Applying econometric methods to decision cost models for Expected Utility violations

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Abstract

Recent work proposes a decision cost argument for the occurrence of expected utility violations. These cost models suggest measures over the risky pairs that define these decision costs, but there is more than one form of these cost measures. These proposed measures are furthermore non-nested. This paper assesses, using information criteria, the relative modeling success of these candidate measures in explaining risky choice behavior giving rise to violations. Although the candidate models exhibit some degree of substitutability, our results indicate support for a candidate model that uses relatively simple measures to instrument for decision costs.

Keywords: risk, expected utility, similarity, information criteria

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Applying econometric methods to decision cost models for EU violations

Maximization of expected utility (EU) has been the dominant explanation for risky choices. However, the violations were first suggested by Allais (1953), famously addressed by Kahneman and Tversky (1979), and have become a topic of considerable interest in economics, psychology, and management science, and led to a search for alternative models. One major approach, the non-expected utility models (see the survey article by Starmer)., has modified the assumptions regarding decision-making preferences, but has assumed one-step maximizing behavior

Another approach to EU violations is the similarity approach (Rubinstein, Leland, Buschena & Zilberman, Loomes (2006)), which assumes that decision makers may use different decision-making criteria that depend on the risky choices they have available. They apply a two-stage decision process: first, selecting a decision rule, and then making the actual risky choice. The decision algorithm is affected by the similarity between the risky choices. A simpler decision algorithm is chosen when the alternatives are more similar, and a more complex algorithm (EU) when outcomes are less similar.

This similarity approach can be interpreted as taking into account the mental transaction costs associated with making risky choices, as well as the cost of making the wrong choice. In this way the similarity approach has much in common with bounded rationality models for risky choice such as Payne, Bettman and Johnson and also Conlisk. These similarity models also relate to a growing literature for error frameworks for risky choice in Ballinger and Wilcox, Hey and Orme, Hey, Loomes and Sudgen, and Loomes (2005) that support heteroscedastic error across decision task as opposed to the constant

(Fechner type) error suggested in Harless and Camerer.

One major challenge for this similarity framework is finding the appropriate measures of similarity that explain observed behavior, i.e., whether similarity can be best explained by intuitive measures based on Euclidean distance, by measures of difference of information content of different distributions, reflected by the cross-entropy measure, or by measures that weight distances non-linearly to reflect decreasing marginal utility, such as Loomes' non-linear definition for similarity.

This paper attempts to develop a methodology to resolve this problem empirically, and to apply it to a large data set, using results of experiments done with 300 undergraduate students facing a series of risky choices, over gambles offering risk-return tradeoffs. The similarity among the alternatives vary for the different choices, resulting in several thousand observations of risky choices that vary in the similarity of the outcomes and other characteristics.

After first assessing the state of the econometric methods used to assess EU violations and the decision limitation explanations for them, we test three candidate models for decision limitation as an explanatory measure for risky choice patterns. These tests utilize maximum likelihood linear and nonlinear in parameter logit models across individual's choices for a large set of respondents. Because the models tested are nonnested, various information criteria are utilized.

I. The pioneers: Simple means tests across two risky pairs

Von Neumann and Morgenstern's (1953) EU model provided an axiomatized model to describe risk-return tradeoffs commonly observed in a host of economic settings. Their model of choice provides the EU for a probability vector $\underline{p}$ that defines a risky alternative over a vector of outcomes $\underline{x}$ as:

$$EU(\underline{p}) = \sum_{i=1}^{n} p_i \bullet u(x_i)\,. \qquad (1)$$

The EU model remains a powerful and widely used tool for the analysis of choice under risk.

Shortly after von Neuman and Morgenstern's 1944 paper, Allais (1953) published findings of systematic departures from EU. Allais' results and a body of supporting evidence have become known as independence violations. These violations, in addition to other behavior inconsistent with EU, came to the fore in economics by the publication of Kahneman and Tversky's (1979) seminal paper. The nature of these EU violations, and especially what to do about them, remains the topic of considerable interest within economics, psychology, management science, and other fields.

Kahneman and Tversky's (1979) common consequence pairs, often referred to as common-ratio effect pairs, illustrate the nature of these violations:

Pair 1: Choose between gamble A and B:

A:	gives \$3000 with probability 1.0	B:	gives \$4000 with probability .8 gives \$ 0 with probability .2

Pair 2: Choose between gamble C and D:

C:	gives \$3000 with probability .25 gives \$ 0 with probability .75	D:	gives \$4000 with probability .2 gives \$ 0 with probability .8.

Most experimental subjects select lotteries A and D, a pattern that violates EU (von Neumann and Morgenstern, 1953; Jensen, 1967; and Fishburn, 1988). This violation is shown clearly by rewriting the gambles comprising the second pair as linear combinations of gambles A and B plus a gamble (\$0) that denotes a gamble giving a zero payoff with certainty. Under EU, if A is preferred to B, then C must be preferred to D because:

$C = 1/4 * A + 3/4 * (\$0)$ and $D = 1/4 * B + 3/4 * (\$0)$.

