

A Note on absolute and relative risk aversion:

A commonly used measure to indicate the intensity of risk aversion for an individual is the Arrow-Pratt absolute risk aversion coefficient, which can be computed as follows

$$r(x) = -\frac{u''(x)}{u'(x)} \quad (1)$$

Notice that if an individual is risk averse, it is implied that utility is increasing at a decreasing rate with respect to x . For this reason, a risk averse individual will have the following properties: (1) $u'(x) > 0$ and (2) $u''(x) < 0$.

Let's use the example where $u(x) = \log(x)$. In this case, $r(x) = -\frac{\frac{1}{x^2}}{\frac{1}{x}} = \frac{x}{x^2} = \frac{1}{x}$. Notice that for all levels of x , this will be positive. Further, let's assume that $x = 10,000$. In this case, $r = .0001$. How do we interpret this? We can rewrite equation (a) above in the following way

$$r(x) = -\frac{\left(\frac{du'(x)}{dx}\right)}{u'(x)} \quad (2)$$

$$= -\frac{d}{dx} \ln(u'(x)) \quad (3)$$

$$= -\frac{\left(\frac{du'(x)}{u'(x)}\right)}{dx} \quad (4)$$

This result indicates that $r = .0001$ indicates that the decision-maker's marginal utility is falling at a rate of 0.01% per dollar change in income. Another point here is that as x increases, $r(x)$ decreases, meaning this individual has **decreasing absolute risk aversion**. This implies that the individual becomes less risk averse for a dollar bet for increasing levels of x .

Notice that an individual who is risk neutral will have $u''(x) = 0$, meaning $r(x) = 0$. Thus, a higher $r(x)$ indicates a more intense aversion to risk.

Relative risk aversion can be computed in the following way

$$r(x) = -\frac{u''(x)}{u'(x)} x \quad (5)$$

For the function we used above, the relative risk aversion coefficient is equal to 1. This can be interpreted similarly to the absolute value, except that the change in x is by 1%, rather than 1 unit. Notice that the relative risk aversion coefficient is constant across all levels of wealth, implying **constant relative risk aversion**.