Composite Materials for Aircraft Structures: A Brief Review of Practical Application

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Montana State University

ME 480 Introduction to Aerospace, Spring 2010
Introduction

- Composite materials are used more and more for primary structures in commercial, industrial, aerospace, marine, and recreational structures.
From Last Time

• Composite parts used for aircraft applications are defined by
  – Material, process, and manufacturing specifications.
  – Material allowable (engineering definition).

• All of these have a basis in regulatory requirements.

• Most efficient use of advanced composites in aircraft structure is in applications with
  – Highly loaded parts with thick gages.
  – High fatigue loads (fuselage and wing structure, etc).
  – Areas susceptible to corrosion (fuselage, etc).
  – Critical weight reduction (empennage, wings, fuselage, etc).

• Use must be justified by weighing benefits against costs.
Composition of Composites

Fiber/Filament Reinforcement

- High strength
- High stiffness
- Low density
- Carbon, Glass, Aramid, etc

Matrix

- Good shear properties
- Low density
- Thermoset & Thermoplastic
- Epoxy, Polyester, PP, Nylon, Ceramics, etc.

Composite

- High strength
- High stiffness
- Good shear properties
- Low density
- Anisotropic!
Overview

• **Micromechanics**
  – Study of mechanical behavior of a composite material in terms of its constituent materials

• **Ply Mechanics**
  – Study of mechanical behavior of individual material plies based on variations from global coordinate system

• **Macromechanics**
  – Study of mechanical behavior utilizing ply mechanics of a homogenized composite material

• **Failure Theories**
CADEC: Introduction

**Computer Aided Design Environment For Composites**

Compliment to "Introduction to Composite Materials Design", Taylor&Francis (1999) by Ever J. Barbero


Software free online—search keywords CADEC & Barbero

- Chapter 6 - Macromechanics
- Chapter 7 - Failure Theory
- Chapter 8 - Thin Walled Beams

**Field Types:**

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Micromechanics: Assumptions

• **Lamina:**
  – Macroscopically homogeneous
  – Linearly elastic
  – Macroscopically Orthotropic
  – Initially stress free

• **Fibers:**
  – Homogeneous
  – Linearly elastic
  – Isotropic/Orthotropic
  – Regularly spaced
  – Perfectly aligned

• **Matrix:**
  – Homogeneous
  – Linearly elastic
  – Isotropic
  – Assumptions in Micromechanics of Composites

Carbon/epoxy (AS4/3501-6) composite (Vf=.70)
Micromechanics: Rule of Mixtures

Assumption: Axial (linear) strain in fiber and matrix

Strain in the composite: \( \varepsilon_1 = \frac{\Delta L}{L} = \varepsilon_f = \varepsilon_m \)

Total force in composite: \( \sigma_1 A_c = \sigma_f A_f \) + \( \sigma_m A_m \)

Stress in the composite: \( \sigma_1 = \sigma_f \frac{A_f}{A_c} \) + \( \sigma_m \frac{A_m}{A_c} \) = \( \sigma_f V_f \) + \( \sigma_m V_m \)

\[ \varepsilon_1 E_1 = \varepsilon_1 E_f V_f + \varepsilon_1 E_m V_m \]

\[ E_1 = E_f V_f + E_m V_m \]  

Rule of mixtures

V_{f,max} \text{ approximately 78%} 
Common range = 55-67%
Micromechanics: Determining Properties

Chapter 4 - Micromechanics

- E₁ - Rule of Mixtures
- E₂ - Rule of Mixtures
- E₂ - Halpin-Tsai
- V₁₂ - Rule of Mixtures
- G₁₂ - Rule of Mixtures
- G₁₂ - Cylindrical Assemblage
- G₂₃ - Stress Partitioning
- Coefficient of Thermal Expansion
- Coefficient of Moisture Expansion
- Transport Properties
- E₁, E₂ - Periodic Microstructure
- V₁₂, V₂₃ - Periodic Microstructure
- G₁₂, G₂₃ - Periodic Microstructure
- Continuous Strand Mat - E, G, V
- Continuous Strand Mat - Strengths
- Continuous Strand Mat - α
- Continuous Strand Mat - β
- Longitudinal Tensile Strength
- Longitudinal Compressive Strength
- Transverse Tensile Strength
- Transverse Compressive Strength
- Inplane Shear Strength
Micromechanics: Rule of Mixtures ($E_1$)