The empirical analysis of these original EU violations consisted of means tests of choice proportions between pair AB and CD. These tests are simple, and in a sense powerful. These tests, however, offer a limited view of the nature of these violations, and in particular how robust they are to gambles that differ in the probabilities of the alternative outcomes.

II. The developers: Multiple choices per person and panel data

In response to the simple EU violations above, a substantial number of competing Generalized EU models (GEU) weaken the von Neumann-Morgenstern axioms to allow for choice patterns violating EU. These GEU models essentially introduced a nonlinear treatment, through a function $\pi(\cdot)$, of the probabilities for the valuation of risky alternatives. This nonlinear function $\pi(\cdot)$ is introduced into the EU formulation in equation (1), defining:

$$GEU(\underline{p}) = \sum_{i=1}^{n} \pi(p_i)u(x_i) \tag{2}$$

The specific nature of $\pi(\cdot)$ distinguishes the competing GEU models. Virtually all of these models retain some of the EU axioms (chiefly transitivity), while weakening others in order to allow for common patterns of EU violations. Particularly good summaries of these models can be found in Fishburn (1988), and in Harless and Camerer (1994).

In order to test between the many competing GEU models, Harless and Camerer (1994), Hey and Orme (1994), Hey (1995), Wilcox (1993), Ballinger and Wilcox (1997), and others extended the set of risky choices considered. Some of these empirical estimations apply maximum likelihood methods and information criteria to the analysis of the competing GEU and related models, providing evidence for the relative predictive power of the alternative GEU models. Ballinger and Wilcox and also Hey additionally explored the role of heteroscedastic error structures for explaining risky choice.

III. A decision cost explanation: Similarity models

In contrast to the "reduced form" treatment of choice patterns through the weighting function $\pi(\cdot)$ under the GEU models, an alternative approach establishes instruments for decision costs and benefits based on the importance and/or the difficulty of risky choice selection. To motivate this approach, consider again the Kahneman and Tversky pairs AB and CD. The similarity approach (Rubinstein, 1988, 2003; Leland, 1994; Buschena and Zilberman, 1995, 1999, 2000), and related work in Coretto (2002) posits that the difference in the relative importance in choice between pair AB (important or dissimilar) and pair CD (unimportant or similar) explains patterns of choice violating EU.

In this similarity approach, respondents are held to place less importance on the choice between pair CD relative to the choice between pair AB, with a resulting higher likelihood of selection of the more risky alternative D over C than for the more risky alternative B over A. The similarity approach holds that, when choices between dissimilar pair AB (which is taken to more accurately reflect actual attitudes regarding risk return tradeoffs) and pair CD are compared, the increased likelihood of selection of alternative D relative to alternative B appears to reflect non-EU preferences. Rather than changing the preference axioms as in the GEU models, these similarity models suggest a structural approach whereby choice models are augmented with a measure that reflects the importance and difficulty of choice. Specifically, assuming that computation extracts mental cost, and the decision maker can select from two algorithms, one more effort intensive (EU), and the other less effort intensive (e.g. choosing the outcome with the highest expected value). The similarity approach can be interpreted as the outcome of an optimization process where a decision maker maximizes expected utility of choice less mental cost of calculation. When outcomes are similar, and the loss for making the wrong choice is small, the decision maker may prefer the less effort intensive approach. On the other hand, when outcomes are dissimilar, and the loss for making the wrong choice is large, the decision maker may prefer the more effort intensive, and more accurate, approach.

Thus, the similarity approach assumes that optimization includes two elements. First, selection of an algorithm, and then, the actual choice using the selected algorithm. This two-stage procedure may be consistent with some of the new findings of neuroeconomics (Camerer, et. al). By observing where actions occur in the brain, it was found that distinct brain processes may serve the same function in different situations, one of the processes being a control process (effortful), the other being an automatic process (effortless). Studies also found that the brain is able to do assigned tasks efficiently, using the specialized systems it has at its disposal. The neuroeconomic literature also suggests that switching algorithms may result in non-optimal outcomes in certain situations. That implies a decision making approach that is consistent with our similarity model. If both the cost of effort and the cost of mistakes are taken into account, the decision rule resulting in these mistakes may be optimal.

Hey (1995) and Ballinger and Wilcox (1997) found some support for similarity effects in their exploration of heteroscedastic error structures and risky choice. Buschena and Zilberman (1995, 1999) tested a similarity model against GEU models, and found support for similarity effects through a structural approach. Buschena and Zilberman (2000) additionally found that incorporating similarity measures in an EU model with a heteroscedastic error framework.

One difficulty with the similarity approach is in defining the similarity measure. Rubinstein (1988) considers fairly simple gambles that offer only one nonzero outcome. In his model, the more risky gamble offers a p chance at outcome x, while the safer gamble offers a q chance of y ($p < q$ and $x > y$). Rubinstein provides an axiomatized model for choice under EU preference with similarity effects, and considers two measures for similarity on both outcomes and probabilities (defined here for probabilities).

