

Take-home Practice Problems 2

Not required to be turned in.

- For each of the following demand functions, find the own-price elasticity of demand when  $P = 5$ ; then repeat if  $Q = 10$ . For each case, interpret what the elasticity implies.

(a)  $P = 200 - 10Q$

*We must first solve for the demand function (i.e., solve for  $Q$ ) to get the equation into the right form. Doing so yields  $Q = 20 - 0.1P$ . At  $P = 5$ ,  $Q = 19.5$ . Therefore, the elasticity is  $\varepsilon = \frac{\Delta Q}{\Delta P} \frac{P}{Q} = -0.1 \times \frac{5}{19.5} = -0.026$ . That is, for a 1% increase in price, there is just a 0.026% decrease in quantity demanded.*

*Since we've already solved for the demand function and know that the slope is  $-0.1$ , we can simply plug in for  $Q$  and solve for the associated price. That is, when  $Q = 10$ ,  $P = 100$ . At these values, the elasticity is  $\varepsilon = -0.1 \times \frac{100}{10} = -1$ . That is, there is a 1% change in quantity demanded associated with a 1% change in price. It is unit-elastic.*

(b)  $Q = 250 - 5P$

*We already have the equation in the appropriate form, so we can directly determine the slope to be  $-5$ . At  $P = 5$ ,  $Q = 225$ , and the elasticity is  $\varepsilon = -5 \times \frac{5}{225} = -0.111$ . Therefore, for a 1% increase in price, there is a 0.11% decrease in quantity demanded.*

*At  $Q = 10$ ,  $P = 48$ , and the elasticity is  $\varepsilon = -5 \times \frac{48}{10} = -24$ . Therefore, for a 1% increase in price, there is a 24% decrease in quantity. Obviously, this is a very elastic product.*

- For each of the following demand functions, find the own-price flexibility of demand when  $P = 10$ ; then repeat if  $Q = 5$ . Interpret what the flexibility implies.

(a)  $2Q = 50 - 2P$

We must first solve for the inverse demand function (i.e., solve for  $P$ ) to get the equation into the right form. Doing so yields  $P = 25 - Q$ . At  $P = 10$ ,  $Q = 15$ . Therefore, the flexibility is  $\phi = \frac{\Delta P}{\Delta Q} \frac{Q}{P} = -1 \times \frac{15}{10} = -1.5$ . That is, for a 1% increase in quantity demanded, there is a 1.5% decrease in price.

At  $Q = 5$ ,  $P = 20$ , and the flexibility is  $\phi = -1 \times \frac{5}{20} = -0.25$ . That is, for a 1% increase in quantity demanded, there is a 0.25% decrease in price.

(b)  $P = 90 - 10Q$

We already have the equation in the appropriate form, so we can directly determine the slope to be  $-10$ . At  $P = 10$ ,  $Q = 8$ , and the flexibility is  $\phi = -10 \times \frac{8}{10} = -8$ . That is, for a 1% increase in quantity demanded, there is an 8% decrease in price.

At  $Q = 5$ ,  $P = 40$ , and the flexibility is  $\phi = -10 \times \frac{5}{40} = -1.25$ . That is, for a 1% increase in quantity demanded, there is a 1.25% decrease in price.

3. For each of the following supply functions, find the *own-price* and *cross-price* elasticities of demand; then, interpret what the elasticity implies.

(a)  $Q_{legs} = 15 - 2P_{legs} + 5P_{breasts}$

$$P_{legs} = 2 \quad P_{breasts} = 4$$

First, determine the  $Q_{legs}$ . That is,  $Q_{legs} = 15 - 2 \cdot 2 + 5 \cdot 4 = 21$ . Then find the *own- and cross-price* elasticities.

$$\varepsilon_{Q_{legs}, P_{legs}} = \frac{\Delta Q_l}{\Delta P_l} \frac{P_l}{Q_l} = -2 \cdot \frac{2}{21} = -\frac{4}{21}$$

$$\varepsilon_{Q_{legs}, P_{breast}} = \frac{\Delta Q_l}{\Delta P_b} \frac{P_b}{Q_l} = 5 \cdot \frac{4}{21} = \frac{20}{21}$$

A 1% increase in the price of chicken legs leads to a 4/21% decrease in the quantity demanded of chicken legs. A 1% increase in the price of chicken breasts leads to a 20/21% increase in the quantity demanded of chicken legs.

