Lossy Transmission Lines

- **Skin Effect**
- **Dielectric Loss**

**Textbook Reading Assignments**
1. 9.1-9.11

**What you should be able to do after this module**
1. Describe the physical phenomenon behind Skin Effect and Dielectric Loss
2. Use a modern CAD tool to simulate the behavior of lossy transmission lines

**Lossy Transmission Lines**

- **Circuit Model**
  - If the line is lossy, we need to include the series resistance and shunt conductance back into our equivalent T-line circuit model.

\[
\begin{align*}
L_{\text{seg}} &= L' \cdot dx \\
R_{\text{seg}} &= R' \cdot dx \\
C_{\text{seg}} &= C' \cdot dx \\
G_{\text{seg}} &= G' \cdot dx
\end{align*}
\]

- We enter the segment at \(x\) and we exit the line at \((x+dx)\)

- The input voltage can be described as: \(V(x,t)\)
- The input current can be described as: \(I(x,t)\)
- The output voltage can be described as: \(V(x+dx,t)\)
- The output current can be described as: \(I(x+dx,t)\)
Circuit Model

- We can write an expression for the voltage drop across the inductor and resistor using KVL:
  \[ V(x,t) = -R \frac{dI(x,t)}{dt} - L \frac{d^2I(x,t)}{dx^2} \]
- Which is rewritten as:
  \[ V(x,t) = R \frac{dI(x,t)}{dt} + L \frac{d^2I(x,t)}{dx^2} \]
- If we let \( dx \to 0 \), we are left with a differential equation:
  \[ \frac{dV(x,t)}{dx} - RI(x,t) + L \frac{dI(x,t)}{dt} \]

Wave Equations

- Let's put these into a more usable form using phasor representation where \( (d/dt \to j \omega ) \)

- If we let \( dx \to 0 \), we are left with another differential equation:
  \[ \frac{dI(x,t)}{dt} = I(x,t) + G \frac{dV(x,t)}{dx} \]

Lossy Transmission Lines

- Now we can write an expression for the output current using KCL:
  \[ I(x,t) = I(x,t) - G \frac{dV(x,t)}{dx} \]
- Which can be rewritten as:
  \[ I(x,t) = -G \frac{dV(x,t)}{dx} + C \frac{d^2I(x,t)}{dx^2} \]

Waves Equations

- We can put the voltage expression into the form of the Wave Equation by differentiating the first Telegrapher equation with respect to \( x \)

- We can now plug in our expression for the derivative of current:
  \[ \frac{d^2V(x,t)}{dx^2} = (G + j \omega C) V(x) \]

- Rearranging, we get:
  \[ \frac{d^2V(x,t)}{dx^2} = (G + j \omega C) V(x) \]

- If we define \( \gamma \) as:
  \[ \gamma = \sqrt{(R + j \omega L)(G + j \omega C)} \]

- We can rewrite the voltage expression in Wave Equation form as:
  \[ \frac{d^2V(x,t)}{dx^2} = \gamma^2 V(x) = 0 \]
Lossy Transmission Lines

- **Wave Equations**
  - Now let's put the current expression into the form of the Wave Equation by differentiating the second Telegrapher equation with respect to $x$
  
  \[
  \frac{d^2 I(x)}{dx^2} = (G + j\omega C) \frac{dV(x)}{dx}
  \]

- We call the Imaginary part of the Complex Propagation Constant $\beta$

\[\gamma = \alpha + j\beta\]

where $\gamma$ has units of $\text{rad/m}$

- This quantity has units of $\text{Np/m}$

- We use $\gamma$ to describe ratios of voltages and currents.

- Complex Propagation Constant for Passive T Lines
  - We choose the square root values of $\gamma$ that give positive values for $\alpha$ and $\beta$.
  \[\gamma = \alpha + j\beta\]
  - For passive T lines, $\alpha \geq 0$

**Rearranging, we get:**

\[\frac{d^2 I(x)}{dx^2} - (R + j\omega L) I(x) = 0\]
Lossy Transmission Lines

- Characteristic Impedance
  - The Wave Equations have traveling wave solutions for a lossy medium in the form of:
    \[ V(x) = V^e e^{\gamma x} + V^r e^{-\gamma x} \]
    \[ I(x) = I^e e^{\gamma x} + I^r e^{-\gamma x} \]
  - Forward Traveling Wave
  - Reverse Traveling Wave

- We can now plug in our equations for \( V(x) \) and \( I(x) \), and only consider the forwarding traveling waves:

- Now we have our expression of the characteristic impedance of a lossy transmission line:
  \[ Z = \frac{V}{I} \]
  \[ Z = \frac{V^e e^{\gamma x}}{I^e e^{\gamma x}} = \frac{V^e}{I^e} e^{\gamma x} \]

