ECNS 204 – Microeconomics
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Homework 3
Due Tuesday, February 26

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1. The following questions are concerning the ABC tree planting company.
   a. The ABC tree planting company has the following relationship between labor inputs (L) in terms of labor hours and the amount of trees that can be planted (Q). Workers are paid $10 per hour. Fill out the table below to determine the marginal product of labor (MPL) and average product of labor (APL).

Q	L	MPL	APL
10	3	**3.33**	**3.33**
20	8	**2.00**	**2.50**
30	16	**1.25**	**1.88**
40	28	**0.83**	**1.43**
50	45	**0.59**	**1.11**
60	68	**0.43**	**0.88**

   b. Fill out the rest of the chart below with regard to the demand curve and cost of planting trees for ABC tree planting Company. There are no fixed costs.

Demand Schedule						
Q	P	TR	MR	TC	MC	Profit
10	15	**150**	**15**	**30**	**3**	**120**
20	13	**260**	**11**	**80**	**5**	**180**
30	11	**330**	**7**	**160**	**8**	**170**
40	9	**360**	**3**	**280**	**12**	**80**
50	7	**350**	**-1**	**450**	**17**	**-100**
60	5	**300**	**-5**	**680**	**23**	**-380**

   c. How many trees should they plant, and what price should be charged? Answer using both method I and method II.

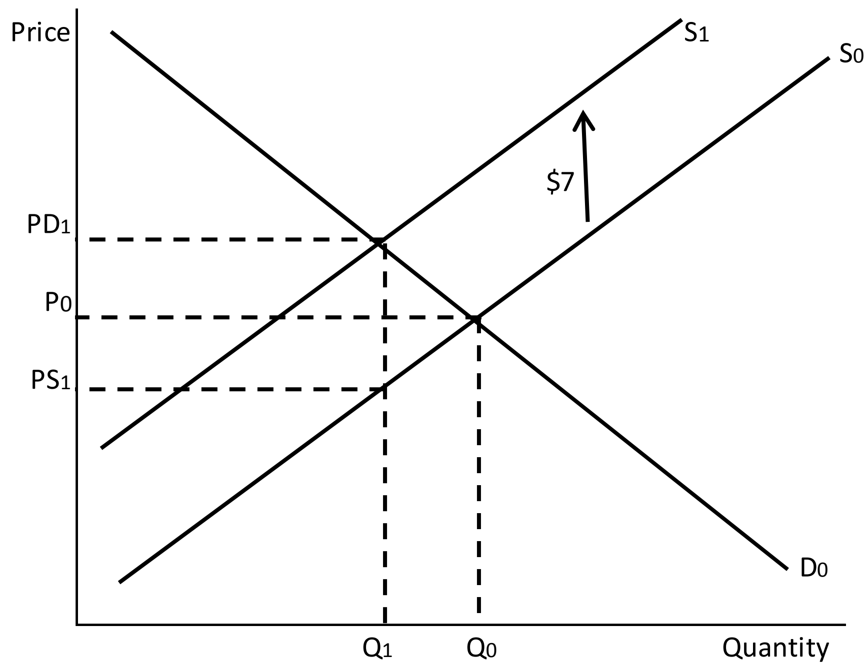
      method I: Q* = 20, P* = 13
      method II: Q* between 20,30, P* between 11 and 13

   d. If a $50 annual licensing fee is required to operate this company, what happens to TC? What happens to MC? What happens to the optimal amount of trees that should be planted?

      TC increases by $50 for all quantity levels. MC does not change. The optimal amount of trees to be planted is unchanged.

e. Now assume a $7 excise tax per tree planted ($70 per 10 trees) is imposed on ABC tree planting company. Show graphically how a tax would impact the equilibrium price and quantity. Also, show the effective price received by the firm.

MC increases by $7 per tree. This effectively shifts the supply curve upward by $7. Based on Method I, $P_0 = 13$ and $Q_0 = 20$. After the excise tax is imposed, the equilibrium quantity, $Q_1 = 10$ while the effective price to consumers (PD_1) is $15 per tree, which translates into an effective firm price (PS_1) of $8 per tree [=$15 - $7].



f. In an effort to increase the amount of new trees planted, the local government agrees to reimburse tree planting companies 80% of the costs, effectively lowering MC by 80%. What is the impact on the equilibrium quantity and price of trees planted?

