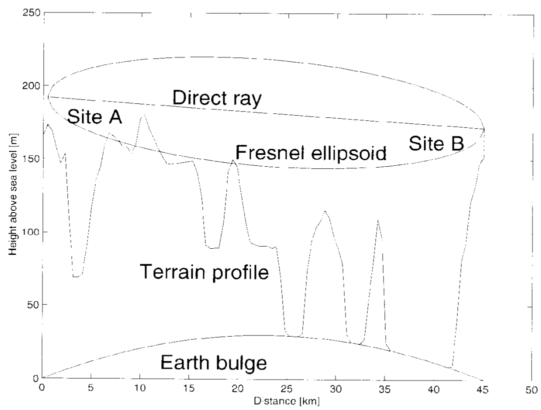

Figure 6.7: Path profile as Figure 6.2 but with Earth radius corrected to account for atmospheric refractive index gradient. The Fresnel ellipsoid represents 0.6 × the first Fresnel zone at 900 MHz

- Non-LOS communication involves an additional loss due to Diffraction

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Earth Bulge with Fresnel Diffraction

http://www.cisco.com/univercd/cc/td/doc/product/wireless/bbfw/ptop/p2pspg02/spg02ch2.htm#xtocid19

Minimum Antenna Height

The minimum antenna height at each end of the link for paths longer than seven miles (for smooth terrain without obstructions) is the height of the First Fresnel Zone plus the additional height required to clear the earth bulge. The formula would be:

$$H = 43.3\sqrt{\frac{D}{4F}} + \frac{D^2}{8}$$

where

H = Height of the antenna (in feet)

D = Distance between antennas (in miles)

F = Frequency in GHz

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Tower Height

Line of Sight Distance Between Antenna Towers	Height of Tower to Avoid Flat Earth Curvature	Tower Height Required Over Tallest Obstacle In Line-of-Sight to Provide 60% Fresnel Zone Clearance	
		2.4GHz 802.11b/g (Fresnel Zone Radius = 39 Feet)	5.8 GHz 802.11a (Fresnel Zone Radius = 25 Feet)
8 Miles	10 feet	33	25
10 Miles	15 feet	38	30
12 Miles	20 feet	43	35
14 Miles	25 feet	48	40
16 Miles	30 feet	53	45
18 Miles	40 feet	63	55
20 Miles	50 feet	73	65
22 Miles	60 feet	83	75
24 Miles	70 feet	93	85
26 Miles	80 feet	103	95
28 Miles	100 feet	123	115
32 Miles	125 feet	148	140

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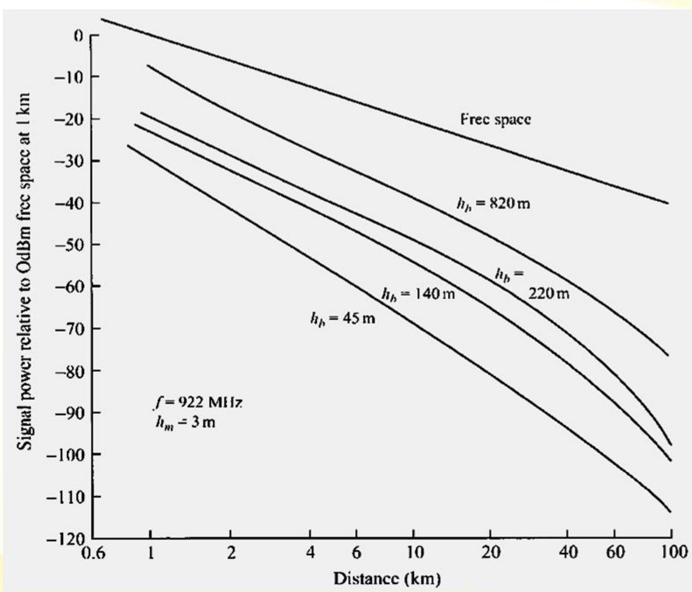
Path Loss: Free Space vs Ground effects