Longitudinal Modulus - Rule of Mixtures

\[ E_1 = E_f V_f + E_m V_m \]

- $E_f = 72.345$
- $V_f = 0.5$
- $E_m = 3.4$
- $E_1 = 3.78725 \times 10^1$

Design and Analysis of Aircraft Structures
Micromechanics: Rule of Mixtures ($E_2$)

Transverse Modulus - Rule of Mixtures

\[ \frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \]

- $E_f = 72.345$
- $V_f = 0.5$
- $E_m = 3.4$
- $E_2 = 6.49477 \times 10^0$

Graph showing the relationship between $E_2$ and fiber volume fraction.
Micromechanics: Determining Properties

Chapter 4 - Micromechanics

- $E_1$ - Rule of Mixtures
- $E_2$ - Rule of Mixtures
- $E_2$ - Halpin-Tsai
- $V_{12}$ - Rule of Mixtures
- $G_{12}$ - Rule of Mixtures
- $G_{12}$ - Cylindrical Assemblage
- $G_{23}$ - Stress Partitioning
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Design and Analysis of Aircraft Structures
Micromechanics: Rule of Mixtures ($\nu_{12}$)

Inplane Poisson's Ratio - Rule of Mixtures

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

\[\begin{align*}
\nu_f &= 0.22 \\
\nu_m &= 0.38 \\
V_f &= 0.5 \\
V_{12} &= 0.30000
\end{align*}\]
Micromechanics: Rule of Mixtures ($G_{12}$)

\[ \frac{1}{G_{12}} = \frac{V_m}{G_m} + \frac{V_f}{G_f} \]

- $G_f = 2.96496 \times 10^1$
- $V_f = 0.5$
- $G_m = 1.23188 \times 10^0$
- $G_{12} = 2.36548 \times 10^0$

Inplane Shear Modulus - Rule of Mixtures

Design and Analysis of Aircraft Structures
Micromechanics: Other Methods & Strengths

- $E_1$: Rule of Mixtures
- $E_2$: Rule of Mixtures
- $E_2$: Halpin-Tsai
- $V_{12}$: Rule of Mixtures
- $G_{12}$: Rule of Mixtures
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Micromechanics: Halpin-Tsai ($E_2$)

Halpin-Tsai: “Semiempirical (1969) version to obtain better prediction”—Barbero

$\zeta \equiv$ empirical curve fitting parameter, commonly $2a/b$
Micromechanics: Determining Properties

Chapter 4 - Micromechanics

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Micromechanics: Longitudinal Tensile Strength

Longitudinal Tensile Strength

- $E_f = 72.345$
- $E_m = 3.4$
- $V_f = 0.5$
- $F_{1t} = 9.03035 \times 10^{-1}$
- $\sigma_{fa} = 1.725$

Design and Analysis of Aircraft Structures
Micromechanics: Thermal & Electrical Conductivity

Transport Properties

\[ k_f = 1.05 \quad a/b = 1 \]
\[ k_m = 0.2 \quad V_f = 0.5 \]
\[ k_1 = 0.25000 \times 10^{-1} \quad k_2 = 3.36000 \times 10^{-1} \]

Note: \( k_f \) and \( k_m \) stand for either thermal conductivity, electrical conductivity, or mass diffusivity.

