Abstract

In micromechanical models for glass fibre reinforced polymer (GFRP) composite fatigue, fibre fatigue behavior can be a key input variable since fibres are the main constituents to bear applied loads on composites. In the description of fatigue behavior, the fibre S-N curve can be obtained by a reverse-engineering technique by matching a composite S-N curve using commercial FEM codes [1]. However, the literature data required for verification of this S-N curve are rare, except for a few studies on fibre bundles [2][3]. Moreover, a reasonable description of scatter of single fibre fatigue behavior is required for reliable micromechanical modelling. In this work, a test set-up and method are developed for testing fibre fatigue behavior, the experimental results and S-N curves of single fibre fatigue tests are compared to fatigue tests of bundles consisting of 45 fibres made by the composite group of Montana State University. Furthermore, a fracture based model is used to give reasonable estimations on the fatigue behavior of glass fibres and fibre bundles. 

1 Introduction

Fatigue failure of GFRP composites is one of the key issues in wind turbine rotor blades which are designed for ca. 20 years of service. Composite fatigue behaviour is affected by many factors such as fibre and matrix properties, fibre/matrix interface properties, laminate stacking orientations, etc. One example to show the effect of fibre properties on composite fatigue behavior is given in Figure 1. This figure shows the comparison of fatigue test results between 4-layer unidirectional glass fibre reinforced epoxy resin composites and pure epoxy resin. The run-out tests (which are fatigue tests stopped before specimen failure) are indicated by arrows. It can be seen that composites give a longer and, in this figure, slightly more scattered fatigue life diagram comparing to the pure epoxy resin.
Figure 1. Comparison of fatigue behavior of unidirectional composites and the epoxy resin

The understanding of fibre effects on composite fatigue behavior requires the knowledge of fibre fatigue characteristics. For the characterisation of fibre fatigue, a test set-up and method are developed for testing fibre fatigue behavior, to validate a fracture based model to estimate the fatigue behavior of glass fibres. Moreover, fatigue characteristics between single fibres and fibre bundles consisting of 45 fibres impregnated in the CoRezyn 63-AX-051 orthophthalic polyester matrix [4] are compared and discussed.

2 Experiments

The same glass fibres as used in the fibre bundles [4] were employed in this paper. The glass fibres have an average diameter of 10.66 µm of which the standard deviation is 0.93 µm, and a reference Young’s modulus of 72.4 GPa, according to the manufacturer [4]. Fibre specimens with 25 mm gauge length were prepared according to the ASTM D 3379-75 standard [5].

During the tests, an LCL-113g 1 N maximum capacity load cell manufactured by OMEGA Corporation was mounted on a 25 kN maximum capacity servo-hydraulic test machine to monitor applied fatigue loads. The fibre specimen consists of a paper frame, in which a single fibre has been fixed with adhesive, see Figure 2. The specimen was fixed on a steel ring connected with the load cell at the top end using a drop of KYOWA CC-33A adhesive (below the folded line in Figure 2). Then, before the start of the test, the two supporting paper edges were cut. Figure 3 shows a mounted specimen. An external Solartron displacement sensor with 10 mm measurement range was used to measure displacements. This sensor was also used to control the test. In all fatigue tests, the R ratio (the ratio between minimum and maximum applied strains) was set to be 0.1. The test frequency was set to be 1 Hz at the beginning and then increased to 20 Hz gradually over the first few hundred cycles.
Since the employed load cell involves a C-shaped set of flexible arms, additional displacements due to load cell bending should be carefully checked during fatigue tests. To find the relationship between load cell bending displacements and applied loads, a dozen tensile tests (not to failure) on a steel foil specimen were carried out prior to fibre tests. The results from those tests show a linear relationship of applied load versus load cell bending displacement. Three LCL113-g load cells were used during the test programme described in this paper. Because of the slight change of the steel ring mounting location of each load cell, the average slopes of load-displacement curves from the three load cells were 459 mN/mm, 465 mN/mm, and 515 mN/mm, respectively.

Before each fatigue test, a pre-tension test (up to 150 mN) was performed to determine the slope of the load-displacement curve of the specimen. Then maximum and minimum applied loads can be calculated using Equation (1).

\[
\frac{F_{\text{max}}}{S_{\text{total}}} - \frac{1}{S_{\text{loadcell}}} = \frac{d_{\text{max}}}{1} \quad \text{and} \quad \frac{F_{\text{min}}}{S_{\text{total}}} - \frac{1}{S_{\text{loadcell}}} = \frac{d_{\text{min}}}{1}
\]

in which, \(F_{\text{max}}\), \(F_{\text{min}}\), \(d_{\text{max}}\), and \(d_{\text{min}}\) are maximum and minimum applied loads and fibre displacements, respectively; \(S_{\text{total}}\) is the slope of load-displacement curve from the fibre pre-tension test, and \(S_{\text{loadcell}}\) is the average slope of load-displacement curves from load cell bending tests.