Rubinstein's first measure is the absolute difference measure between the probabilities, $|p - q|$. His relative difference measure is the ratio of the probabilities, p/q. Rubinstein also includes a qualitative similarity criterion; probability q is dissimilar to p if $q = 1$ and $p < 1$; that is, degenerate probabilities that offer an outcome with certainty are viewed as dissimilar to probabilities with less than certainty.

Because many risky pairs offer more than one nonzero outcome, a few more general similarity measures have been proposed. Three similarity models are considered in this paper, because the definition of "similarity" is subjective. All of the models used nonlinear measures over the probability vectors to define similarity and differ in how they handle qualitative similarity (lotteries offering certainty of a positive payoff), with one model including it directly as an additional and separate explanatory term and the other two models incorporating certainty into their functional form.

Candidate Model 1: Distance-based similarity. This similarity model is relatively simple, using the Euclidian distance between the probability vectors $\underline{p}$ and $\underline{q}$ <u>to describe differences between gambles</u> (Buschena and Zilberman, 1995, 1996, 2000). This distance measure is reasonably applied for the set of gambles considered here that offer nontrivial risk-return tradeoffs. We have also explored a more generalized difference measure based on absolute differences in the distributions' Cumulative Distribution Functions, with this CDF measure is empirically comparable to the Euclidian distance measure.

In this paper's empirical estimation below, both the distance measure and its square will be considered. In addition to the distance measure, some of these estimates also include 0 - 1 similarity measure, quasi-certainty, that generalizes the qualitative component in Rubinstein's similarity definition. Quasi-certainty in this definition takes the value 1 if the less risky of the two gambles in the pair gives greater than a zero payoff with certainty.

Candidate Model 2: (Dis)similarity as defined through cross entropy. Entropy provides a useful alternative measure for describing the differences between probability distributions. Specifically, the cross-entropy measure described below over the distributions serves to define the similarity between risky alternatives.

Shannon's (1948) entropy stems from information theory and serves as a measure of how a particular distribution differs from a uniform distribution. The entropy measure for a discrete distribution is:

$$Ent(\underline{p}) = -\sum_{i=1}^{n} p_i \ln(p_i)\,. \tag{3}$$

A uniform distribution is in a sense the least informative over a range of outcomes, so the smaller a distribution's entropy, the more information it contains. A uniform distribution has maximum entropy, while a degenerate (p_i = 1, 0) distribution has minimum entropy and is clearly most informative. Coretto (2002) develops a risky choice model combining EU with Shannon's entropy as an explanation of EU violations.

More useful than Shannon's entropy for our application is the Kullback-Leiber (1951) cross entropy that measures how two distributions (our $\underline{p}$ and $\underline{q}$) differ from one another:

$$CrossEnt(\underline{p},\underline{q}) = -\sum_{i=1}^{n} p_i \ln(p_i / q_i) \,. \tag{4}$$

Two identical (extremely similar) distributions would have a cross-entropy value of 0. As the distributions differ more from one another (are dissimilar), the cross-entropy measure increases.

Under this candidate model for similarity, the cross entropy of distributions is used to define the similarity of the pairs in our empirical estimations. For our empirical application, q_i never takes the value 0. If p_i takes the value 0, the value inside the sum is set to zero for our estimations.

The monograph by Golan, Judge, and Miller (1996) provides a very thorough treatment of the use of entropy in econometrics. Preckel (2001) provides a useful summary of entropy and cross entropy. To our knowledge, this paper offers the first extensive test of cross entropy for explaining EU independence violations. In our empirical application, we consider subsets of models including the risky pairs' cross entropy, its square, and the previously defined quasi-certainty measure.

Candidate Model 3: Loomes' nonlinear similarity. Loomes (2006) recently introduced a nonlinear similarity measure (ϕ). This measure allows for both the ratio of the gamble probabilities and the difference in the probabilities to affect choice. The relative effects of the probability ratios and differences can vary across individuals in Loomes' measure, with these relative effects defined through two parameters, α and β, in the following formulation. Loomes' model allows for gambles over as many as three outcomes, with these gambles defined through their probability vectors p and q:

$$
\begin{aligned}
&f = [1-(p_1/q_1)] \\
&g = [1-(q_2/p_2)] \\
&h = [1-(p_3/q_3)] \\
&a_I = (q_1 - p_1) \\
&a_J = (q_3 - p_3) \\
&\phi(\alpha,\beta,\underline{p},\underline{q}) = (fgh)^{\beta}[(a_I/a_J)^{(a_I+a_J)^{\alpha}}].
\end{aligned} \tag{5}
$$

Loomes development of $\phi\ (\cdot)$ includes a discussion of its two component parts, relating to the two coefficients α and β. The coefficient α is restricted to be nonpositive and defines the degree of divergence between the perceived (subjective) and objective (actual) probability ratios. If α = 0, there is no difference between these ratios; as α declines, this perceived vs. objective difference becomes larger. In Loomes' development, α differs across individuals.