Design and Analysis of Aircraft Structures 13-22
Ply Mechanics

- So what happens if we vary the fiber direction angle away from the 1-direction?
- CADEC uses Micromechanics results and fiber angle
  - Plane Stress
  - Transform stress/strain
  - Off-Axis Compliance/Stiffness
  - 3D Constitutive Equ’s

Figure 10.1 Idealized representation of a unidirectional fiber-reinforced material.
## Ply Mechanics: CADEC

### Chapter 5 - Ply Mechanics

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### Stress and Strain Transformations | 3D Constitutive Equations

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</table>
Ply Mechanics: Compliance Plane Stress

Compliance Equations - [S] Matrix

\[
\begin{align*}
\sigma_1 &= 1.4e6 & \varepsilon_1 &= 2.74606E+04 \\
\sigma_2 &= 1.2e6 & \varepsilon_2 &= 9.53771E+04 \\
\sigma_6 &= .8e6 & \gamma_6 &= 2.40128E+05 \\
\varepsilon_1 &= 3.78725E+01 & E_1 &= 3.78725E+01 \\
E_2 &= 1.12711E+01 & E_2 &= 1.12711E+01 \\
G_{12} &= 3.33156E+00 & \nu_{12} &= 0.30000
\end{align*}
\]

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6
\end{bmatrix} = \begin{bmatrix}
1/E_1 & -\nu_{12}/E_1 & 0 \\
-\nu_{12}/E_1 & 1/E_2 & 0 \\
0 & 0 & 1/G_{12}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}
\]
### Ply Mechanics: CADEC

#### Chapter 5 - Ply Mechanics

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Design and Analysis of Aircraft Structures
Ply Mechanics: Transformations

Transform Material Stresses to Global Stresses

\[
\begin{align*}
\sigma_1 &= 100 \\
\sigma_2 &= 10 \\
\sigma_6 &= -5 \\
\sigma_4 &= 6.42909E04 \\
\sigma_5 &= 6.42909E04 \\
\sigma_x &= 3.49106E+01 \\
\sigma_y &= 7.50894E+01 \\
\sigma_{xy} &= -4.05761E+01 \\
\sigma_{yz} &= 8.95398E+04 \\
\sigma_{xz} &= -1.57883E+04
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} &=
\begin{bmatrix}
m^2 & n^2 & -2mn \\
n^2 & m^2 & 2mn \\
mn & -mn & m^2 - n^2
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}
\]

\[
\begin{align*}
\begin{bmatrix}
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix} &=
\begin{bmatrix}
m & -n \\
n & m
\end{bmatrix}
\begin{bmatrix}
\sigma_4 \\
\sigma_5
\end{bmatrix}
\end{align*}
\]

\[
m = \cos(\theta) \\
n = \sin(\theta)
\]
**Ply Mechanics: Off-Axis Stiffness Matrices**

### Off-Axis Stiffness Matrix - $[\bar{Q}]$

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>$1.50706 \times 10^0$</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>$2.05733 \times 10^0$</td>
</tr>
<tr>
<td>$\gamma_{xy}$</td>
<td>$6.42909 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>$4.95793 \times 10^1$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$5.43611 \times 10^1$</td>
</tr>
<tr>
<td>$\sigma_{xy}$</td>
<td>$3.13561 \times 10^1$</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>$0.30000$</td>
</tr>
<tr>
<td>$\theta^o$</td>
<td>$45$</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} = [\bar{Q}] \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.76928 \times 10^1 & 9.00294 \times 10^0 & 6.83338 \times 10^0 \\
9.00294 \times 10^0 & 1.76928 \times 10^1 & 6.83338 \times 10^0 \\
6.83338 \times 10^0 & 6.83338 \times 10^0 & 1.08868 \times 10^1
\end{bmatrix}
\]

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# Ply Mechanics: CADEC

## Chapter 5 - Ply Mechanics

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</table>
Ply Mechanics: Stress-Strain Relationships

- Stress-Strain Relationship: \( \sigma_{ij} = C_{ij} \epsilon_{ij} \)

- With 3 planes \( \rightarrow C_{ij} \) has 81 terms, but since: \( \sigma_{ij} = \sigma_{ji} \) and: \( \epsilon_{ij} = \epsilon_{ji} \) only 36 terms

- Orthotropic material (2 planes of symmetry) reduces to 9 terms:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yx} \\
\sigma_{xy} \\
\sigma_{xz}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\epsilon_{yx} \\
\epsilon_{xy} \\
\epsilon_{xz}
\end{bmatrix}
\]
Ply Mechanics: Orthotropic Material

