

EE581 Fall 2007  
**Homework #Last**

**Problem #1. Effect of aberrations on the point spread function and OTF**

Aberrations can have a complicated effect on the quality of an image forming instrument. Treating these effects analytically is possible, but it is involved, tedious, and available in a number of textbooks if you need it. On the other hand, calculating the effect of aberrations on the point spread function  $h$  is very straight forward using standard FFT techniques. Once we know  $h$  it is easy to calculate the intensity PSF  $|h|^2$  and the OTF.

To save on toner and time, you may want to copy your various plots into a document and scale them to put many on a page.

**a) Baseline “Perfect” point spread function**

Set up a two-dimensional matrix representing the pupil plane of your system, using a circular aperture. Calculate the point spread function  $h$ . For assigning an appropriate numerical scale, assume an optical system consisting of a single lens with pupil diameter of 25 mm and focal length 250 mm, illuminated with a plane wave with  $\lambda = 0.5 \mu\text{m}$ .

*Plot a 2-D gray-scale image of the **magnitude** of the amplitude point spread function, normalized to full white at the maximum.*

*Plot a cross section that goes through the maximum, along the x-axis, and label the x-value for the first minima on your plot.*

*For the intensity PSF  $|h|^2$ , plot a cross section that goes through the maximum.*

*Compute the OTF as the inverse Fourier transform of the intensity PSF, and plot a cross section that goes through the maximum along the  $f_x$  axis.*

**Hints:**

To get enough detail in the point spread function, a ratio of matrix size to pupil diameter of 8 seems to work well.

In Matlab, you may find the following series of commands useful:

*abs(h)* – returns the absolute value for complex  $h$

*fftshift(h)* – shifts zero frequency component of FFT to center (swaps left and right halves of a vector, or swaps quadrants 1 and 3 and quadrants 2 and 4 of a matrix)

*colormap(gray(64))* – sets imaging colormap to go from 0=black to 64=white.

*image(64\*abs(h)./max(max(abs(h))))* – plots 2-D image of  $abs(h)$  normalized to maximum value of  $h = 1$

*axis('square')* – adjusts image so that scale for  $x$  and  $y$  are the same

*grid on* – turns on the grid for plots, if you want to have a grid

For the rest of these plots, I am interested in the relative change to the PSF – don't worry about the scale values – just have fun and observe the effects.

**b) Spherical Aberration**

*Plot a 2-D gray-scale image of the **magnitude** of the PSF when you have one wave of spherical aberration.*

*Plot a cross section that goes through the maximum, along the x-axis. Show the unaberrated PSF on the same plot for reference.*

*Estimate how many waves of spherical aberration result in the maximum value of the PSF decreasing by 10% relative to the case with no aberration.*

Estimate how many waves of spherical aberration result in a doubling of the full width at half maximum of the PSF.

For one wave, two waves and three waves of spherical aberration:

For the intensity PSF  $|h|^2$ , plot a cross section that goes through the maximum.

Compute the OTF and plot a cross section that goes through the maximum along the  $f_x$  axis.

For one wave of spherical aberration,  $W = \lambda(r/w)^4 = \lambda(x^2+y^2)^2/w^4$ . The pupil function should be multiplied by  $\exp(jkW)$  to calculate the effect of the aberration.

### c) **Balanced Spherical**

Plot a 2-D gray-scale image of the **magnitude** of the PSF when you have one wave of balanced spherical aberration.

Plot a cross section that goes through the maximum, along the  $x$ -axis. Show the unaberrated PSF on the same plot for reference.

Estimate how many waves of balanced spherical aberration result in the maximum value of the PSF decreasing by 10% relative to the case with no aberration.

For one wave, two waves and three waves of balanced spherical aberration:

For the intensity PSF  $|h|^2$ , plot a cross section that goes through the maximum.

Compute the OTF and plot a cross section that goes through the maximum along the  $f_x$  axis.

For one wave of balanced spherical aberration,  $W = \lambda[(r/w)^4 - (r/w)^2] = \lambda[(x^2+y^2)^2/w^4 - (x^2+y^2)/w^2]$ . The effect of the spherical aberration is “balanced” by adding some defocus. This is the minimum spot you could find by adjusting your observation plane along the optical axis. The best balance occurs with one wave of defocus balancing one wave of spherical aberration.

### d) **Coma**

Plot a 2-D gray-scale image of the **magnitude** of the PSF when you have one wave of coma.

Plot a cross section that goes through the maximum, along the  $x$ -axis. Show the unaberrated PSF on the same plot for reference.

For one wave, two waves and three waves of coma:

For the intensity PSF  $|h|^2$ , plot a cross section that goes through the maximum.

Compute the OTF and plot a cross section that goes through the maximum along the  $f_x$  axis.

For one wave of coma,  $W = \lambda[(x^2+y^2)x/w^3]$ .

### e) **Astigmatism**

Plot a 2-D gray-scale image of the **magnitude** of the PSF when you have one wave of astigmatism.

Plot a cross section that goes through the maximum, along the  $x$ -axis. Show the unaberrated PSF on the same plot for reference.

For one wave, two waves and three waves of astigmatism:

For the intensity PSF  $|h|^2$ , plot a cross section that goes through the maximum.

Compute the OTF and plot a cross section that goes through the maximum along the  $f_x$  axis.

For one wave of astigmatism,  $W = \lambda(x^2/w^2)$ .

Investigate the effect of defocus combined with astigmatism, as you did for balanced spherical.

**Problem #2. Goodman problem 7-8.**