The $(fgh)^{\beta}$ component of φ(·) "scales down" the bracketed portion that relates to how close the less risky alternative is to certainty. In particular, Loomes' is working to define qualitative certainty-type effects more generally than in the candidate models using the Euclidian distance or the cross-entropy measures. A value of β = 0 in (5) indicates that the ratio of probabilities does not affect choice, while β values of larger absolute value indicate a person strongly affected by these ratios, and thus more affected by the less risky gamble's relationship to certainty. As for α, the β coefficient likely differs across individuals in Loomes' formulation.

A priori, if we assume that decision makers account for the computational effort associated with the choice of algorithm, then perhaps and the more easily interpreted Euclidian distance similarity measure has additional merit over the cross-entropy and power function measures such as Loomes. [David Z, I'm still not sure that we can say much about this, and wonder if it's necessary.]

IV. The data: "Industrial strength" probability triangle with real payoffs

Our experiment was designed to test for similarity effects on EU violations intensively. The risky pairs allow for clear tests of both quantitative (such as measured by Euclidian distance) and quantitative effects (such as measured by quasi-certainty) across a large set of risky pairs. The set of gamble pairs given to each respondent was devised so that parametric estimation of the three candidate similarity models in explaining EU violations should be possible. Each respondent faced both hypothetical and "real" gambles, but we found no significant effects of real payoffs on choices for estimation using individual choice.

More than 300 undergraduate students faced a series of risky choices over gamble pairs offering a risk/return tradeoff for which EU predicts the same qualitative choice. EU would predict that (1) either the less risky choice would always be selected for every choice pair by an individual, or (2) the more risky choice would always be selected. Only a few (27, 8.5%) of subjects had choices completely consistent with EU in that they always selected the least risky option in every pair (no subject always selected the most risky option). For the other (288) subjects who had some choices violating EU, we use the candidate similarity models to explain when the riskier choice is more likely. With this approach, we focus on measuring the characteristics of the risky pairs that give rise to the risk-taking behavior and thus for many respondents EU violations when their choices over similar pairs is compared with their choices over dissimilar pairs. This experiment and data are additionally described in Buschena and Zilberman (1999).

The risky pairs in the experimental design differed considerably in their quantitative and qualitative similarity, allowing an extensive test of how similarity (defined through our three candidate measures) relates to the occurrence of EU violations. The gamble pairs used in the design are illustrated in Figure 1. Gamble $p = (p_1, p_2, p_3)$ and $q = (q_1, q_2, q_3)$ were defined for each subject over a common set of outcomes $x = (x_1, x_2, x_3)$ for all questions.

The data set considered from each subject consisted of their choices over 26 risky pairs. Gambles are defined by a choice between two vectors of discrete probabilities over a common set of outcomes, either ($0, $15, $20) or ($0, $30, $40). The pattern for these gambles began with Kahneman and Tversky's (1979) certainty effect pairs described at the outset of our paper. Kahneman and Tversky's four original gambles are included in the set of risky pairs we use, but are augmented by an additional 104 gamble pairs. In each of these pairs, one gamble (s) is less risky (lower expected value and variance) than the other gamble (r) is. An important design feature is that EU predicts the same choice pattern for each pair, the same prediction made use of by Kahneman and Tversky in showing their well-known patterns of EU violations.

The gamble pairs are illustrated in Figure 1. Lotteries on the borders of the unit triangle are listed in the table below Figure 1, with each border pair defined by the probability vectors $\mathbf{b^1}$ and $\mathbf{b^2}$. Kahneman and Tversky's pairs are *RV* and *kℓ* in this Figure. Every other lottery on the locus of points is defined using a scalar $\alpha \in (0, 1)$, as a linear combination of the border pairs: $\alpha\mathbf{b^1} + (1-\alpha)\mathbf{b}^2$. For example, gamble *B* in the figure is a combination of border pairs *A* and *C* where $\alpha = 0.5$. For interior pairs on the *DH, RV*, and *fj* loci, α takes values 0.25, 0.5, and 0.75. For interior pairs on the *IQ* and *We* loci, α takes values 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, and 0.875. A risky pair was defined for every possible combination of points on each locus—e.g., additional pairs on the *DH* locus were *DE, DF, DG, EF, EG, EH, FG, FH,* and *GH*. There were 106 gamble pairs in total to be selected from. This set of risky pairs varies considerably in their values for the candidate similarity measures. The EU model predicts consistent choice for each one of the 106 pairs—either always **r** or always **s**.

V. What do decision makers view as similar?

Our three candidate models for risky pair similarity were designed to explain choice patterns violating EU, and that explanation is our primary interest. Another subject of interest is to what degree these measures relate to how respondents view the pairs. To assess these views, we make use of a set of subjective responses regarding the similarity of the gamble pairs, elicited using a visual scale on the computer screen over a subset of risky choice pairs (5-6 per person). Respondents moved a cursor over a computer screen to indicate their subjective similarity level on the scale below:

1		10
very dissimilar	moderately similar	very similar

Estimation. The three candidate models were used to assess the subjective responses via OLS regression, n = 1672. The error structure did not exhibit any apparent heteroscedasticity or time series problems.