Orthotropic Material - Stresses to Strains

| $\sigma_1$  | $\sigma_2$  | $\sigma_3$  | $\sigma_4$  | $\sigma_5$  | $\sigma_6$  | $E_1$       | $E_2 = E_3$ | $G_{12} = G_{13}$ | $G_{23}$ | $\nu_{12} = \nu_{13}$ | $\nu_{23}$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\gamma_4$ | $\gamma_5$ | $\gamma_6$ |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------------------|-----------|----------------------|-------------|---------------|---------------|---------------|-----------|-----------|
| 0.1         | 0.1         | 0           | 0           | 0           | 0           | 3.78725E+01 | 1.12711E+01 | 3.33156E+00        | 3.03426E+00 | 0.30000              | 0.57335     | 1.84831E-03   | 8.08012E-03  | -5.87904E-03 | 0.00000E+00 | 0.00000E+00 |

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_4 \\
\gamma_5 \\
\gamma_6 
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\]
Macromechanics

• What if there are multiple lamina at differing angles?

• CADEC uses Micromechanics and Ply mechanics to determine:
  – Stiffness and Compliance Equations
  – Laminate Moduli
  – Global and Material Stresses and Strains
  – Strains and Curvatures
  – Thermal and Hygroscopic loads
  – For both Intact and Degraded materials

• Assumes:
  – Plane sections remain plane
  – Symmetry about a neutral surface
  – No shear coupling
  – Perfect bonding
Shorthand Laminate Orientation Code

Tapes or Undirectional Tapes

- Each lamina is labeled by its ply orientation.
- Laminae are listed in sequence with the first number representing the lamina to which the arrow is pointing.
- Individual adjacent laminae are separated by a slash if their angles differ.
- Adjacent laminae of the same angle are depicted by a numerical subscript indicating the total number of laminae which are laid up in sequence at that angle.
- Each complete laminate is enclosed by brackets.
- When the laminate is symmetrical and has an even number on each side of the plane of symmetry (known as the midplane) the code may be shortened by listing only the angles from the arrow side to the midplane. A subscript “S” is used to indicate that the code for only one half of the laminate is shown.
Shorthand Laminate Orientation Code

Fabrics and Tapes and Fabrics

- When plies of fabric are used in a laminate. The angle of the fabric warp is used as the ply direction angle. The fabric angle is enclosed in parentheses to identify the ply as a fabric ply.
- When the laminate is composed of both fabric and tape plies (a hybrid laminate). The parentheses around the fabric plies will distinguish the fabric plies from the tape plies.
- When the laminate is symmetrical and has an odd number of plies, the center ply is overlined to indicate that it is the midplane.
Computer Aided Design Environment For Composites

Complement to "Introduction to Composite Materials Design", Taylor&Francis (1999) by Ever J. Barbero

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Macromechanics: Defining Laminate

File: C:\CADEC\TROUBLE.DEF

Laminate Definition for: Problem 5.3

Number of Layers: 1
Number of Materials: 1
Total Thickness: 1.000E-3

Layer Thicknesses: 0.001
Layup Angles: 90
Layer Materials: 1

Loading:
Nx = -1e-9
Ny = 0
Nxy = 0
Mx = 0
My = 0
Mxy = 0
Qx = 0
Qy = 0

Temperature Change: 0
Moisture Concentration: 0
Safety Factor: 1

Use Current Laminate Properties
Set Up Laminate Mat. Properties

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Macromechanics: Defining Laminate

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<th>Problem 5.3</th>
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<td>Safety Factor:</td>
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Use Current Laminate Properties ➔
Set Up Laminate Mat. Properties ➔
Macromechanics: Material Properties

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<td>G₁₂</td>
<td>3.33156E+00</td>
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<td>3.03426E+00</td>
<td>F₁₃</td>
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<td>F₁₄</td>
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Design and Analysis of Aircraft Structures 13-40
Macromechanics: CADEC Quirkiness

