

$$V_{OUT} = V_{IN} \left( 1 - e^{-\frac{t}{RC}} \right)$$
(1)

Since rise-time is defined as the time it takes to transition from 10% to 90% of  $V_{OUT}$ , we can solve the equation to get two separate solutions. The first solution is the time it takes to transition from 0 volts to 10% of  $V_{OUT}$ . To accomplish this,  $V_{IN}$  is set to 1 volt and  $V_{OUT}$  is set to 0.1 volts. The second solution is obtained in the same way except that  $V_{OUT}$  is set to 0.9 volts. Since rise-time is defined as the time between these two solutions, the results are simply subtracted and yield a rule of thumb for the rise-time of an RC circuit.

$$t_{rise} = 2.2 \cdot RC \quad (2)$$

The second step of this derivation is to solve the same RC circuit in the frequency domain. The general solution to this circuit is given by:

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + RCs}$$
(3)

Since bandwidth is defined as the frequency at which the magnitude of the response is attenuated by 30%, then this expression can be solved to generate a rule of the thumb.

$$BW = \frac{1}{2\pi RC}$$
 (4)

Now that we have general expression for rise-time and bandwidth in terms of resistance and capacitance, we can combine the two expressions to yield a single linear relationship. This expression can now be used to quickly convert between the rise-time of a digital signal and the frequency components that the rise-times possesses.

$$t_{rise} \cdot BW = 0.35 \tag{5}$$

#### Translating Toggle-Rate to Bandwidth

The digital toggle rate of a signal can be converted into a frequency representation using the Fourier Transform. In Fourier analysis, there are a set of basic transforms that can be scaled according to the application. In Fourier representation, a periodic signal is represented as a series of impulses occurring at the desired periodicity in the time domain. This type of representation is called the Shaw function (III(t)). Its transform into the frequency domain is another Shaw function (III(s)). The scaling between the two domains is accomplished by the Similarity Theorem which has an inverse relationship between the two domains (as in, closer spaced impulses in the time domain will result in wider spaced impulses in the frequency domain).

$$\mathbb{E}\{III(t \cdot period)\} = \left(\frac{1}{period}\right)III\left(\frac{s}{period}\right)$$
(6)

A rule of thumb commonly used in digital systems is that the system must have enough bandwidth to capture the 3<sup>rd</sup> harmonic of the digital pulse train. When relating this to the Shaw function, the third harmonic refers to the third impulse in the frequency domain. **Figure 2** illustrates the transform and how the rule of thumb relates.



**Figure 2:** A periodic signal in the time domain is represented as a series of impulses spaced at the period of the signal. This translates into a series of impulses in the frequency domain that occur at the integer evaluation of (1/period).

The total bandwidth needed for a periodic signal can be expressed as:

$$BW_{periodic\_signal} = 3 \cdot \left(\frac{1}{period}\right)$$
(7)

#### **Translating Pulse-Width to Bandwidth**

Pulse width can also be converted into a frequency representation using the Fourier Transform. In Fourier representation, a pulse in the time domain is described as the rectangle function ( $\Pi$ (t)). When this function is translated into the frequency domain, it yields the sinc function (sinc(s)). Again, the scaling between the two domains is accomplished by the Similarity Theorem which has an inverse relationship between the two domains (as in, a narrower the pulse in the time domain yields a wider sinc envelope in the frequency domain).

$$\Im\{\Pi(t \cdot width)\} = \left(\frac{1}{width}\right) \sin c \left(\frac{s}{width}\right)$$
(8)

The sinc function produces a series of envelopes as the frequency increases. The zero crossings of the sinc function will occur at integer evaluations of the sinc function argument. In this case it will be evaluated at integer evaluations of (1/width). **Figure 3** shows how a time domain pulse is represented in the frequency domain.





As mentioned before, a common rule of thumb is that the system needs enough bandwidth to capture the third harmonic of the digital pulse train. In Fourier representation, a digital pulse train is described as a rectangle function convolved with the Shaw function. In the time domain, this produces a series of pulses repeating at the maximum toggle rate of the data stream. The pulse is represented using

the rectangle function and the periodicity of the data stream is represented with the Shaw function. The convolution operator combines the two functions in the time domain. In the frequency domain, the rectangle function transforms into the sinc function, the Shaw function transforms into another Shaw function, and the convolution operator transforms into a multiplication operation. The multiplication of the Shaw and sinc functions in the frequency domain has the effect of producing Shaw impulses that are bound by the envelope of the sinc function.