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Plane Earth Loss

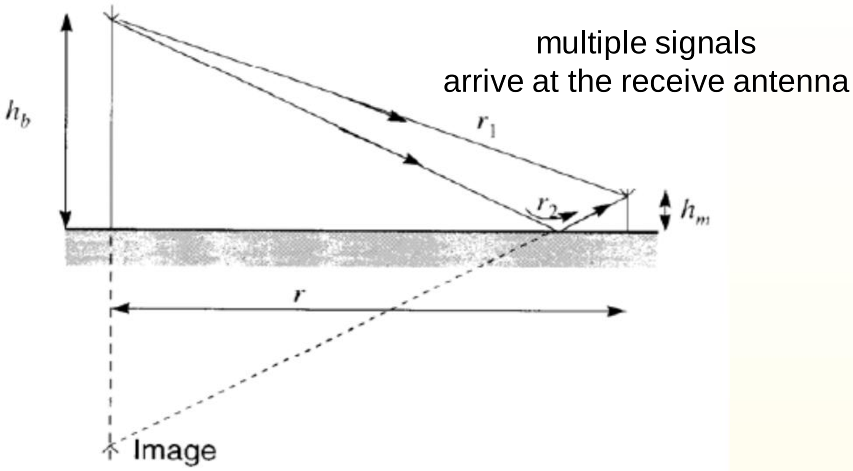

Figure 5.5: Physical situation for plane earth loss

Plane Earth Loss

$$r_1 = \sqrt{(h_b - h_m)^2 + r^2}$$
$$r_2 = \sqrt{(h_b + h_m)^2 + r^2}$$

$$(r_2 - r_1) = r\left[\sqrt{\left(\frac{h_b + h_m}{r}\right)^2 + 1} - \sqrt{\left(\frac{h_b - h_m}{r}\right)^2 + 1}\right]$$

$$(1 + x)^n \approx 1 + nx$$

$$(r_2 - r_1) \approx \frac{2h_m h_b}{r}$$

Plane Earth Loss

The overall amplitude of the result (electric or magnetic field strength) is then

$$A_{\text{total}} = A_{\text{direct}} + A_{\text{reflected}} = A_{\text{direct}} \left| 1 + R\exp\left(jk \frac{2h_m h_b}{r} \right) \right| \tag{5.28}$$

where k is the free space wavenumber.

$$\frac{P_r}{P_{\text{direct}}} = \left(\frac{A_{\text{total}}}{A_{\text{direct}}} \right)^2 = \left| 1 + R\exp\left(jk \frac{2h_m h_b}{r} \right) \right|^2 \tag{5.29}$$

where P_r is the received power.

Plane Earth Loss

The direct path is itself subject to free space loss, so it can be expressed in terms of the transmitted power as

$$P_{direct} = P_T \left(\frac{\lambda}{4\pi r} \right)^2 \tag{5.30}$$

so the path loss can be expressed as

$$\frac{P_r}{P_T} = \left(\frac{\lambda}{4\pi r} \right)^2 \left| 1 + R\exp\left(jk \frac{2h_m h_b}{r} \right) \right|^2 \tag{5.31}$$

$$\frac{P_r}{P_T} = 2\left(\frac{\lambda}{4\pi r} \right)^2 \left[1 - \cos\left(k \frac{2h_m h_b}{r} \right) \right]$$

Plane Earth Loss - PEL

Plane earth loss includes ground reflections

$$\frac{P_r}{P_t} = L_{pel} \approx \left(\frac{\lambda}{4\pi r} k \frac{2h_m h_b}{r} \right) \approx \frac{h_m^2 h_b^2}{r^4} \qquad k = \frac{2\pi}{\lambda}$$

Plain earth loss, dB: $L_{peldB} = 40\log r - 20\log h_m - 20\log h_b$

compared to,

Free space loss, dB: $L_{pdB} = 20Log\left(\frac{4\pi r}{\lambda}\right) = 20Log\left(\frac{4\pi r f}{c}\right)$

$$L_{pdB} = 92.4 + 20Log(f_{GHz}) + 20Log(R_{Km})$$

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Plane Earth Loss: Example

Example 5.5

Calculate the maximum range of the communication system in Example 5.1, assuming $h_m = 1.5$ m, $h_b = 30$ m, $f = 900$ MHz and that propagation takes place over a plane earth. How does this range change if the base station antenna height is doubled?