The Euclidian distance model included a constant, distance, its square, and the quasi-certainty measure. The cross-entropy model included a constant, the pairs' cross entropy, its square, and the quasi-certainty measure. The Loomes' power function model included a constant and the function $\varphi(\cdot)$ in (5).

The results of the estimation for the subjective similarity responses under the Euclidian distance model are given in Table 1. The subjects viewed a pair as increasingly less similar at a decreasing rate as the pair's distance increases.

The results of the cross-entropy model in Table 2 are comparable to those for the Euclidian distance, with the pairs decreasing at a decreasing rate in subjective similarity as the cross entropy increases.

The results of the Loomes' model reported in Table 3 indicate some estimation problems. The estimated alpha and beta coefficients were outside the range allowed (alpha must be nonnegative and beta must be nonpositive). Estimations were unsuccessful when alpha and beta were restricted to the ranges required in Loomes. These results were robust to an alternative model excluding the constant, and to a model restricting beta to zero.

In short, the candidate models for subjective similarity use the Euclidian distance and the cross-entropy terms we estimated with the anticipated signs, while Loomes' Power function model was estimated with parameter values outside their allowable range. Information criteria measures such as Akaike's show a slight explanatory advantage for the Euclidian distance model over the cross-entropy model.

VI. Estimated effects of candidate models on choice

A. The model

Variants of each of the three candidate similarity models were estimated in a logit framework for discrete choice (0 = the less risky lottery selected, 1 = the more risky lottery selected). These were estimated separately for each respondent's choices, where there were 26 observations per respondent.

Estimations were carried using the public domain software R. This has become a powerful statistical tool that is particularly good at data management and model diagnostics. We estimated (not always successfully for each respondent) a total of 10 different models based on the three candidate models above. This fairly extensive coverage of the various models was used in order to give as extensive a test of each three candidate models' predictive power. The 10 candidate models are given in Table 4, with the variable list omitting the constant terms that were included for each estimation. The standard logit estimation was carried out for the linear in parameter models 1-8. We carried out a maximum likelihood grid search for the nonlinear in parameter models based on Loomes' similarity approach for models 9 and 10. Model 10 was selected after previous runs over model 9 gave rise to numerous cases of β estimates near zero. Diagnostic tests showed that a comparable grid search approach provided virtually exact results as did the standard estimation for models 1-8. There were some subjects who had no risky lotteries on the border of Figure 1, so none of their observed choices had a value other than 0 for the quasi-certainty variable. For these subjects, models 3, 4, 7, and 8 were estimated omitting that variable; for example, for these subjects, model 1 was estimated instead of model 3.

Information criteria provide a useful method of fit comparison for the multiple non-nested models we consider. These information criteria are based on the estimated log-likelihood functions for each model, but also include a penalty for the number of parameters in each model.

We report two information criteria for completeness, with relatively consistent results. Akaike's (1973) criteria for model j, respondent i is for a sample size n and model parameter k, is:

$$AIC_{ji} = 2*[-LLF_{ji}(\ ) + k_{ji}]/n. \tag{6}$$

The lower the AIC, the higher the model's rank. The AIC measure is asymptotically efficient, but we have only 26 observations. This measure is biased toward models with higher dimension (Hannan and Quinn, 1979; Hurvich and Tsai, 1989; Schwartz, 1978).[1] Suguira (1978) suggests a bias corrected version of the AIC. This corrected measure was extended to nonlinear models by Hurvich and Tsai as:

$$AIC\ C_{ji} = \mathrm{A}IC + [2*(\ k_{ji} + 1)(\ k_{ji}\ + 2)]\backslash[n - k_{ji} - 2]. \tag{7}$$

Schwartz develops an asymptotically optimal Bayesian-based information criterion. This SIC criterion favors models with fewer parameters relative to the AIC (Schwartz, 1978). Note that for our data the penalty relative to the log-likelihood under the SIC is approximately twice that of the AIC C. The Schwartz information criterion is[2]:

$$SIC_{ji} = [-LLF_{ji}(\ ) + 1/2k_{ji} * log(n)]. \tag{8}$$

Clearly, all of the information criteria are asymptotically equivalent. Their differences may be important for small samples such as our relatively small sample of 26 observations.

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[1] Note however that the AIC is justified under somewhat weaker assumptions regarding the underlying model through a maximum entropy approach (White, 1982; Bozdogan, 2000).
[2] We break with convention by using "SIC" to acknowledge Schwartz, rather than the commonly used "BIC" for Bayesian information criteria.