- Characteristic Impedance
  - Remembering the definition for impedance is the ratio of either the forward or reverse traveling waves:
  \[ Z = \frac{\sqrt{\frac{R + j\omega L}{G + j\omega C}}}{\sqrt{\frac{R^e + j\omega L^e}{G^e + j\omega C^e}}} \]

- When we talk about Lossy Transmission Lines, we tend to focus on items 4 & 5
  - Reflections & Coupling can be modeled using our standard LC model of a T-line.
  - Conductor Loss and Dielectric Loss can be modeled using our complete RLCG transmission line circuit.
  - When people talk about Lossy Lines, they are typically referring to Conductor & Dielectric Loss

Sources of Loss

- Sources of Loss
  - Loss refers to the amount of signal transmitted that does not reach the receiver.
    - Loss can occur due to many sources:
      1) Impedance Mismatches Leading to Reflected Energy
      2) Coupling to adjacent Traces
      3) Radiation Loss
      4) Conductor Loss
      5) Dielectric Loss
    - When we talk about Lossy Transmission Lines, we tend to focus on items 4 & 5
      - Reflections & Coupling can be modeled using our standard LC model of a T-line.
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Conductor Loss

- Skin Depth
  - At DC, the resistance of a conductor is proportional to:
    \[ R = \frac{\rho L}{W \cdot H} = \frac{\rho L}{Area} \]
  - At AC, the charge is equally distributed across the cross-section of the conductor:
Conductor Loss

- Skin Depth
  - When AC current flows through the conductor, the charge is not equally distributed within the cross-sectional area of the conductor. At AC, the current will attempt to find the path of least impedance.
  - This results in two trends:
    1) The current within the conductor will spread out as far as possible in order to minimize the partial self-inductance of the conductor.
    2) The current within the conductor will try to move as close as possible to the oppositely directed return current in order to maximize the partial mutual inductance between the two currents.

Conductor Loss

- Skin Depth
  - The phenomenon of the current flowing through this reduced cross-section of the conductor is described using a quantity called Skin Depth.
  - The reduced cross-section has the effect of increasing the series resistance of the conductor as the frequency increases.

Skin Depth is described by \( \delta \) and is expressed as:

\[
\delta = \frac{2 \rho}{\pi \mu} = \frac{\rho}{\sqrt{\mu \pi f}} = \frac{1}{\sqrt{\mu \pi f} \cdot \sigma}
\]

where:
- \( \mu \) is the permeability of the conductor
- \( \sigma \) is the conductivity of the conductor (1/\( \rho \))

Skin depth has units of meters and is the definition of the depth below the surface of a conductor with depth \( d \) at which the current density \( J \) decays to 1/e (~0.37) of the current density of the surface \( J_S \):

\[
J = J_S e^{-\delta/d}
\]

Conductor Loss

- Total Conductor Loss
  - The complete expression for conductor loss should also include any DC loss due to the resistivity of the bulk material:

\[
R_{\text{conductor}} = R_{AC} + R_{DC} \sqrt{f}
\]

- Note that it is hard to define a complete expression for the AC conductor resistance because it depends on the geometry of the conductor. (i.e., round, square, rectangle, etc.)

- For a given cross-sectional shape, the skin depth is then applied to that shape in order to predict the new cross-sectional area that the current flows through.
Lossy Transmission Lines

• Dielectric Loss
  - Now we move to the 2nd main effect in lossy lines, the dielectric.
  - We use the G’ element in our model to account for dielectric loss.

\[
C = \frac{Q}{V}
\]
- The capacitor is constructed using two conducting surfaces separated by a dielectric.
- In the ideal case, this structure does NOT allow DC current to flow between the two terminals of the device.

An ideal capacitor will store a particular amount of charge depending on the voltage applied.

The capacitor is constructed using two conducting surfaces separated by a dielectric.

In reality, the charge in the capacitor is held by electric dipoles within the material.

If an AC voltage is applied to the capacitor, the dipoles will align to the direction of the applied electric field.

This movement of charge results in AC current flow.

\[
I = C \frac{dV}{dt}
\]
- The amount of current that flows is proportional to the rate of change of voltage across the capacitor.

Lossy Transmission Lines

• Dielectric Loss
  - Since there is no DC current (or no in-phase current), there is no real power dissipation in an ideal capacitor.
  - This allows us to say that the resistivity of an ideal capacitor is \(\infty\).
  - However, real capacitors do have some resistance associated with them.
  - This means that current will flow through the capacitor that is in phase with the voltage.
  - We model this resistance with a resistor component in parallel with the capacitor.

The density of available dipoles in a material to hold the charge is reflected in the dielectric constant (i.e., the electric permittivity).

If a parallel plate capacitor was constructed using an air dielectric, the capacitance would be given by:

\[
C_A = \varepsilon_0 \frac{A}{t}
\]
- If the air was then replaced with a different insulating material with \(\varepsilon_r > 1\), the new capacitance would be described as:

\[
C = \varepsilon_r C_A
\]
- The dipoles in the material do not re-orient instantaneously upon a change in voltage.