Solution

Assuming that the range is large enough to use the simple form of the plane earth model (5.34), then

$$\log r = \frac{L_{PEL} + 20\log h_m + 20\log h_b}{40} = \frac{148.3 + 3.5 + 29.5}{40} \approx 4.53$$

Hence $r = 34$ km, a substantial reduction from the free space case described in Example 5.4. If the antenna height is doubled, the range may be increased by a factor of $\sqrt{2}$ for the same propagation loss. Hence $r = 48$ km.

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Empirical Path Loss

Table 2.1 *Empirical power drop-off values*

City	n_1	n_2	$d_b(m.)$
London	1.7–2.1	2–7	200–300
Melbourne	1.5–2.5	3–5	150
Orlando	1.3	3.5	90

$$g(d) = d^{-n_1}\left(1 + \frac{d}{d_b}\right)^{-n_2}$$

See text eq 8-47

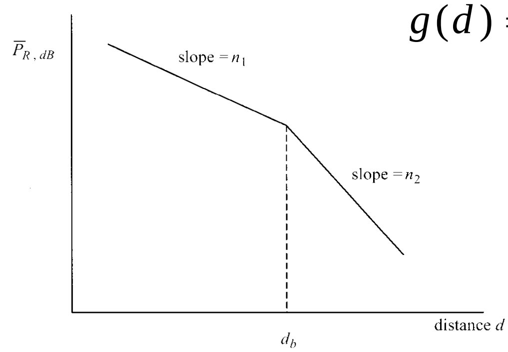

Figure 2.2 Two-slope received signal model, wireless communication

Mobile Signal

The received power of a mobile wireless signal is not constant

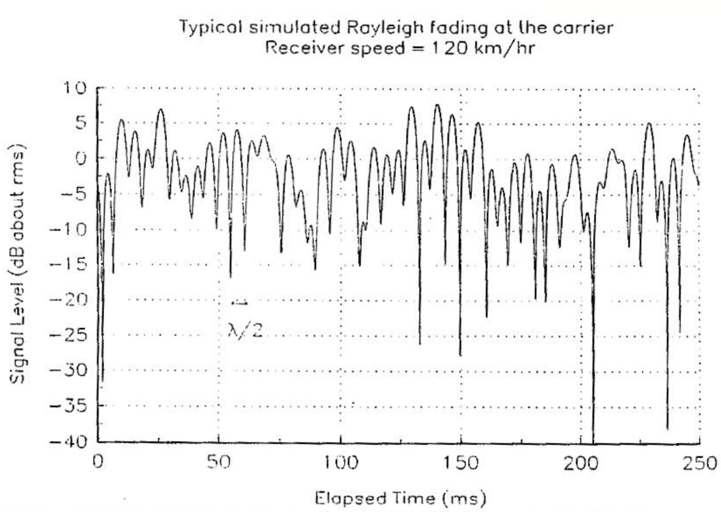

Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

Lecture 39
BER- Bit Error Rate

EELE445

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Figure 7–2 Error probability for binary signaling.