B. Results

Information from the information criteria rankings for each model over every individual's choices is given in Tables 5 and 6 for the AIC C, and the SIC criterion. We provide the results of both of these rankings to illustrate how they differ for evaluation of these relatively small sample estimations. Note also that although the AIC and AIC C values themselves differed, their rankings did not.

For both ranking methods, we report (1) the average ranking across individuals for the 10 models; (2) the percentage of subjects for which each model had the first, second, and third ranks; and (3) the percentage of subjects for which the model ranked in the top five models.

A. AIC C tanking results, Table 5.

The AIC C results from the Logit models provide a fairly clear picture of the relative predictive power of the risky pairs' Euclidian distance over the probabilities to describe choice patterns. This distance measure has low average rank when used singly in model 1. Loomes' restricted power function model with $\beta = 0$ (model 10) and the model using cross entropy (model 4) also perform well in terms of AIC C ranks. The evidence regarding the full version of Loomes' model (model 9) is very clear; the restricted version of this model has considerably more support under both the AIC C ranking and the SIC ranking below. Under any of the three candidate formulations, higher dimension models are penalized quite heavily by the AIC C measure. This penalization is likely related to our relatively small sample size, but note that the unreported AIC rankings were virtually identical to the AIC C rankings that include an additional penalty for higher dimension models.

B. SIC ranking results, Table 6.

In contrast to the AIC C ranking results in Table 5, the SIC results offer less clarity in terms of model selection. The uninformative prior and the apparent close substitutability between models tend to show less of a distinction between models. All of the models save one (model 9) are now favored by at least some subjects. As a candidate model, the family of Euclidian distance-based models (1-4) receives support from the SIC rankings in that they have somewhat lower average ranks than other models do. The cross-entropy model in a quadratic form (model 6) also has relatively low average rank. The restricted Loomes' power formulation model is not compared as favorably to the other models under the SIC ranking.

C. Posterior probabilities using the SIC measures.

The Bayesian structure of the SIC statistic in equation (8) allows construction of an approximate measure of the (posterior) probability for observing the estimated SIC conditional on a uniform (uninformative) prior. This interpretation makes the SIC quite attractive in terms of model selection, as these posterior probabilities provide information on the models beyond the rankings in Tables 5 and 6. We construct and estimate these estimated posterior probabilities for the 10 candidate models.

The constructed approximate posterior conditional probabilities, the SIC weights, for model j and respondent i are constructed as:

$$SIC\ Weight_{ji} = \sum_{k} exp(\text{-}SIC_{ji}) \,/\, exp(SIC_{ki})\,. \tag{9}$$

As shown in Schwartz, these SIC weights are the approximate posterior probabilities of observing the sample conditional on an uninformative prior (see also Ramsey and Schafer, 1978). Strictly speaking, this interpretation is developed for linear models such as the Euclidian distance and cross-entropy candidate models. We provide summary statistics for these SIC weights across the set of respondents for each of the candidate models in Table 7a. The minimums for these approximate posterior probabilities were uniformly zero for each model.

The posterior probabilities again illustrate the substitutability between the candidate models, particularly for the Euclidian distance based models (1-4), and the models using cross entropy (6 and 7). There is more limited support for the full cross-entropy model (model 8), and the restricted Loomes power formulation (model 10).

In order to better assess the posterior probabilities of these candidate models, we recalculated them using a selected subset of models, with one model from each of these three nested "families." We selected what appeared (from Tables 6 and 7a) to be the most promising models from (a) those using the Euclidian distance measure (model 1), (b) those using the cross-entropy measure (model 4), and (c) the most promising from those using the Loomes power formulation (model 10).

The summary statistics for the posterior probabilities for this restricted set of models are given in Table 7b. The models again appear to substitute for one another on average, with model 1 from the Euclidian distance family having a somewhat higher mean posterior probability. The restricted Loomes power function model (model 10) has somewhat more support in this table than was illustrated in Table 7a. Note also that each model appears to have strong support for particular subjects, as evidenced by the maximum posterior probabilities in Table 7b.

In order to reduce the potential for selection bias in favor of the Euclidian distance- and cross-entropy-based model families, we reproduced the subset approach by selecting what appeared to be the "worst" two models from these families. The summary statistics for these models (models 2 and 5) compared with Loomes' restricted model (model 10) are given in Table 7c. Clearly the potential for selection bias based on the results in Table 7a is imperfectly understood. There is again support for the Euclidian distance family, here model 2.

D. Summary of the information criteria rankings.

Somewhat as expected, the three-candidate models for measuring similarity effects on choice appear to serve as substitutes. There is support (particularly under the AIC C criteria) for the relatively simple Euclidian distance-based models over the cross entropy and the Loomes' power formulation. The AIC C criteria provided quite clear ranking results, while the SIC criteria indicated that the models within one and to some extent between the three-candidate families showed considerable substitution with one another.

These findings may depend on the particular set of experimental data. In particular, we estimate the underlying likelihood functions using a per-respondent sample size of 26 for a large number (n = 288) respondents. Additional analysis using alternative data sets may provide more refinement of these results.