Chapter 6 - Macromechanics

Analysis of Intact Material
- Mid-Strains & Curvatures
- Thermal & Hygroscopic Loads
- Reduced Stiffness Matrices
- [A], [B], [D] and [H] Matrices
- Global Strains
- Material Strains
- Global Stresses
- Material Stresses

Plate Stiffness and Compliance
- Stiffness Equations
- Compliance Equations
- Laminate Moduli

Analysis of Degraded Material
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- Material Strains
- Global Stresses
- Material Stresses

Laminate Definition

To Chapter 7
### Macromechanics: Global Stresses

#### Global Stresses - Intact Material

<table>
<thead>
<tr>
<th>Ply</th>
<th>Face</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\sigma_{xy}$</th>
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<th>$\sigma_{xz}$</th>
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<tr>
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<td>0.000</td>
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<td>0.000</td>
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#### Mechanical Stress

<table>
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<tr>
<th>Ply</th>
<th>Face</th>
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<td>6.35E-07</td>
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<td>BOT</td>
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<td>6.35E-07</td>
<td>3.87E-23</td>
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<td>-3.32E-07</td>
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</table>

#### Total Stress

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<th>$\sigma_{xy}$</th>
<th>$\sigma_{yz}$</th>
<th>$\sigma_{xz}$</th>
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</thead>
<tbody>
<tr>
<td>8</td>
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<td>-6.84E-07</td>
<td>6.35E-07</td>
<td>3.83E-23</td>
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<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>BOT</td>
<td>-6.84E-07</td>
<td>6.35E-07</td>
<td>3.87E-23</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
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<td>-9.18E-07</td>
<td>-3.32E-07</td>
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<tr>
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<td>-2.97E-07</td>
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</tr>
</tbody>
</table>
• **Stiffness of composite**
  where:
  - \([A]\) = in-plane stiffness.
  - \([D]\) = bending stiffness.
  - \([B]\) relates in-plane strains to bending moments and curvatures to in-plane forces—bending-extension coupling.
  - \([H]\) relates transverse shear strains to transverse forces.

\[
\{N_x, N_y, M_x, M_y, M_{xy}\} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{44} & B_{41} & B_{42} & B_{46} \\ B_{11} & B_{21} & B_{41} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{42} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{46} & D_{16} & D_{26} & D_{66} \end{bmatrix} \{e_{1x}, e_{2y}, \psi_{11}, \psi_{22}, \gamma_{12}, \kappa_{11}\}
\]

\[
\{V_1, V_2\} = \begin{bmatrix} H_{44} & H_{45} \\ H_{45} & H_{55} \end{bmatrix} \begin{bmatrix} \gamma_{12} \\ \gamma_{22} \end{bmatrix}
\]

\[
A_{ii} = \sum_{k=1}^{N} (\Omega_{ij})_k (z_k - z_{k-1}) = \sum_{k=1}^{N} (\Omega_{ij})_k k_i \quad i, j = 1, 2, 6
\]

\[
B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\Omega_{ij})_k (z_k^2 - z_{k-1}^2) = \sum_{k=1}^{N} (\Omega_{ij})_k k_{ik} \quad i, j = 1, 2, 6
\]

\[
D_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\Omega_{ij})_k (z_k^3 - z_{k-1}^3) = \sum_{k=1}^{N} (\Omega_{ij})_k \left( k_{ik}^2 z_k + \frac{k_{ik}^4}{12} \right) \quad i, j = 3, 2, 6
\]

\[
H_{ij} = \frac{5}{2} \sum_{k=1}^{N} (\Omega_{ij})_k \left[ \frac{4}{15} \left( k_{ik}^3 + \frac{k_{ik}^5}{12} \right) \right] \quad i, j = 4, 5
\]
Macromechanics: ABD Matrices

Analysis of Intact Material
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- Global Stresses
- Material Stresses

Analysis of Degraded Material
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- Material Stresses

Plate Stiffness and Compliance
- Stiffness Equations
- Compliance Equations
- Laminate Moduli
Macromechanics: ABD Matrices