MC is reduced by 80% at every quantity, giving the following relationship.

Q	P	MR	MC	Profit
10	15	15	**0.6**	**144**
20	13	11	**1**	**244**
30	11	7	**1.6**	**298**
40	9	3	**2.4**	**304**
50	7	-1	**3.4**	**260**
60	5	-5	**4.6**	**164**

This results in a new equilibrium quantity at 40 trees with a price of $9. Price falls and quantity increases.

2. A firm discovers that when it uses K units of capital and L units of labor, it is able to produce given the following production function

$$Q = K^{1/3}L^{2/3}$$

   a. Suppose that the firm produces 20 units of output with 40 units of labor and 5 units of capital.
      i. Compute the $MRTS_{LK}$.

      The easiest way to do this is to solve for K in terms of L and Q. this can be done as follows:

$$\boldsymbol{K^{\frac{1}{3}} = \frac{Q}{L^{\frac{2}{3}}}}$$
$$\boldsymbol{K = Q^3/L^2}$$

      We know that if $\boldsymbol{L = 40}$ and $\boldsymbol{K = 5}$, then $\boldsymbol{Q = 20}$. If we increase L by one-unit and keep Q constant, what will K be. This can be found as $\boldsymbol{K = 20^3/(41^2) = 4.759}$. So, as we add one-unit of labor, we give up 0.241 capital units. Thus, $\boldsymbol{MRTS = 0.241}$. We can exactly find this by using calculus, where $\boldsymbol{\frac{dK}{dL} = -\frac{Q^3}{L^3} = -0.25}$.

      ii. Compute the MPL.

      If we increase labor units by one and hold constant capital, we obtain the following:

$$\boldsymbol{Q = K^{1/3}L^{2/3} = 5^{1/3}41^{2/3} = 20.332}$$

      Thus, $\boldsymbol{MPL = 20.332 - 20 = 0.332}$. Using calculus, we find that $\boldsymbol{\frac{dQ}{dL} = \frac{2}{3}K^{1/3}\,L^{-1/3} = \frac{1}{3}}$

      iii. Compute the MPK.

      If we increase capital units by one and hold constant labor, we obtain the following:

$$\boldsymbol{Q = K^{1/3}L^{2/3} = 6^{1/3}40^{2/3} = 21.253}$$

      Thus, $\boldsymbol{MPK = 21.253 - 20 = 1.253}$. Using calculus, we find that $\boldsymbol{\frac{dQ}{dK} = \frac{1}{3}K^{-2/3}\,L^{2/3} = \frac{2}{3}}$

      iv. Is the equation $MRTS_{LK} = \frac{MPL}{MPK}$ approximately true?

$$\boldsymbol{0.241 \approx \frac{0.332}{1.253} = 0.265}$$

      This relationship is exactly true when we use calculus but close when we approximate numerically.

b. Suppose that capital and labor units can be employed for $1 and $2 per unit, respectively, and that the firm uses 5 units of capital in the short run. What is the short-run total cost to produce 20 units of output?

$$\boldsymbol{TC = P_K K + P_L L = (1 * 5) + (2 * 40) = 85}$$

c. If a firm produces 20 units of output, it will employ 20 units of capital and 20 units of labor.

   i. Show that the total cost is lower in the long run than in part (b).

$$\boldsymbol{TC = P_K K + P_L L = (1 * 20) + (2 * 20) = 60}$$

   ii. Show that $MPK/P_K = MPL/P_L$

MPK = $21^{1/3}20^{2/3} - 20 = 0.328$ (or 1/3 using calculus)
MPL = $20^{1/3}21^{2/3} - 20 = 0.661$ (or 2/3 using calculus)
MPK/P_K = 0.328/1 = 0.328
MPL/P_L = 0.661/2 = 0.331

   iii. Does this production function exhibit constant, increasing, or decreasing returns to scale?

If both capital and labor are increased by 5%, then $K_1 = 20 * (1 + 0.05) = 21$ and $L_1 = 21$. Using the production function above, we find that $Q = 21^{1/3}21^{2/3} = 21$, which is a 5% increase. Since a 5% increase in inputs lead to a 5% increase in outputs, we observe constant returns to scale.