ECNS561, Fall 2011

Take-home Graded Practice Opportunity 2

Due date: September 30, 2011, 5:00 p.m.

1. Is it the case that if $\lim_{n\to\infty} \widehat{\theta}_n = \theta$ implies $\text{plim}_{n\to\infty} \widehat{\theta}_n = \theta$? Prove.

2. Suppose that a sequence of random variables Y_n follows the probability density function:

$$Y_n = \begin{cases} 0 & \text{with probability } (n-1)/n \\ n^2 & \text{with probability } 1/n \end{cases}$$

Show that $\text{plim}_{n\to\infty} Y_n = 0$, but that the $\lim_{n\to\infty} E[Y_n] = \infty$.

3. Suppose that $\overline{Y}$ and $S^2_{\overline{Y}}$ are the mean and sample variance associated with a random sample $\{Y_1, Y_2, \ldots, Y_n\}$, which is drawn from a population with some finite variance, σ^2_Y.
   (a) What is the expected value of $S^2_{\overline{Y}}$?
   (b) Prove that $E[S_{\overline{Y}}] \leq \sigma_Y$. *Hint:* For a concave function $h(\cdot)$, Jensen's inequality states that $E[h(Z)] \geq h(E[Z])$.

4. Consider the table 1, which describes wheat prices in four regions and the protein levels associated with each price. In general, protein levels indicate the quality of wheat. the table represents random samples from each of the four regions.

   Complete the following:

   (a) First, investigate the data. What might they tell you about the supply and demand dynamics in each region? What do they tell you about the effect of quality? Justify your answers.
   (b) Estimate the mean population wheat price across all protein levels.
   (c) Estimate the sampling variance and sampling standard deviation? Interpret these results – what inferences can we make about the population?
   (d) Compute a 95% confidence interval around the population mean of the wheat price. Then compute a 95% confidence interval around the population mean of 15% protein wheat price. What inferences can you make when comparing the two confidence intervals?
   (e) Set up, test, and interpret each of the following hypotheses:
      i. At a 95% confidence level, the price of wheat is less than \$5.85/bu.

ii. At a 99% confidence level, the average price of 12% and 13% protein wheat is greater than \$5.80/bu.

iii. At a 90% confidence level, the average price of 14% and 15% protein wheat is not equal to the average price of 12% and 13% protein wheat.

(f) For each of the tests in part (e), compute p-values. Interpret.

Table 1: Table of wheat prices and protein levels in four regions

Region	**Wheat price per bushel**	**Protein level**
1	\$ 5.80	12%
1	\$ 5.83	13%
1	\$ 5.89	14%
1	\$ 5.98	15%
2	\$ 5.79	12%
2	\$ 5.84	13%
2	\$ 5.88	14%
2	\$ 5.95	15%
3	\$ 5.84	12%
3	\$ 5.89	13%
3	\$ 5.92	14%
3	\$ 5.99	15%
4	\$ 5.76	12%
4	\$ 5.80	13%
4	\$ 5.87	14%
4	\$ 5.94	15%

5. Consider the following three vectors:

$$\boldsymbol{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{(3\times1)} \qquad \boldsymbol{B} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}_{(2\times1)} \qquad \boldsymbol{C} = \begin{bmatrix} 1 & 4 \end{bmatrix}_{(1\times2)}$$

Calculate:

(a) $\boldsymbol{A} \cdot \boldsymbol{B}'$

(b) $\boldsymbol{C}' + \boldsymbol{B}$

(c) $\boldsymbol{D} = \boldsymbol{B}' \cdot \boldsymbol{B}$

(d) $\boldsymbol{D}^{-1}$

(e) $\boldsymbol{B}'\boldsymbol{C}'$

(f) $\boldsymbol{D}^{-1} \cdot \boldsymbol{B}'\boldsymbol{C}'$

6. Applied practice. Please turn in your code and a summary of your output. If you are going to do the assignment in anything other than SAS, please also email me your code. You don't have to print the entire output window; just provide the relevant statistics asked for in the problems.

   Part of this week's applied practice will have you use simulations to prove several properties and theorems that we discussed in class. The other part will have you perform hypothesis tests.

   Simulation Analysis

   (a) Simulate random samples from the distribution $Y \sim N(2, 2^2)$. Show that $\lim_{n \to \infty} E[\overline{Y}_n] = \mu_Y$ (asymptotic unbiasedness) and plim $(\overline{Y}_n) = \mu$ (law of large numbers). *Hint*: the SAS example I showed in class has code for simulating from a normal distribution.

   (b) Prove graphically the central limit theorem using simulated values from a uniform distribution $Y \sim U[0, 4]$. *Hint*: To simulate a uniform distribution in SAS, use the function `rand(''Uniform'')*b`, where `b` is the upper bound of the uniform distribution (lower bound, `a`, is assumed to be 0).

   Hypothesis Tests

   1 Using the data presented in problem 4, perform the hypothesis tests in part (e). In SAS, you can use the `proc ttest` procedure, but note that the SAS default is to perform a one-sided hypothesis test. Use the "sides" option to alter the default setting. Interpret the results, including the p-values.