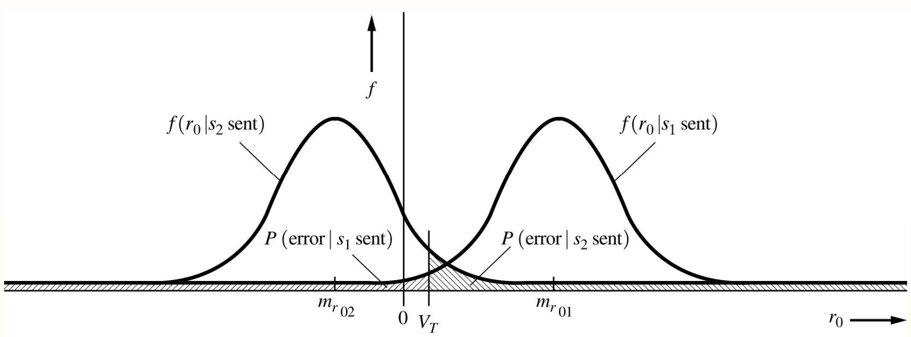

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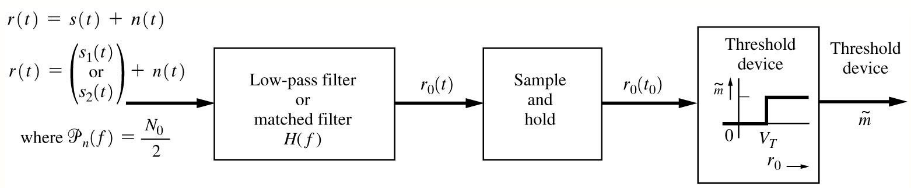
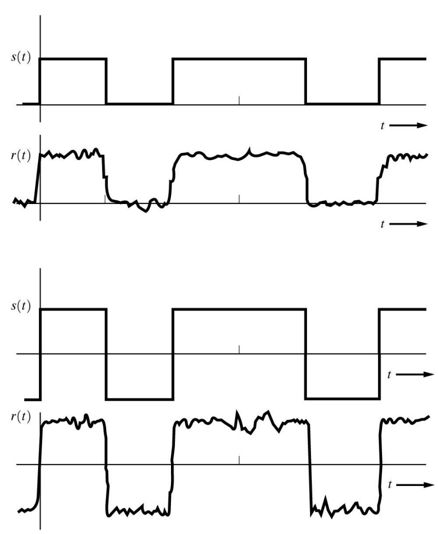
Figure 7–4 Receiver for baseband binary signaling.



(a) Receiver

Figure 7–4 Receiver for baseband binary signaling.



(b) Unipolar Signaling

(c) Polar Signaling

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Couch, *Digital and Analog Communication Systems*, Seventh Edition
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Figure 6–17 Integrate-and-dump realization of a matched filter.

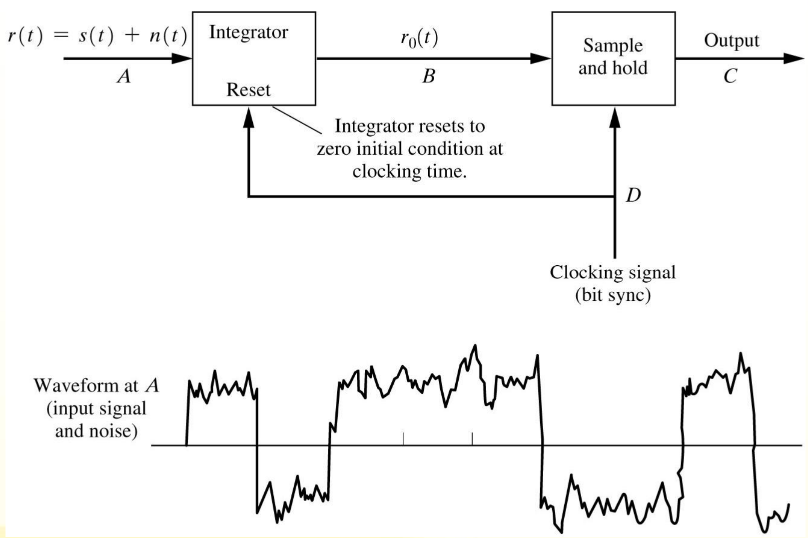
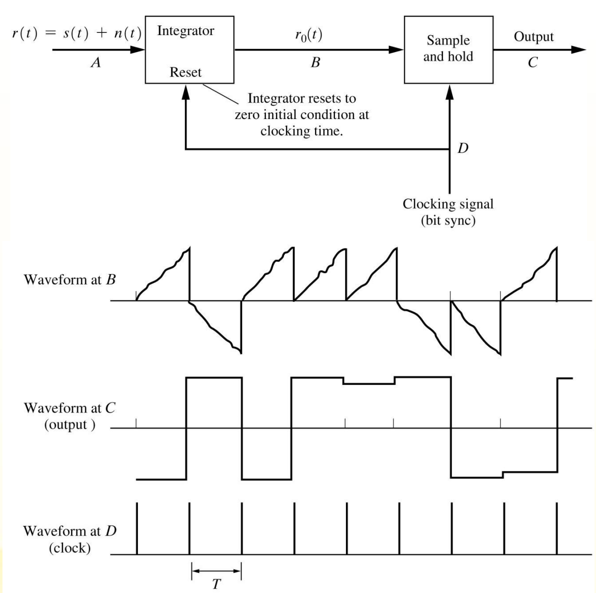


Figure 6–17 Integrate-and-dump realization of a matched filter.



