

An Empirical Test of Rubinstein's Similarity Definitions for Choice Under Risk

David Buschena®

Associate Professor

Agricultural Economic and Economics Department

Montana State University - Bozeman

phone: (406)994-5623

e-mail: dbuschena@montana.edu

David Zilberman

Professor

Agricultural and Resource Economics

University of California Berkeley

October, 2006

An Empirical Test of Rubinstein's Similarity Definitions

Abstract

This paper tests two alternative definitions of probability similarity using experimental data. These definitions are key components of Rubinstein's [14] larger axiomatic model for risky choice under similarity effects. The results of our tests strongly support absolute difference similarity measure over relative (ratio) similarity. As developed by Rubinstein, the evidence argues against an axiomatic reduced-form model that treats preference and similarity jointly.

Keywords: expected utility model, expected utility violations, similarity

JEL Classification: C91, D81

Rubinstein [14] presents a rigorously developed model for risky choice in which people use multiple processes to select between risky alternatives. These processes are selected through an algorithm that sorts decision problems into (1) important (dissimilar) decisions that require considerable decision effort and (2) less important (similar) decisions that are solved using a simple decision rule. Rubinstein's choice sorting and decision process develops an axiomatic structure for and thoroughly explores earlier work by Luce [12] and Tversky [15] and relates to bounded rationality models for risk in Payne, Bettman and Johnson [13] and in Conlisk [5].

A key component of a choice algorithm selection model is the appropriate measure of similarity. Rubinstein offers two measures, one of which proves critical for his axiomatic model incorporating both similarity and preference. We test these two alternative definitions, although some aspects of Rubinstein's model are abstracted from. To our knowledge such explicit tests of these definitions for Rubinstein's similarity definitions have not been published previously.

This paper tests two candidate measures for similarity. The implications of our findings are farther-reaching, applying to the influential generalized- EU models summarized extensively in Starmer [15] as well as in earlier work. We find little empirical support for the type of relative similarity that is critical for an axiomatic structure that simultaneously models similarity effects and preference.

1. The empirical challenge

Many researchers have found systematic departures of choice from EU, the most well known being independence violations. These departures are illustrated by Kahneman and Tversky's [8] very well-known common consequence pairs over 1970's Israeli pounds:¹

¹ In 2006 U.S. dollars, the value of a 1977 Israeli pound is approximately \$.093, so 3000 pounds in 2006 dollars is approximately \$279.

Pair 1: Choose between gamble A and B:

A: gives 3000 with probability 1.0 B: gives 4000 with probability .8
gives 0 with probability .2

Pair 2: Choose between gamble C and D:

C: gives 3000 with probability .25 D: gives 4000 with probability .2
gives 0 with probability .75 gives 0 with probability .8.

Most experimental subjects in Kahneman and Tversky's work, and in subsequent experiments, selected lotteries A and D, a pattern that violates EU which requires the choice of either both A and C or both B and D. These common consequence pairs provide a useful empirical starting-point for our tests of Rubinstein's similarity definitions.

2. A proposed solution: pair similarity

Rubinstein provides an important and theoretically impressive contribution to risky choice modeling. In addition to proposing the similarity concept for EU violations, Rubinstein provides a rigorous axiomatization of how similarity and preference jointly affect choice. This axiomatization hinges on the way similarity is defined, with the success of this preference/similarity axiomatization requiring similarity to affect decision makers in a relative rather than an absolute sense. Rubinstein proves that if similarity enters the decision process in a relative (absolute) sense, its effects on choice under risk can (cannot) be incorporated into an axiomatic model combining preference and similarity effects.