3 Fatigue characteristics of glass fibres and fibre bundles

Experimental single glass fibre characteristics are shown by red oblique cross symbols in Figure 4, in which run-out tests are indicated by arrows in red. Large scatter of fibre fatigue life is observed. For instance, at the maximum strain \(\varepsilon_{\text{max}}\) level from 0.025 to 0.035, the fatigue lives scatter spans approximately a factor of 1000. In addition, extremely long fatigue
tests (longer than 10 million cycles) have already appeared at $\varepsilon_{\text{max}}$ of 0.025, which is approximately 78% of the average fibre tensile strain. Surprisingly, a 1 cycle fatigue test can also be found at $\varepsilon_{\text{max}}$ of 0.022.

The fatigue characteristics of fibre bundles consisting of 45 fibres impregnated in the CoRezyN 63-AX-051 orthophthalic polyester matrix are also shown by black dots in Figure 4, in which run-out tests are indicated by arrows in black. The fatigue lives of fibre bundles were tested by the composites group of Montana State University. It can be seen that fatigue life scatter of glass fibres is much larger than that of fibre bundles. This phenomenon agrees with the larger scatter in single glass fibres static strengths (see Table 1 below, the Weibull shape parameter $\beta$ of glass fibres is much smaller than that of fibre bundles).

![Figure 4. S-N curves of fibre bundles and single fibres](image)

S-N curves of fatigue lives of both fibres and fibre bundles excluding runouts and 1 cycle fatigue tests are extracted using the least square method and drawn in green and pink lines in Figure 4, respectively. Looking at these regression lines (which are partly extrapolated into the low and high cycle regions), it appears that the glass fibres have a flatter S-N curve compared to the fibre bundles. That means, single glass fibres have longer fatigue lives in the high cycle fatigue region and shorter fatigue lives in the low cycle fatigue region. Additionally, it is found that fatigue life scatter of single glass fibres is also much larger than that of fibre bundles.
However, more fatigue tests of fibre bundles at $\varepsilon_{\text{max}}$ above 0.035 and glass fibres at $\varepsilon_{\text{max}}$ below 0.02 are required before the above observations can be verified. In addition, damage observations in the fibre tests are recommended to facilitate explanation of any differences in behaviour and to explain the difference in scatter. There is the possibility that, for $\varepsilon_{\text{max}}$ below 0.02, the S-N curve of glass fibres becomes “steeper” and gets closer to the S-N curve of fibre bundles.

4 Fibre fatigue model

On the basis of fracture mechanics [6][7], fibre fatigue damage evolution is simplified by the propagation of a dominant elliptical flaw perpendicular to the loading axis. A strength degradation model derived from Griffith’s crack theory and Paris’ Law, is shown in Equation (2). For the detailed derivation, refer to [7].

$$\sigma_{\text{res}} (N) = \sigma_{\text{res}} - k \sigma_{\text{max}}^{(c+2)N}$$  \hspace{1cm} (2)

in which $\sigma_{\text{res}}$ and $\sigma_{\text{f}}$ are fibre residual and initial tensile strength respectively, $\sigma_{\text{max}}$ is the maximum applied stress, $N$ is the number of elapsed fatigue cycles and $c$ and $k$ are model parameters given by Equation (3).

$$c = m - 2 \hspace{0.2cm}, \hspace{0.2cm} k = \frac{\pi C}{2} \left(1 - R^{(c+2)}\right)K_{\text{lc}}^c$$  \hspace{1cm} (3)

in which $K_{\text{lc}}$ is fibre toughness under mode I, $R$ is the stress ratio between applied maximum and minimum loads, $m$ and $C$ are material constants in Paris’ Law, which is.

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (4)

in which $\Delta K$ is the range of stress intensity factor between applied maximum and minimum loads.

When $\sigma_{\text{res}}$ reduces to $\sigma_{\text{max}}$, the fibre is assumed to be broken. Then Equation (2) turns to

$$\sigma_{\text{f}} = \sigma_{\text{res}} + k \left(\sigma_{\text{max}}\right)^{(c+2)N}$$  \hspace{1cm} (5)

Due to the high ratio of length-to-diameter of glass fibres, any effect of stresses perpendicular to the length direction is neglected. Thus Equation (5) can be rewritten by replacing stress by strain using Hooke’s Law

$$\varepsilon_{\text{f}} = \varepsilon_{\text{res}} + k E^2 \left(\varepsilon_{\text{max}}\right)^{(c+2)N}$$  \hspace{1cm} (6)

in which $\varepsilon_{\text{f}}$ is fibre tensile strain, $\varepsilon_{\text{max}}$ is the strain at maximum stress, $E$ is the fibre Young’s modulus. Tensile strain $\varepsilon_{\text{f}}$ is described by a 2-parameter Weibull distribution:
in which \( \beta \) and \( \varepsilon_0 \) are shape and scale parameters, respectively; \( P \) is fibre survival probability under \( \varepsilon_t \). Substituting Equation (6) into Equation (7) gives

\[
P(N) = \exp \left\{ - \left[ \frac{N + \frac{1}{kE^2\varepsilon_{\max}^2}}{\frac{\varepsilon_0^c}{kE^2\varepsilon_{\max}^{c+2}}} \right]^\beta \right\}
\]