VII. Conclusion

Both the experimental studies and neuroeconomics suggest that the traditional frameworks for analyzing decision making under uncertainty have to be modified. The similarity approach can explain some of the behavioral paradoxes detected by experimental studies, and it also suggests that different algorithms are used for different choices under uncertainty, which is consistent with the new findings in neuroeconomics. In particular, this similarity approach suggests that EU is used when choices are dissimilar and the stakes are higher, and a simpler rule is used when outcomes are more similar, and the stakes are correspondingly lower. The task, then, is to identify which measure of similarity is triggering the algorithm choice. The empirical analysis suggests that the intuitive Euclidean distance measure is most consistent with the observed data. The cross-entropy measure, which measures relative distributional information content, also performs well, while the Loomes' measure, which combines probabilities and outcomes in a non-linear fashion is least likely to predict observed choices.

The econometric methodology presented here allows for selection between similarity measures using (1) robust grid search maximum likelihood estimation routines over the full set of choices made by a respondent, and (2) several information criteria. These methods are to our mind superior to the analysis of subsets of choices taken to support or reject a particular theory. In this regard we extend work by Hey and Orme, and by Hey, to which risk analysts are substantially indebted.

[David Z, I deleted the above because it seemed to repeat the first paragraph in the conclusions, but maybe you had a separate point in mind. I would like to acknowledge Hey here as I see his work as providing some statistical honesty to this issue].

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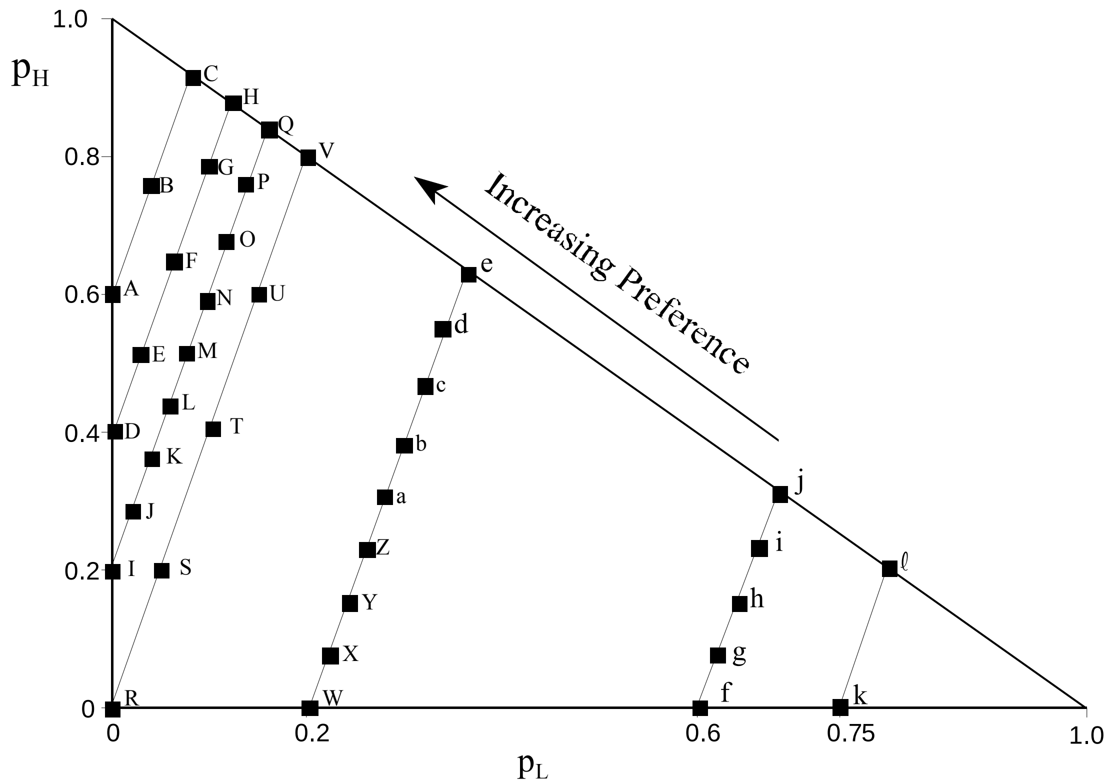
Fig. 1. Probability triangle for experimental risky choice pairs



Probabilities of the border pairs

Pair	p_L	p_M	p_H	q_L	q_M	q_H
AC	.00	.40	.60	.08	.00	.92
DH	.00	.60	.40	.12	.00	.88
IQ	.00	.80	.20	.16	.00	.84
RV	.00	1.00	.00	.20	.00	.80
We	.20	.80	.00	.36	.00	.64
fj	.60	.40	.00	.68	.00	.32
Kl	.75	.25	.00	.80	.00	.20

Table 1
Subjective similarity and Euclidian distance

Variable	Coefficient est.	Standard error
Constant	6.42***	.065
Distance	-5.84***	.361
Distance SQ	3.06***	.291
Quasi certainty	-.674***	.151