Rubinstein proposes two similarity measures—absolute and relative. Define lotteries $G_1=(x, p)$ and $G_2=(y, q)$, where G_1 has a p probability of receiving positive outcome x (\$0 otherwise) and G_2 has a q probability of positive outcome y (\$0 otherwise). These lotteries are

“prospects” in Kahneman and Tversky’s [8] terminology, each offering the opportunity to win only one non-zero outcome. Rubinstein defines similarity for both outcomes and for probabilities analogously. Probabilities are similar in Rubinstein’s ε -difference (absolute distance) sense if $|p - q| \leq \varepsilon$, for some critical ε . Alternatively, probabilities are similar in a λ -ratio (relative difference) sense if $1/\lambda \leq p/q \leq \lambda$ for some other critical scalar λ . Rubinstein proves that the λ -ratio measure is the only type of similarity that can be used in a joint axiomatic model of preference and similarity. Rubinstein’s preference relation is weaker than that for many of the generalized-EU models, requiring only transitivity and reflexivity (it is a weak order), monotonicity, continuity, and that gamble $(x,0)$ for $x>0$ be indifferent to gamble $(0,p)$ for $p>0$ both of which are indifferent to gamble $(0,0)$.

In addition to his quantitative measures, Rubinstein defines similarity qualitatively, where a probability of zero is dissimilar to any positive probability, and an outcome of zero is dissimilar to any non-zero outcome.

3. Empirical test of Rubinstein’s similarity measures

We developed risky pairs designed to test the predictive power of Rubinstein’s absolute vs. relative probability similarity as part of a larger experiment designed to test similarity effects more generally.² Respondents were given varying likelihoods (including a control group that had a zero likelihood of cash payments based on play) of playing their selected risky choice for actual cash payments in the experiment.³ Respondents faced 26 risky pairs that were randomly assigned from a larger set of gamble pairs. See Buschena and Zilberman [2, 3] for a description.

² We also have risky pairs with different relative and absolute similarity over outcomes, but cannot separate the effects of outcome similarity from preference such as through the curvature of the utility function.

³ An increasing likelihood of playing for cash decreased risk taking by a respondent in our larger experiment. A probit regression on choice corrected for this and other experimental factors.

Each respondent's survey consisted of risky pairs defined for non-zero outcomes of either $x=\$15$ and $y=\$20$, or $x=\$30$ and $y=\$40$ in 1993 U.S. dollars. In every risky pair the respondent choose between G_1 giving a p chance of outcome $\$x$ (otherwise $\$0$) or G_2 giving a q chance of $\$y$ (otherwise $\$0$). Each pair offered a risk-return tradeoff; lottery G_1 uniformly offers a lower mean and variance than lottery G_2 does. EU predicts that the respondent will either always select G_1 or always select G_2 for every pair.

Figure 1 illustrates the risky pairs used for our test of absolute and relative similarity measures over probabilities. The occurrence of the risky pairs in the respondent's survey was determined via a random number generator. The probability vectors for every pair differ in an ϵ -difference sense, but not in a ratio sense.

INSERT FIGURE 1 HERE

Under Rubinstein's ϵ -difference similarity on the probabilities, a greater proportion of the respondents should select the "riskier" lottery G_2 (carrying the higher likelihood of a zero outcome) as the difference between the probabilities of winning a positive amount ($p-q$) decreases. Alternatively, if Rubinstein's λ -ratio similarity holds, the proportion of respondents selecting the riskier lottery should be stable across the pairs in Figure 1 since they all have the same λ -ratio.

The ϵ -difference and λ -ratio similarity measures, and the choice likelihoods are given in Table 1 for each outcome population and in total. T-statistics are provided for tests of differences in the proportion of respondents selecting the riskier alternative in each pair. The choice proportions clearly support ϵ -difference similarity over λ -ratio similarity for both outcome set populations. As predicted under ϵ -difference similarity, the proportions selecting the riskier lottery in both pairs GH and CD were both statistically significantly higher than those selecting the riskier lottery for both pairs AB and EF for the total sample.

