ME 480 Introduction To Aerospace
Midterm Test, Closed Notes, 15 min max.
(20 pts total)

Name____________________________

1. (1 pt) T / F The source of all aerodynamic forces on a body is the pressure distribution and the shear stress distribution over the surface.

2. (1 pt) Write down the relation between absolute altitude \( h_a \) and geometric altitude \( h_G \). \[ h_a = h_G + \text{radius of the earth} \]

3. (1 pt) T F The Standard Atmosphere is defined by gas dynamics from physics.

4. (2 pts) What are the two criteria for an isentropic process?
   \[ \text{adiabatic} \quad \text{reversible} \]

5. (2 pts) Euler’s Equation is a statement of the \( \text{momentum} \) equation, while Bernoulli’s Equation is derived from the \( \text{energy} \) equation.

6. (1 pt) T / F Pressure Drag is typically more for laminar flow.

7. (1 pt) T / F Turbulent boundary layer shear stress is less than that in a laminar boundary layer.

8. (3 pts) Which variables remain constant for isentropic flow? \( T_0, p_0, \text{and} \rho_0 \).

9. (3 pts) Reynold’s number is a ratio of what forces acting on a body as a consequence of flow? \text{Inertial to Viscous}

10. (2 pts) Why are swept wings advantageous for high Mach numbers?
    \text{Increases the critical Mach number because of normal velocity component to leading edge.}

11. (1 pt) T F Systems Engineering typically occurs early in the life cycle of a project when 85% of the budget is committed.

12. (1 pt) As a fluid element flows through a shock wave, what happens to the total pressure \( P_0 \)? \text{decreases}

13. (1 pt) T/F The Rayleigh Pitot Tube Formula assumes isentropic flow.
1. In a gas turbine engine, the pressure of the incoming air is increased by flowing through a compressor; the air then enters a combustor that looks vaguely like a long can (often called the “combustion can”). Fuel is injected into the combustor and burns with the air, and the burned fuel-air mixture exits the combustor at a higher temperature than the air coming into the combustor. The pressure of the flow through the combustor remains relatively constant; that is the combustion process is at constant pressure. Consider the case where the gas pressure and temperature entering the combustor is $4 \times 10^6$ N/m$^2$ and 900 K. The gas temperature exiting the combustor is 1500 K. Calculate the gas density at:

a) The inlet to the combustor and  
b) The exit of the combustor

(assume that the specific gas constant for the fuel-air mixture is the same as that for pure air)
Solution

Let $p_3$ and $T_3$ denote conditions at the inlet to the combustor, and $T_4$ denote the temperature at the exit. Note: $p_3 = p_4 = 4 \times 10^6$ N/m\(^2\)

(a) $\rho_3 = \frac{p_3}{RT_3} = \frac{4 \times 10^6}{(287)(900)} = 15.49 \text{ kg/m}^3$

(b) $\rho_4 = \frac{p_4}{RT_4} = \frac{4 \times 10^6}{(287)(1500)} = 9.29 \text{ kg/m}^3$
2. An airplane is flying at a pressure altitude of 10 km at a velocity of 596 m/s. The outside air temperature is 200 K. What is the pressure measured by a Pitot tube mounted on the nose of the airplane?

Solution

\[ a_1 = \sqrt{\gamma RT_1} = \sqrt{(1.4)(287)(220)} = 297 \text{ m/sec} \]

\[ M_1 = \frac{V_1}{a_1} = \frac{596}{197} = 2.0 \]

The flow is supersonic. Hence, the Rayleigh Pitot tube formula must be used.

\[
\frac{p_{0_2}}{p_1} = \left[ \frac{(\gamma + 1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)} \right]^{\gamma-1} \left[ \frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1} \right]^{\frac{\gamma}{\gamma-1}}
\]

\[
\frac{p_{0_2}}{p_1} = \left[ \frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)} \right]^{3.5} \left[ \frac{1 - 1.4 + 2(1.4)(2)^2}{2.4} \right]^{\frac{1.4}{1.4}}
\]

\[
\frac{p_{0_2}}{p_1} = 5.64
\]

\[ p_1 = 2.65 \times 10^4 \text{ N/m}^2 \text{ from Appendix A.} \]

Hence,

\[ p_{0_2} = 5.64(2.65 \times 10^4) = 1.49 \times 10^5 \text{ N/m}^2 \]
3. The wing of the Fairchild Republic A-10A twin-turbofan close support airplane is approximately rectangular with a wingspan of 17.5m and a chord of 3m. The airplane is flying at standard sea level with a velocity of 200 m/s.