The best we can do:

Bit Error Rate for Polar baseband, BPSK, QPSK :

$P_e = Q\left(\sqrt{2(E_b / N_0)}\right)$ *The Best we can do*!

for unipolar baseband, OOK, coherent FSK :

$P_e = Q\left(\sqrt{E_b / N_0}\right)$

$E_b = AT_b = \int_{T_b} v(t)^2 dt$ the energy per bit

N_0 is the noise density $watts / Hz$

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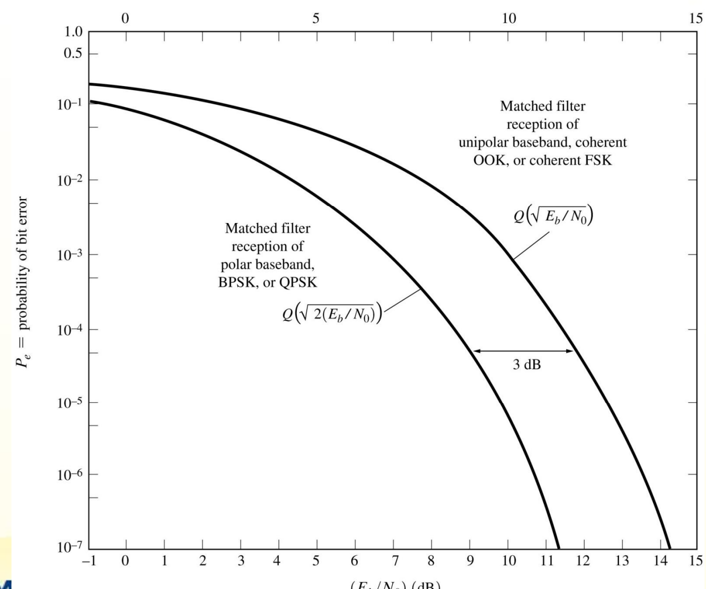
Figure 7–5 P_e for matched-filter reception of several binary signaling schemes.



Couch, *Digital and Analog Communication Systems*, Seventh Edition

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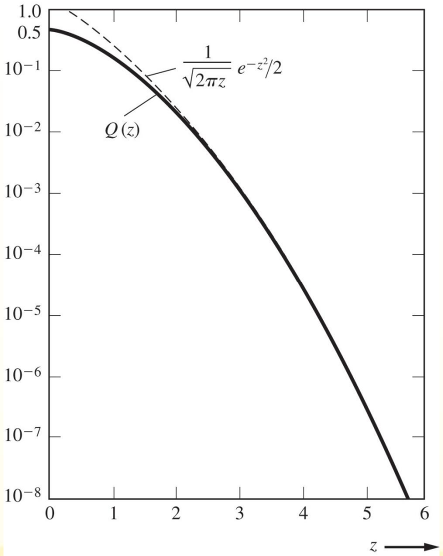
Figure B–7 The function *Q*(*z*) and an overbound,



for practical BER numbers :

$$Q(z) \approx \frac{1}{\sqrt{2\pi}\, z} e^{-z^2/2}$$

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Q, ERF, and ERFC functions

F.1 The *Q*-Function

Computation of probabilities that involve a Gaussian process require finding the area under the tail of the Gaussian (normal) probability density function as shown in Figure F.1.

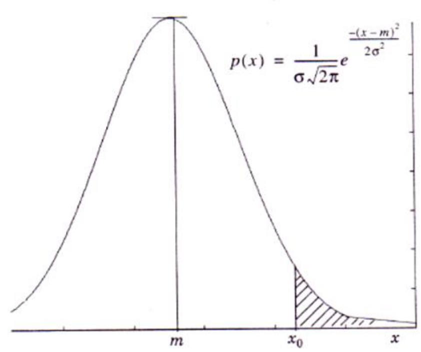

Figure F.1 Gaussian probability density function. Shaded area is $Pr(x \geq x_0)$ for a Gaussian random variable.