INSERT TABLE 1 HERE

4. Conclusions

Rubinstein pioneered the formal modeling of decision making under as a two step algorithm which considers complexity and mental transaction costs. However, the empirical support for Rubinstein's absolute similarity measure over his relative measure in both probabilities and outcomes bodes ill for a fully axiomatized reduced-form choice model that combines preference and similarity effects. Alternatively, our empirical results show the empirical value of incorporating absolute similarity into risk choice modeling. Such incorporation may require a structural model for both preference and similarity effects jointly. Such structural models have been explored in work by Leland [9], Buschena and Zilberman [2,3] and Loomes [10]. A parallel development to this similarity literature is found in stochastic models for choice by Ballinger and Wilcox [1], Hey and Orme [7], Hey [6], Buschena and Zilberman [4], and Loomes and Sudgen [11].

References

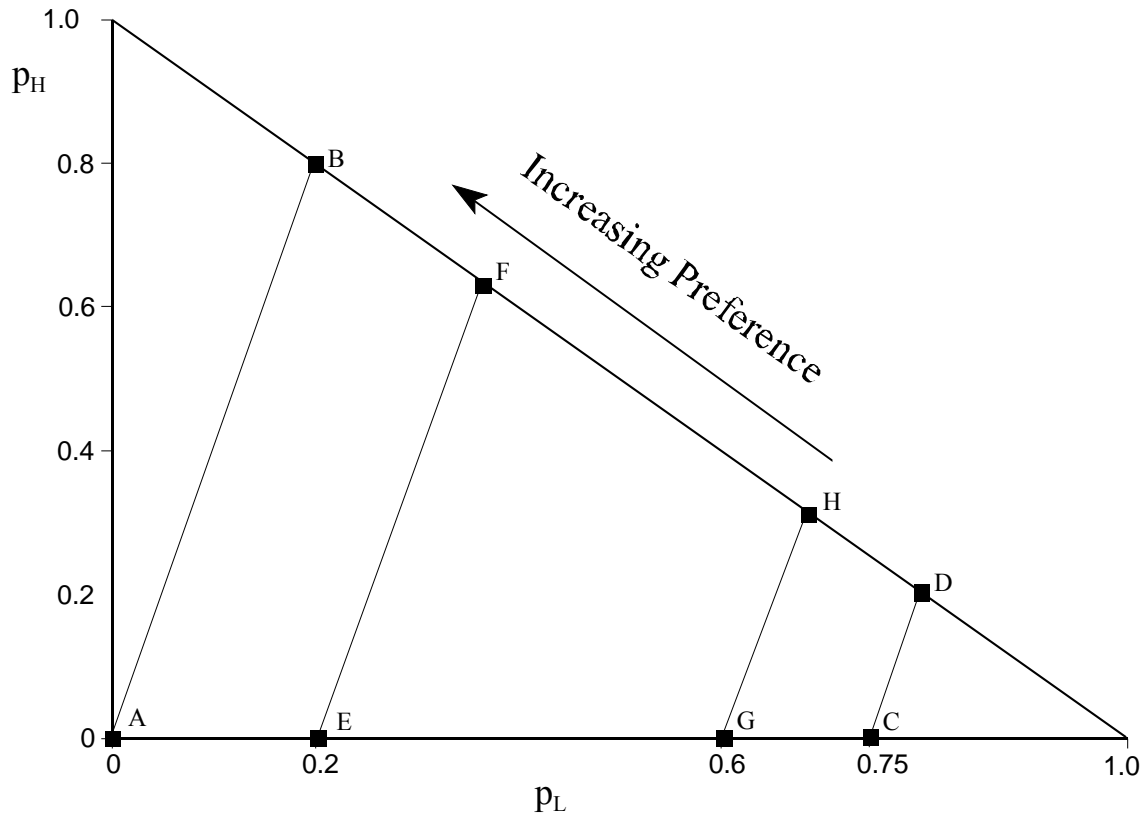
- [1] Ballinger, T. Parker, and Nathaniel T. Wilcox. (1997). "Decisions, Error, and Heterogeneity," *The Economic Journal* 107, 1090-105.
- [2] Buschena, David E., and David Zilberman. (1999a). "Testing the Effects of Similarity and Real Payoffs on Choice." In M. Machina and B. Munier (eds.), *Beliefs, Interactions, and Preferences in Decision Making*. Kluwer Academic Publishers, Boston, MA.
- [3] Buschena, David E., and David Zilberman. (1999b). "Testing the Effects of Similarity on Risky Choice: Implications for Violations of Expected Utility," *Theory and Decision*, 46, 253-80.
- [4] Buschena, David E. and David Zilberman. (2000). "Generalized Expected Utility, Heteroscedastic Error, and Path Dependence in Risky Choice." *Journal of Risk and Uncertainty* 20, 67-88.
- [5] Conlisk, John. (1996). "Three Variants on the Allais Example." *American Economic Review*. 79, 392-407.
- [6] Hey, John D. (1995). "Experimental Investigations of Errors in Decision Making Under Risk", *European Economic Review* 39, 633-40.
- [7] Hey, John D. and C. Orme (1994). "Investigating generalizations of expected utility theory using experimental data. *Econometrica* 62, 1291-1326.
- [8] Kahneman, Daniel, and Amos Tversky. (1979). "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica* 47, 263-91.
- [9] Leland, John W. (1994). "Generalized Similarity Judgments: An Alternative Explanation for Choice Anomalies," *Journal of Risk and Uncertainty* 9, 151-72.
- [10] Loomes, G., 2006, The improbability of a general, rational and descriptively adequate