$$\Im\{\Pi(t \cdot width) * III(t \cdot period)\} = \left[ \left(\frac{1}{width}\right) \cdot \sin c \left(\frac{s}{width}\right) \right] \cdot \left[ \left(\frac{1}{period}\right) \cdot III\left(\frac{s}{period}\right) \right]$$
(9)

For a 50% duty cycle pulse train, the Shaw impulses will occur at every integer evaluation of (1/period). The envelope of the sinc function will have zero crossings at every integer evaluation of (1/width), which will cancel out the Shaw impulses at these frequencies. What is left are the Shaw impulses evaluated at every odd integer evaluation of (1/period). The first odd integer impulse evaluation is called the fundamental frequency. The remaining odd integer impulse evaluations are called harmonics. As stated before, the system needs enough bandwidth to capture the third harmonic of the pulse train. In the special case just described, this occurs in the middle of the second sinc envelope. The assumption that this is sufficient bandwidth can be extended as the pulse width decreases. It can now be said that a system needs enough bandwidth to capture 1/2 of the second sinc envelop to reliably deliver the associated pulse. This relationship can be written as:

$$BW_{pulse} = 1.5 \cdot \left(\frac{1}{width}\right)$$
(10)

## **Evaluating the Logic Analyzer Bandwidth**

Logic analyzers specify the bandwidth of their front-end circuitry similar to oscilloscope front-ends. When deciding whether a logic analyzer has adequate performance for debugging a particular digital signal, the previously mentioned transforms can be used. The best way to illustrate this is through an example. A system has the following specifications for its digital signals. Each of these specifications can be converted to their analog bandwidths. The logic analyzer needs to have enough bandwidth to accommodate the highest of these analog frequencies.

	System Specifications	Corresponding Bandwidth		
Maximum Toggle Rate	600 MHz (1.2 Gb/s)	1.800 GHz		
Minimum Pulse Width	800ps (48% duty cycle)	1.875 GHz		
Rise-Time	250ps	1.4 GHz		
Logic Analyzer Bandwidth Needed		1.875 GHz		

## **Evaluating the Logic Analyzer Probe Loading**

Logic Analyzers also specify the loads of their various probing options. This is typically in the form of a lumped capacitance and/or an impedance vs. frequency profile. For a quick analysis of whether the probe load will break the system, the lumped capacitance can be used. When considering the probe load, the frequency at which its capacitance begins to shunt out the target signal must be high enough as to not effect the three previously mentioned metrics. The probe capacitance forms a RC filter with the transmission lines in the system (typically  $50\Omega$ ). This will have a response given by:

$$f_{3dB-probe} = \frac{1}{2\pi (Z_0 // Z_0) C_{probe}}$$
(11)

If we use the example in the previous section, we can determine the maximum capacitance that the

probe can present on the system without sever degradation. In the above example, the system had 1.875 GHz of bandwidth present in its digital signals. Plugging this into the above expression returns the maximum probe capacitance that can be tolerated by the system. In this case it is 3.4pF.



**Figure 4:** The capacitive load of the logic analyzer probe forms an RC filter with the impedance of the transmission line. In order to not disturb the signal being probed, the 3dB frequency of the probe's RC load must be larger than the highest analog frequency present in the digital pulse train.

#### Conclusion

As data rates increase in digital systems, engineers are being forced to understand the limitations of their validation tools. By understanding the frequency components present in their digital signals, engineers can quickly evaluate the performance ability of their logic analyzer and the logic analyzer probe load. By taking the time upfront to evaluate the performance of their tools, engineers can increase the probability of making a successful measurement and dramatically reduce their turn-on and validation time.

# **About the Author**

Brock J. LaMeres received his BSEE from Montana State University in 1998 and his MSEE from the University of Colorado in 2001. He is currently working on his Ph.D. at the University of Colorado where his research focus is High-Speed IO for next generation IC's. LaMeres is a hardware design engineer for Agilent Technologies in Colorado Springs where he designs logic analyzer probes and high-speed transport systems.

◀ Worse   Better ►						
Rate This Article	O	igodot	igodot	igodot	igodoldoldoldoldoldoldoldoldoldoldoldoldol	Submit
Add Comments	1	2	3	4	5	

Home | Site Map | Prefs | FAQs | About Us | Contact Us | Tech Groups | Ed Resources | Tech Topics | Company Advertising Opportunities | Privacy Statement | Terms and Conditions

Copyright 1996 - 2004 TechOnLine, Bedford, MA, All Rights Reserved.