If a given time varying voltage, there are going to be dipoles that are perfectly aligned with the applied electric field. These dipoles contribute to the capacitance of the structure and result in an \textit{out of phase} current (\(-90\))

Due to the finite speed at which the dipoles can change, there will be dipoles that are aligned perpendicular to the applied electric field which produce an \textit{in phase} current.

This \textit{in phase} current is the source of the leakage current in a capacitor.
Dielectric Loss

The Complex Dielectric Constant

The Loss Tangent (Dissipation Factor)

The Loss Tangent, \( \tan(\delta) \)

Lossy Transmission Lines

The Complete Dielectric Constant

The Loss Tangent (Dissipation Factor)

At this point, things can get a little confusing. Remember that:

- the dielectric constant is described as a complex quantity in order to describe both:
  - the out of phase current due to an applied voltage
  - the in phase current due to an applied voltage

- The out of phase current that results from an applied voltage is actually the expected response of an ideal capacitor. That is why this quantity is described using the real part of the complex dielectric constant.

- The in phase current that results from an applied voltage is due to the loss in a real capacitor and is described using the imaginary part of the complex dielectric constant.
Lossy Transmission Lines

• **The Loss Tangent (Dissipation Factor)**
  - We can now put his expression for \( R_{leakage} \) into terms of bulk conductivity of the material:
    \[
    R_{leakage} = \frac{1}{\omega \mu \sigma C}
    \]
  - The definition of resistance & capacitance of a test structure is (looking from the top to bottom):
    \[
    R = \frac{\rho \text{Length}}{\sigma \text{Area}} \quad \text{and} \quad C = \varepsilon_r \varepsilon_0 \frac{\text{Length}}{\varepsilon_r \varepsilon_0}
    \]
  - These two expressions can be combined to form a relationship between the loss tangent & the bulk conductivity of the material:
    \[
    R = \frac{\rho}{\varepsilon_r \varepsilon_0} = \frac{1}{\sigma} \frac{1}{\tan(\delta)}
    \]

- Low Loss Dielectric
  - We model the conductor loss due to skin depth using the shunt conductance in our RLGC T-line model.
  - This conductance value is dependent on frequency:
    \[
    G_{shunt} = 2\pi f \tan(\delta) C
    \]
  - When stimulating the lossy transmission line with a digital signal, each frequency component of the signal will experience a different shunt conductance.

Lossy Transmission Lines

• **Dielectric Loss**
  - Now we have the bulk conductivity (and resistivity) in terms of the Loss Tangent and frequency:
    \[
    \sigma = \varepsilon_r \varepsilon_0 \frac{1}{\tan(\delta)}
    \quad \rho = \varepsilon_r \varepsilon_0 \frac{1}{\tan(\delta)}
    \]
  - This can we used to describe the conductance or resistance of the shunt resistor in our model:
    \[
    G \propto f
    \]
  - We typically use \( G' \) to model the dielectric loss because it scales directly with length. That way all 4 of our T-line parameters will scale with length.

Lossy Transmission Lines

• **Total Loss**
  - We now see that there are two frequency dependent sources of loss in our lossy T-line.
  - This results in higher frequencies being attenuated more than lower frequencies.
  - The two frequency dependent sources are specified in terms of unit length (i.e., \( G', R' \))
    \[
    G_{losses} = 2\pi f \tan(\delta) C
    \quad R'' = R_{0} + R_{leakage} \sqrt{f}
    \]

- Low Loss Regime
  - when \( R_{leakage} \ll X_c \) and \( G_{leakage} \ll X_c \)
    - we can use the lossless equations for \( Z \) and velocity
  - Good Conductor
    - when \( R_{leakage} \ll X_c \) (i.e., skin effect is negligible)
      \[
      a = \frac{\omega}{\beta}
      \quad b = \frac{\sqrt{\mu \sigma}}{\beta}
      \]
  - Low Loss Dielectric
    - when \( G_{leakage} \ll X_c \) (i.e., the dielectric leakage is negligible)
      \[
      a = \frac{\sigma}{2 \varepsilon_{leak} \varepsilon_0}
      \quad b = a \sqrt{\mu_{leak} \varepsilon_0}
      \]

- Approximations
  - There are a couple approximations that can be made to simplify the analysis of lossy lines:
    - Low Loss Regime
      - when \( R_{leakage} \ll X_c \) and \( G_{leakage} \ll X_c \)
        - we can use the lossless equations for \( Z \) and velocity
    - Good Conductor
      - when \( R_{leakage} \ll X_c \) (i.e., skin effect is negligible)
      - Low Loss Dielectric
        - when \( G_{leakage} \ll X_c \) (i.e., the dielectric leakage is negligible)