theory of decision under risk. Working Paper. School of Economics, University of East Anglia.

- [11] Loomes, G. and R. Sugden. “Testing Different Stochastic Specifications of Risky Choice.” *Economica* 65: 581-598.
- [12] Luce, R. Duncan. (1956). “Semi-Orders and a Theory of Utility Discrimination,” *Econometrica* 24:178-91.
- [13] Payne, John W., James R. Bettman, and Eric J. Johnson. (1993). “The Adaptive Decision Maker.” Cambridge: Cambridge University Press.
- [14] Rubinstein, Ariel. (1988) “Similarity and Decision-making Under Risk (Is There a Utility Theory Resolution to the Allais Paradox?),” *Journal of Economic Theory* 46, 145-53.
- [15] Starmer, Chris. (2000). “Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice Under Risk.” *Journal of Economic Literature* XXXVIII, 332-382.
- [16] Tversky, Amos. (1977). “Features of Similarity,” *Psychological Review* 84(4), 327-52.

Table 1: Testing ϵ -and Relative Difference Measures for Probability Similarity.

Pair	ϵ -Difference Similarity	Relative Difference Similarity	Relative Difference		Proportion
			(\$0, \$15, \$20)	(\$0, \$30, \$40)	Selecting the Riskier Lottery
AB	0.2	0.08	.105 (n=38)	.152 (n=33)	0.127 (n = 71)
EF	0.16	0.08	.208 (n=24)	.100 (n=30)	0.148 (n = 54)
GH	0.08	0.08	.362 (n=69)	.302 (n=43)	0.339 (n = 112)
CD	0.05	0.08	.485 (n=33)	.480 (n=25)	0.483 (n = 58)
T-statistics for proportional differences					
AB	vs. EF		-1.12	.619	-.339
	vs. GH		-2.26	-1.52	-3.20
	vs. CD		-3.55	-2.72	-4.41
EF	vs. GH		-1.39	-2.05	-2.58
	vs. CD		-2.14	-3.15	-3.76
GH	vs. CD		-1.19	-1.47	-1.79



Probabilities of the Border Pairs

Pair	p_L	p_M	p_H	q_L	q_M	q_H
AB	0	1	0	0.2	0	0.8
EF	0.2	0.8	0	0.36	0	0.64
GH	0.6	0.4	0	0.68	0	0.32
CD	0.75	0.25	0	0.8	0	0.2

Figure 1: Probability Triangle