in which the location parameter \( \frac{1}{kE^2\varepsilon_{\max}^2} \) can be neglected in long cycle fatigue tests since it is much smaller comparing to \( N \), and Equation (8) becomes a 2-parameter Weibull distribution \([8]\). Thus the scale parameter \( \frac{\varepsilon_0^c}{kE^2\varepsilon_{\max}^{c+2}} \) is related to the average fibre fatigue life \( N_{avg} \) by

\[
N_{avg} = \frac{\varepsilon_0^c}{kE^2\varepsilon_{\max}^{c+2}} \times \Gamma \left( 1 + \frac{c}{\beta} \right)
\]

in which \( \Gamma \) indicates the gamma function. By performing the logarithms on both side of Equation (9), an S-N form description of fibre fatigue characteristics is shown in Equation (10).

\[
\log N_{avg} = -(c + 2) \log \varepsilon_{\max} + \left( c \log \varepsilon_0 - \log(kE^2) + \log \left( \frac{\varepsilon_0}{\varepsilon_{\max}^{c+2}} \right) \right) \times \Gamma \left( 1 + \frac{c}{\beta} \right)
\]

If the experimental S-N curve is assumed to be formed by the average fibre life at every \( \varepsilon_{\max} \) level, parameters \( c \) and \( k \) can be obtained by curve fitting of the experimental S-N curve. The Weibull parameters in Equation (7) of 25 mm glass fibre and fibre bundles are given in Table 1 \([4][9]\).

<table>
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<tr>
<th></th>
<th>( \beta )</th>
<th>( \varepsilon_0 )</th>
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<tr>
<td>Single glass fibre</td>
<td>6.34</td>
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<tr>
<td>Fibre bundle</td>
<td>22.70</td>
<td>4.47e-2</td>
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**Table 1** Weibull parameters of single glass fibres and fibre bundles

Then the parameters \( c \) and \( k \) from the fibre S-N curves (see the green and pink lines in Figure 5) are extracted and shown in Table 2. Comparing to glass fibres, fibre bundles have a smaller \( c \) but a larger \( k \).
Using the extracted parameters $c$ and $k$, Equation (6) indicates that the residual strain of glass fibres almost never degrades during most of its fatigue life but drops suddenly in the end (“sudden-death” strength degradation). From the test results of 30 fibre tension tests, the maximum fibre tensile strain is found to be 0.041 [9]. On one occasion, the residual tensile strain of the fibre after 10 million cycles at $\varepsilon_{\text{max}}$ of 0.023 was measured in a post tension test to still be as high as 0.038. This indicates that fibre tensile strain is not severely affected by fatigue and supports the earlier observation of sudden-death degradation.

The estimated fatigue lives of glass fibres and fibre bundles subjected to different maximum applied strain $\varepsilon_{\text{max}}$ are shown in Figure 5 and Figure 6, respectively. At each $\varepsilon_{\text{max}}$ level in both graphs, 100 fatigue lives are calculated following Equation (6) with the parameters shown in Table 2. The initial strains $\varepsilon_i$ of glass fibres and fibre bundles are generated following Equation (7) with the parameters shown in Table 1 using the Monte-Carlo method. The survival probability $P$ in Equation (7) is randomly generated between 0 and 1 by a uniformly distributed function [10]. It can be seen that in Figure 5 the fatigue life estimations follow the test data reasonably well apart from 1 cycle tests indicated by the values displayed at 1 cycle of each $\varepsilon_{\text{max}}$. In Figure 6 the fatigue life estimations are in good agreement with the test data in the fatigue life region of less than $10^6$ cycles, whereas in the higher fatigue cycle region, the estimations underestimate the fatigue life scatter.

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$k$</th>
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<tbody>
<tr>
<td>Single glass fibre</td>
<td>13.04</td>
<td>5.63e-11</td>
</tr>
<tr>
<td>Fibre bundle</td>
<td>10.44</td>
<td>8.97e-10</td>
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Table 2 Extracted model parameters $c$ and $k$ of single glass fibres and fibre bundles

**Figure 5.** Fatigue life estimations of single glass fibres

**Figure 6.** Fatigue life estimations of fibre bundles
6 Conclusions

R=0.1 fatigue tests on single glass fibres are carried out in this paper. The fatigue characteristics between glass fibres and fibre bundles consisting of 45 fibres are compared. With the available data, it seems that at low strain ranges, glass fibres have longer fatigue lives than fibre bundles, although fibre fatigue data in this region are lacking. Further development of the dataset is recommended although the large fatigue life scatter of glass fibres hinders the generation of fatigue data at low strains.

Furthermore, a fracture based fatigue model is used to describe the fatigue characteristics of glass fibres and fibre bundles. With the model parameters extracted from experimental S-N curves, the fatigue life estimations agree reasonably well with the experimental data on glass fibres and fibre bundles at the fatigue life region of less than 10e6 cycles.

Acknowledgement

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References

[9] Qian C., Static tests on single glass fibres, WMC internal report WMC-2010-25 (2010)