LLF -3285.2, AIC = 3.93
N = 1672
R^2= .201
R^2 adjusted = .200

Table 2
Subjective similarity and cross entropy

Variable	Coefficient est.	Standard error
Constant	6.42***	.065
Cross entropy	-5.84***	.361
Cross entropy squared	3.06***	.291
Quasi certainty	-.674***	.151

LLF -3414.1, AIC = 4.09
N = 1672
R^2 = .068
R^2 adjusted = .067

Table 3
Subjective similarity and Loomes' power function

Variable	Coefficient est.	Standard error
Constant	3.61***	0.202
Alpha	-0.23***	0.011
Beta	2.20***	1.05
		LLF -3286.0, AIC = 36.93
		N = 1672

Table 4
List of candidate models

Model	Basic measure	Variables (in addition to a constant)
1	Euclidian distance	Distance
2	Euclidian distance	Distance, distance squared
3	Euclidian distance	Distance, quasi-certainty
4	Euclidian distance	Distance, distance squared, quasi certainty
5	Cross entropy	Cross entropy
6	Cross entropy	Cross entropy, cross entropy squared
7	Cross entropy	Cross entropy, quasi certainty
8	Cross entropy	Cross entropy, cross entropy squared, quasi certainty
9	Loomes' full	Loomes power function, unrestricted model
10	Loomes' restricted	Loomes power function, β restricted to zero

Table 5
Akaike information criteria (small sample) corrected ranks for logit models explaining choice under risk, N = 288

Model	Proportion of Subjects with AIC C Rank				Average
	Rank = 1	Rank = 2	Rank = 3	Rank 1-5	rank
1. Euclidian distance	40%	44%	15%	100%	1.79
2. Euclidian distance, distance squared				49%	5.51
3. Euclidian distance, quasi-certainty	2%	1%	3%	61%	5.08
4. Euclidian distance, distance squared, quasi-certainty					9.29
5. Cross entropy	33%	17%	46%	100%	2.20
6. Cross entropy, cross entropy squared				47%	5.57
7. Cross entropy, quasi-certainty	3%	0%	0%	45%	5.57
8. Cross entropy, cross entropy squared, quasi-certainty					9.34
9. Loomes' power formulation, full model					8.12
10. Loomes' power formulation, restricted model	27%	32%	38%	100%	2.18

Table 6
Schwartz information criteria ranks for logit models explaining choice under risk, N = 288

Model	Proportion of Subjects with SIC Rank				Average
	Rank = 1	Rank = 2	Rank = 3	Rank 1-5	Rank
1. Euclidian distance	18%	15%	9%	69%	4.11
2. Euclidian distance, distance squared	10%	11%	12%	59%	4.88
3. Euclidian distance, quasi-certainty	12%	13%	13%	14%	4.53
4. Euclidian distance, distance squared, quasi-certainty	5%	15%	9%	69%	5.16
5. Cross entropy	18%	14%	13%	57%	5.41
6. Cross entropy, cross entropy squared	12%	6%	14%	35%	4.84
7. Cross entropy, quasi-certainty	6%	11%	11%	43%	5.54
8. Cross entropy, cross entropy squared, quasi-certainty	17%	11%	8%	57%	5.09
9. Loomes' power formulation, full model					9.89
10. Loomes' power formulation, restricted model	11%	6%	8%	53%	5.21

Table 7a. Schwartz information criteria posterior probabilities for logit models explaining choice under risk, N = 288

Model	Average	Median	Maximum
1. Euclidian distance	.113	.085	.443
2. Euclidian distance, distance squared	.111	.084	.700
3. Euclidian distance, quasi-certainty	.122	.095	.665
4. Euclidian distance, distance squared, quasi-certainty	.120	.087	.995
5. Cross entropy	.082	.054	.391
6. Cross entropy, cross entropy squared	.122	.081	.766
7. Cross entropy, quasi-certainty	.089	.059	.899
8. Cross entropy, cross entropy squared, quasi-certainty	0.133	0.087	0.855
9. Loomes' power formulation, full model	.010	.004	.073
10. Loomes' power formulation, restricted model	.096	.062	.877

Table 7b. Schwartz information criteria posterior probabilities for logit models explaining choice under risk. Subset of Models Selected as the Optimal Model From Each of Three Families, N = 288

Model	Average	Median	Maximum
1. Euclidian distance	.374	.344	.999
8. Cross entropy, cross entropy squared, quasi-certainty	.274	.226	.973
10. Loomes' power formulation, restricted model	.359	.318	.998

Table 7c. Schwartz information criteria posterior probabilities for logit models explaining choice under risk. Subset of Models Selected as the Apparent "Worst" Model From The Euclidian Distance and Cross Entropy Families, N = 288

Model	Average	Median	Maximum
2. Euclidian distance, distance squared	0.384	0.317	0.999
5. Cross entropy	0.296	0.247	0.998
10. Loomes' power formulation, restricted model	0.319	0.301	0.981