Analysis for Intact Material

\[
\begin{bmatrix}
1.71763 & 5.80162E-01 & -8.84037E-19 \\
5.80162E-01 & 1.71763 & 3.49677E-17 \\
-8.84037E-19 & 3.49677E-17 & 5.68736E-01 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
6.77626E-20 & 1.69407E-20 & -3.09413E-21 \\
-1.69407E-20 & 0.00000 & -6.09111E-21 \\
-3.09413E-21 & -6.09111E-21 & 1.69407E-20 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
6.18290E-06 & 2.79199E-06 & 5.46670E-07 \\
2.79199E-06 & 1.27429E-05 & 5.46670E-07 \\
5.46670E-07 & 5.46670E-07 & 2.73105E-06 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
2.09407E-01 & -9.29063E-04 \\
-9.29063E-04 & 2.14981E-01 \\
\end{bmatrix}
\]
### Macromechanics: Stiffness Equations

#### Plate Stiffness Equations

<table>
<thead>
<tr>
<th>$N_x$</th>
<th>$N_y$</th>
<th>$N_{xy}$</th>
<th>$M_x$</th>
<th>$M_y$</th>
<th>$M_{xy}$</th>
<th>$\varepsilon_x^0$</th>
<th>$\varepsilon_y^0$</th>
<th>$\gamma_{xy}^0$</th>
<th>$\kappa_x$</th>
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<td>5.47E-08</td>
<td>-1.69E-21</td>
<td>0.00E+00</td>
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<td>2.73E-07</td>
<td>5.47E-08</td>
<td>2.73E-07</td>
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</table>

Matrices [ABD] and [H] are based on the current intact material, or the last matrix file loaded.
### Macromechanics: Laminate Moduli

<table>
<thead>
<tr>
<th>Laminate Moduli:</th>
<th>Inplane</th>
<th>Bending</th>
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<td>$E_x$</td>
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<td>$E_x^b$</td>
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<tr>
<td>$E_y$</td>
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<td>$\nu_{xy}$</td>
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Macromechanics: Degraded Material

Analysis of Intact Material

- Mid-Strains & Curvatures
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Analysis of Degraded Material

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Plate Stiffness and Compliance

- Stiffness Equations
- Compliance Equations
- Laminate Moduli

Laminate Definition

To Chapter 7
Macromechanics: Degraded Material

• What is a degraded material?

Figure 7.7 Determination of FF load by an incremental and two-step approach.
### Macromechanics: ABD Comparison

#### Analysis for Intact Material

$$
[A] = \begin{bmatrix}
0.171763 & 0.580162E-01 & -0.884037E-19 \\
0.580162E-01 & 0.171763 & 0.349677E-17 \\
-0.884037E-19 & 0.349677E-17 & 0.568736E-01 \\
\end{bmatrix}
$$

$$
[B] = \begin{bmatrix}
0.677626E-20 & 0.169407E-20 & -0.309413E-21 \\
-0.169407E-20 & 0.600000 & -0.609111E-21 \\
-0.309413E-21 & -0.609111E-21 & 0.169407E-20 \\
\end{bmatrix}
$$

$$
[D] = \begin{bmatrix}
0.618050E-06 & 0.291958E-06 & 0.616670E-07 \\
0.291958E-06 & 0.127429E-05 & 0.546700E-07 \\
0.618050E-06 & 0.546700E-07 & 0.273105E-06 \\
\end{bmatrix}
$$

$$
[H] = \begin{bmatrix}
0.209407E-01 & -0.290673E-04 \\
-0.290673E-04 & 0.214981E-01 \\
\end{bmatrix}
$$