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Q, ERF, and ERFC functions

Figure F.1 illustrates the probability that a Gaussian random variable x exceeds x_0, $Pr(x \geq x_0)$, which is evaluated as

$$Pr(x \geq x_0) = \int_{x_0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/(2\sigma^2)} dx \quad \text{(F.1)}$$

The Gaussian probability density function in Equation (F.1) cannot be integrated in closed form.

Any Gaussian probability density function may be rewritten through use of the substitution

$$y = \frac{x-m}{\sigma} \quad \text{(F.2)}$$

to yield

$$Pr\left(y > \frac{x_0 - m}{\sigma}\right) = \int_{\left(\frac{x_0-m}{\sigma}\right)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \quad \text{(F.3)}$$

where the kernel of the integral on the right-hand side of Equation (F.3) is the normalized Gaussian probability density function with mean of 0 and standard deviation of 1. Evaluation of the integral in Equation (F.3) is designated as the Q-function, which is defined as

Q, ERF, and ERFC functions

$$Q(z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \quad \text{(F.4)}$$

Hence Equations (F.1) or (F.3) can be evaluated as

$$P\left(y > \frac{x_0 - m}{\sigma}\right) = Q\left(\frac{x_0 - m}{\sigma}\right) = Q(z) \quad \text{(F.5)}$$

The Q-function is bounded by two analytical expressions as follows:

$$\left(1 - \frac{1}{z^2}\right)\frac{1}{z\sqrt{2\pi}} e^{-z^2/2} \leq Q(z) \leq \frac{1}{z\sqrt{2\pi}} e^{-z^2/2}$$

For values of z greater 3.0, both of these bounds closely approximate $Q(z)$.

Two important properties of $Q(z)$ are

$$Q(-z) = 1 - Q(z) \quad \text{(F.6)}$$

$$Q(0) = \frac{1}{2} \quad \text{(F.7)}$$

A graph of $Q(z)$ versus z is given in Figure F.2.

A tabulation of the Q-function for various values of z is given in Table F.1.

Q, ERF, and ERFC functions

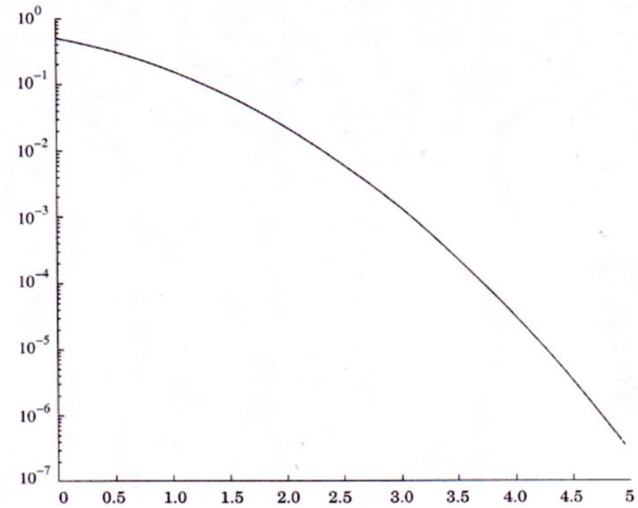

Figure F.2 Plot of the *Q*-function.

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Q, ERF, and ERFC functions

F.2 The *erf* and *erfc* Functions

The error function (*erf*) is defined as

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\,dx \tag{F.8}$$

and the complementary error function (*erfc*) is defined as

$$erfc(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-x^2}\,dx \tag{F.9}$$

The *erfc* function is related to the *erf* function by

$$erfc(z) = 1 - erf(z) \tag{F.10}$$

The *Q*-function is related to the *erf* and *erfc* functions by

$$Q(z) = \frac{1}{2}\left[1 - erf\left(\frac{z}{\sqrt{2}}\right)\right] = \frac{1}{2}erfc\left(\frac{z}{\sqrt{2}}\right) \tag{F.11}$$

$$erfc(z) = 2Q(\sqrt{2}z) \tag{F.12}$$

$$erf(z) = 1 - 2Q(\sqrt{2}z) \tag{F.13}$$

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Figure 7–13 Matched-filter detection of QPSK.