a) If the flow is considered to be completely laminar, calculate the boundary layer thickness at the trailing edge and the total skin friction drag. Assume incompressible flow.

b) Now assume that the flow is completely turbulent and calculate the boundary layer thickness at the trailing edge and the total skin friction drag. Compare this result to part a).
Solution

a)

First, calculate the value of the Reynolds number.

\[ \text{Re}_L = \frac{\rho_{\infty} V_{\infty} L}{\mu_{\infty}} = \frac{(1.225)(200)(3)}{(1.7894 \times 10^{-5})} = 4.10 \times 10^7 \]

The dynamic pressure is

\[ q_{\infty} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 = \frac{1}{2} (1.225)(200)^2 = 2.45 \times 10^4 \text{ N/m}^2 \]

Hence,

\[ \delta_L = \frac{5.2L}{\sqrt{\text{Re}_L}} = \frac{5.2(3)}{\sqrt{4.1 \times 10^7}} = 0.0024 \text{ m} = 0.24 \text{ cm} \]

and

\[ C_f = \frac{1.328}{\sqrt{\text{Re}_L}} = \frac{1.328}{\sqrt{4.1 \times 10^7}} = 0.00021 \]

The skin friction drag on one side of the plate is:

\[ D_f = q_{\infty} Sc_f = (2.45 \times 10^4)(3)(17.5)(0.00021) \]

\[ D_f = 270 \text{ N} \]

The total skin friction drag, accounting for both the top and the bottom of the plate is twice this value, namely

Total \( D_f = 540 \text{ N} \)
b) \[
\delta = \frac{0.37L}{(Re_L)^{0.2}} = \frac{0.37(3)}{(4.1 \times 10^7)^{0.2}} = 0.033 \text{ m} = 3.3 \text{ cm}
\]

From part a) above, we find

\[
\frac{\delta_{\text{turbulent}}}{\delta_{\text{laminar}}} = \frac{3.3}{0.24} = 13.75
\]

The turbulent boundary layer is more than an order of magnitude thicker than the laminar boundary layer.

\[
C_f = \frac{0.074}{(Re_L)^{0.2}} = \frac{0.074}{(4.1 \times 10^7)^{0.2}} = 0.0022
\]

The skin friction drag on one side is then

\[
D_f = q_w \cdot Sc_f = (2/45 \times 10^4)(3)(17.5)(0.0022)
\]

\[
D_f = 2830 \text{ N}
\]

The total, accounting for both top and bottom is

Total \(D_f = 5660 \text{ N}
\]

Comparing to part a) above, we find

\[
\frac{(D_{f_{\text{turbulent}}})}{(D_{f_{\text{laminar}}})} = \frac{5660}{540} = 10.5
\]

The turbulent skin friction drag is an order of magnitude larger than the laminar value.
4. During the 1920s and early 1930s, the NACA obtained wind tunnel data on different airfoils by testing finite wings with an aspect ratio of 6. These data were then “corrected” to obtain infinite wing airfoil characteristics. Consider such a finite wing with an area of 1.5 ft$^2$ and an aspect ratio of 6. The wing was mounted in a wind tunnel where the test section flow velocity was 100 ft/s at standard sea level conditions. When the wing was pitched to $\alpha = -2^\circ$ no lift was measured. When the wing was pitched to $\alpha = 10^\circ$, a lift of 17.9 lb was measured. Calculate the lift slope for the infinite wing airfoil. Assume a span effectiveness factor of 0.96.

Solution

\[ q_{\infty} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 = \frac{1}{2} (0.002377)(100)^2 = 11.9 \text{ lb/ft}^2 \]

at $\alpha = 10^\circ$, $L = 17.9$ lb. Hence

\[ C_L = \frac{L}{q_{\infty} S} = \frac{17.9}{(11.9)(1.5)} = 1.0 \]

at $\alpha = -2^\circ$, $L = 0$ lb. Hence $\alpha_L = 0 = -2^\circ$

\[ a = \frac{dC_L}{d\alpha} = \frac{1.0 - 0}{[10 \cdot (2)]} = 0.083 \text{ per degree} \]

This is the finite wing lift slope.

\[ a = \frac{a_0}{1 + \frac{57.3 a_0}{\pi e AR}} \]

Solve for $a_0$.