#### Analysis for Degraded Material

$$
[A] = \begin{bmatrix}
0.123342 & 0.382818E-01 & -0.481758E-19 \\
0.382818E-01 & 0.123342 & 0.396429E-17 \\
-0.481758E-19 & 0.396429E-17 & 0.401432E-17 \\
\end{bmatrix}
$$

$$
[B] = \begin{bmatrix}
0.847033E-20 & -0.370577E-21 & -0.168615E-21 \\
-0.370577E-21 & 0.100000 & 0.102720E-20 \\
-0.168615E-21 & 0.102720E-20 & 0.168615E-21 \\
\end{bmatrix}
$$

$$
[D] = \begin{bmatrix}
0.267511E-06 & 0.166970E-06 & 0.712518E-07 \\
0.166970E-06 & 0.112553E-05 & 0.712518E-07 \\
0.267511E-06 & 0.712518E-07 & 0.712518E-07 \\
\end{bmatrix}
$$

$$
[H] = \begin{bmatrix}
0.418816E-02 & -0.185812E-04 \\
-0.185812E-04 & 0.185812E-04 \\
\end{bmatrix}
$$
Macromechanics: CADEC Alt Methods

Data can be entered into DAT and DEF files – Easily reloaded into CADEC – More user friendly

Enter laminate

Save

Open CADEC

Load Laminate

Run Laminate Analysis

Analyze Design and Analysis of Aircraft Structures
Failure Theories

- Many failure criteria, most popular:
  - Maximum stress criterion
  - Maximum strain criterion
  - Tsai-Hill failure criterion
  - Tsai-Wu failure criterion

![Figure 7.3](image1.png)  
**Figure 7.3** Failure envelopes using the maximum strain and maximum stress criteria in strain space $\varepsilon_1 = 2\varepsilon_2$ for carbon-epoxy.

![Figure 7.5](image2.png)  
**Figure 7.5** Various failure envelopes in $\sigma_0 - \sigma_2$ space compared with experimental data.
Not Just An Academic Exercise

Consequence of Misalignment in Large, Composite Structure

Design and Analysis of Aircraft Structures
Failure Theories: CADEC

Chapter 6 - Macromechanics

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Laminate Definition
To Chapter 7
### Failure Theories: Max Stress Criterion

#### Maximum Stress Failure Criterion

<table>
<thead>
<tr>
<th>Ply</th>
<th>Angle</th>
<th>Intact Laminate</th>
<th>Rint-Top</th>
<th>Rint-Bot</th>
<th>Degraded Laminate</th>
<th>Rdeg-Top</th>
<th>Rdeg-Bot</th>
</tr>
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<td>1000. (1)</td>
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<td>1000. (1)</td>
<td>1000. (1)</td>
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<tr>
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<td>1000. (1)</td>
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<td>1000. (1)</td>
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<td>1000. (1)</td>
<td></td>
<td>1000. (1)</td>
<td>1000. (1)</td>
<td></td>
</tr>
<tr>
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<td>10.</td>
<td>1000. (1)</td>
<td>1000. (1)</td>
<td></td>
<td>1000. (1)</td>
<td>1000. (1)</td>
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Design and Analysis of Aircraft Structures 13-59
## Failure Theories: Tsai-Wu Criterion

### Tsai-Wu Failure Criterion

<table>
<thead>
<tr>
<th>Ply</th>
<th>Angle</th>
<th>Intact Laminate</th>
<th>Degraded Laminate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rint-Top</td>
<td>Rint-Bot</td>
</tr>
<tr>
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<td>-10.</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>3</td>
<td>10.</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td>-10.</td>
<td>.000</td>
<td>.000</td>
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<tr>
<td>1</td>
<td>10.</td>
<td>.000</td>
<td>.000</td>
</tr>
</tbody>
</table>

Design and Analysis of Aircraft Structures 13-60
Concluding Remarks

• **Composite design fairly simple**
  – Assumptions lead to simplified analysis
  – Idealized
  – Real-world?

• **CADEC**
  – Begin with component properties
  – Micromechanic, Ply and Macromechanic analysis
  – Apply loads and match against failure criteria
  – Simple structures (Not covered)
  – Software options: COMPRO, MSExcel, Matlab, MathCAD, etc.

• **Composites still require significant analysis and physical testing**

• **Parts/Structures are only as good as the manufacturing**
  – “You can never make good parts with bad materials, but you can easily make bad parts with good materials!”