Do problems 4-7, 4-11, 4-15 from Goodman.

Complete the following problem (former midterm exam problem...)

**Problem 1.** An opaque screen has two pinholes, located at \( x = a \) and \( x = -a \). You may treat the screen transmission function \( t(x,y) \) as given by \( t(x,y) = \delta(x+a)\delta(y) + \delta(x-a)\delta(y) \). The screen is placed at the \( z = 0 \) plane and is illuminated by a \( z \)-directed unity amplitude plane wave.

\[
\begin{array}{c}
\text{x-y view} \\
\begin{array}{c}
\uparrow x \\
\bullet a \\
\downarrow y \\
\bullet -a
\end{array}
\end{array}
\quad
\begin{array}{c}
\text{plane wave illumination} \\
\begin{array}{c}
\uparrow x \\
\bullet a \\
\downarrow y \\
\bullet -a
\end{array}
\end{array}
\quad
\begin{array}{c}
\text{x-z view} \\
\begin{array}{c}
\uparrow x \\
\bullet a \\
\downarrow y \\
\bullet -a
\end{array}
\end{array}
\]

a) Write the Fresnel integral relating the observed field \( U(x_o,y_o,z) \) in the plane \( z = z \), in terms of the field \( U(x_1,y_1,0) \) in the \( z = 0 \) plane, and solve for \( U(x_o,y_o,z) \).

b) What is the intensity \( I(z) = |U(0,0,z)|^2 \) **along the \( z \)-axis** \((x=0, y=0)\) behind the screen?

c) Now assume that a \( \pi \) phase delay element is placed in front of the pinhole located at \( x = a \), so that \( t(x,y) = \delta(x+a)\delta(y) - \delta(x-a)\delta(y) \). What is the intensity \( I(z) = |U(0,0,z)|^2 \) **along the \( z \)-axis** behind the screen?