\[ a_0 = \frac{a}{1 - \frac{57.3 a}{\pi e AR}} = \frac{0.083}{1 + \frac{57.3(0.083)}{\pi (0.96)(6)}} \]

\[ a_0 = 0.11 \text{ per degree} \]
1. Consider a thin airfoil (treated as a flat plate) at an angle of attack of 20° in a Mach 2.2 airflow. (Mach 2.2 was the cruising Mach number of the Concorde supersonic transport.) The length of the plate in the flow direction is 202 ft (the length of the Concorde). Assume that the free-stream conditions correspond to a standard altitude of 50,000 ft. The results given in Chapter 4 for skin friction hold for incompressible flow only; there is a compressibility effect on $C_f$, such that its value decreases with increasing Mach number. Specifically, at Mach 2.2, assume that the $C_f$ given in Chapter 4 is reduced by 20%.

a) Given the above, calculate the total drag coefficient for the plate.
   (Note: Estimate the viscosity from the figure provide in Problem 2 above.)

b) If the angle of attack is increased to 50°, assuming that $C_f$ remains the same, calculate the total drag coefficient.

c) For these cases, what can you assume about the relative influence of wave drag and skin friction drag?

### SOLUTION

(a) At 50,000 ft, $\rho_{\infty} = 3.6391 \times 10^{-5} \text{ slug/ft}^3$ and $T_{\infty} = 390^\circ \text{R}$. Hence,

$$a_{\infty} = \sqrt{\gamma R T_{\infty}} = \sqrt{(1.4)(1716)(390)} = 968 \text{ ft/sec}$$

and

$$V_{\infty} = a_{\infty} M_{\infty} = (968)(2.2) = 2130 \text{ ft/sec}$$

The viscosity coefficient at $T = 390^\circ \text{R} = 216.7 \text{ K}$ can be estimated from an extrapolation of the figure provided for Problem 2. The slope of this line is:
\[
\frac{d\mu}{dT} = \frac{(2.12 - 154) \times 10^{-5}}{(350 - 250)} = 5.8 \times 10^{-8} \frac{\text{kg}}{(\text{m})(\text{sec})(\text{K})}
\]

Extrapolating from the sea level value of \( \mu = 1.7894 \times 10^{-5} \text{ kg/(m)(sec)} \), we have at \( T = 216.7 \text{ K} \):

\[
\mu_\infty = 1.7894 \times 10^{-5} - (5.8 \times 10^{-8})(288 - 216.7)
\]

\[
\mu_\infty = 1.37 \times 10^{-5} \text{ kg/(m)(sec)}
\]

Converting to English engineering units, using the information in Chapter 4, we have

\[
\mu_\infty = \frac{1.37 \times 10^{-5}}{1.7894 \times 10^{-5}} (3.7373 \times 10^{-7} \frac{\text{slug}}{\text{ft sec}}) = 2.86 \times 10^{-7} \frac{\text{slug}}{\text{ft sec}}
\]

Finally, we can calculate the Reynolds number for the flat plate:

\[
Re_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{3.6391 \times 10^{-4} (2130)(202)}{2.86 \times 10^{-7}} = 5.47 \times 10^8
\]

Thus, from Eq. (4.100) reduced by 20 percent

\[
C_f = (0.8) \frac{0.074}{(Re_L)^{0.2}} = (0.8) \frac{0.074}{(5.74 \times 10^8)^{0.2}} = 0.00106
\]

The wave drag coefficient is estimated from

\[
c_{d,w} = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}
\]

where \( \alpha = \frac{2}{57.3} = 0.035 \text{ rad} \).

Thus,

\[
c_{d,w} = \frac{4(0.035)^2}{\sqrt{(2.2)^2 - 1}} = 0.0025
\]

Total drag coefficient = 0.0025 + (2)(0.00106) = 0.00462
Note: In the above, \( C_f \) is multiplied by two, because Eq. (4.100) applied to only one side of the flat plate. In flight, both the top and bottom of the plate will experience skin friction, hence that total skin friction coefficient is \( 2(0.00106) = 0.00212 \).

(b) If \( \alpha \) is increased to 5 degrees, where \( \alpha = 5\pi/57.3 \cdot 0.00873 \) rad, then

\[
    c_{d,w} = \frac{4(0.0873)^2}{\sqrt{(2.2)^2 - 1}} = 0.01556
\]

Total drag coefficient = \( 0.01556 + 2(0.00106) = 0.0177 \)

(c) In case (a) where the angle of attack is 2 degrees, the wave drag coefficient (0.0025) and the skin friction drag coefficient acting on both sides of the plate (2 \( \times \) 0.00106 = 0.00212) are about the same. However, in case (b) where the angle of attack is higher, the wave drag coefficient (0.0177) is about eight times the total skin friction coefficient. This is because, as \( \alpha \) increases, the strength of the leading edge shock increases rapidly. In this case, wave drag dominates the overall drag for the plate.