$$(b) \begin{aligned} P_{orange} &= 5 - Q_{orange} + 1.5P_{apples} \\ P_{orange} &= 1.50 & P_{apples} &= 1.25 \end{aligned}$$

$$Q_o = 5 - 1.50 + 1.50 \cdot 1.25 = 5.38$$

$$\varepsilon_{Q_{orange}, P_{orange}} = \frac{\Delta Q_o}{\Delta P_o} \frac{P_o}{Q_o} = -1 \cdot \frac{1.5}{5.38} = -0.28$$

$$\varepsilon_{Q_{orange}, P_{apple}} = \frac{\Delta Q_o}{\Delta P_a} \frac{P_a}{Q_o} = 1.5 \cdot \frac{1.25}{5.38} = 0.35$$

*A 1% increase in the price of oranges leads to a 0.28% decrease in the quantity demanded of oranges. A 1% increase in the price of apples leads to a 0.35% increase in the quantity demanded of oranges.*

$$(c) \begin{aligned} P_{strawberry} &= 15 - 0.5Q_{strawberry} - 2P_{chocolate} \\ P_{strawberry} &= 4 & P_{chocolate} &= 5 \end{aligned}$$

$$Q_s = 30 - 2 \cdot 4 - 4 \cdot 5 = 2$$

$$\varepsilon_{Q_{strawberry}, P_{strawberry}} = \frac{\Delta Q_s}{\Delta P_s} \frac{P_s}{Q_s} = -2 \cdot \frac{4}{2} = -4$$

$$\varepsilon_{Q_{strawberry}, P_{chocolate}} = \frac{\Delta Q_s}{\Delta P_c} \frac{P_c}{Q_s} = -4 \cdot \frac{5}{2} = -10$$

*A 1% increase in the price of strawberries leads to a 4% decrease in the quantity demanded of strawberries. A 1% increase in the price of chocolate leads to a 10% decrease in the quantity demanded of strawberries.*

4. You are a farmer who can produce wheat and barley on the same land. Suppose that the supply of wheat follows the function:

$$Q_{wheat}^S = 100 + 2P_{wheat} + 4 \cdot Rain - 0.5P_{barley} - 1.5P_{fertilizer} + 3Q_{fertilizer}$$

You know that  $P_{wheat} = \$7.00$ ,  $Rain = 8 \text{ in.}$ ,  $P_{barley} = \$4.00$ ,  $P_{fertilizer} = \$20$ ,  $Q_{fertilizer} = 40$ .

Respond to the following:

- (a) How much more wheat will you sell if the market price of wheat increases by 1%?

$$Q_{wheat} = 100 + 2 \cdot 7 + 4 \cdot 8 - 0.5 \cdot 4 - 1.5 \cdot 20 + 3 \cdot 40 = 234$$

$$\varepsilon_{Q_{wheat}, P_{wheat}} = \frac{\Delta Q_w}{\Delta P_w} \frac{P_w}{Q_w} = 2 \cdot \frac{7}{234} = 0.06$$

*A 1% increase in the price of wheat will lead to a 0.06% increase in the quantity supplied of wheat.*

- i. How will producers respond in the short run?

*There will be a slight increase in the quantity supplied of wheat, because farmers are trying to respond to higher prices by selling their stored commodity.*

- ii. How will producers respond in the long run?

*If prices remain higher, then farmers will reallocate land to produce more wheat.*

- (b) If the price of barley increases by 15%, will you plant more or less wheat? How much quantity of wheat will you change?

$$\varepsilon_{Q_{wheat}, P_{barley}} = \frac{\Delta Q_w}{\Delta P_b} \frac{P_b}{Q_w} = -0.5 \cdot \frac{4}{234} = -0.008$$

*A 15% increase in the price of barley will decrease your wheat production by 0.128%, because you will reallocate land to barley production.*

- (c) Suppose that you decided to decrease the price of fertilizer by 10%. How much will your production of wheat change?

$$\varepsilon_{Q_{wheat}, P_{fert}} = \frac{\Delta Q_w}{\Delta P_f} \frac{P_f}{Q_w} = -1.5 \cdot \frac{20}{234} = -0.128$$

*A 10% decrease in fertilizer prices will result in a 12.8% increase in the production of wheat.*