May 30, 2001

Estimating the Speed of Market Reaction to News:

Market Events and Lumber Futures Prices

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Abstract:

With 16 years of daily lumber futures prices, we study the effects of different types of information releases: (1) monthly housing starts estimates, (2) aperiodic administrative and judicial announcements about U.S.-Canada trade disputes, and (3) novel and unprecedented court decisions related to the Endangered Species Act (ESA). The information releases are different in ways that predict their relative speeds of impoundment in prices. We test the predictions using a new event study methodology appropriate to relatively slowly evolving information events. We find that housing starts are absorbed more quickly than trade events, which are absorbed more quickly than ESA events.

JEL Classification numbers: C22, G14, Q23

I. Introduction

Lumber prices in the United States have swung widely in recent years. Between early January and early March of 1993, for example, lumber futures prices more than doubled, from $240 to $490 per thousand board feet (mbf). By July of that year, futures prices had declined again to less than $230.[1] There has been considerable debate over the causes of such volatility.[2] Likely explanations include domestic supply factors, such as cutbacks in logging due to court rulings about the spotted owl and other endangered species, and domestic demand factors such as housing starts. Other possible sources of volatility include trade factors particularly those associated with a protracted dispute between Canada and the United States.

We investigate the impacts of these events on lumber futures prices, focusing on both the size of their impacts and on the time paths of the market's response. Specifically, we compare the impact of three types of events that are monitored and chronicled extensively by lumber industry publications: (1) regular, periodic events in the form of housing start announcements; (2) aperiodic policy decisions related to U.S.-Canada lumber trade disputes; and (3) irregular and unprecedented information releases in the form of court rulings related primarily to the northern spotted owl. We test the hypotheses that the magnitude and speed of market responses vary systematically across these types of events due to systematic differences in the way information enters the market, and in the familiarity of the market with the different information types. We present evidence that regular periodic announcements tend to be incorporated into lumber

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[1] Cash prices showed similar variation. Between October 1992 and March 1993, for example, monthly average cash lumber prices increased from roughly $250 to almost $500 per mbf. By July 1993, prices had declined to about $300. More recently, in January 2001, futures contract prices were about $185 per mbf and by May had almost doubled to $350.

[2] The debate on this issue reached a peak, at least in the popular press, in 1993. Information sources cited by the news media at that time included Gorte (1993), Wilderness Society (undated), and Sierra Club Legal Defense Fund (1993).

futures prices more rapidly than events whose content is multi-dimensional and whose announcement dates are not known in advance.

There is a large literature using event study methods, developed by financial economists to assess the performance of financial markets. The seminal paper in this literature, Fama, Fisher, Jensen and Roll (1969), examines the effects of stock splits on firm returns. Two strands of the event study literature are closely related to the present study: (1) studies of the impacts of regulations on stock values (see, for example, Schwert (1981), Maloney and McCormick (1982), Binder (1988) and Ellison and Mullin (1995 and 2001); and (2) studies of the impacts of scheduled events—for example, the release of various government reports—on market prices for commodities (see, for example, Sumner and Mueller (1989), Fortenbery and Sumner (1993), and Carter and Galopin (1993). The model we use for estimation—the Distributional Event Response Model, or DERM—represents an innovation to previous event study methodology by parameterizing the time path of market responses to events. The DERM posits that market responses to events follow a time path that is unique to the type of event studied and allows a straightforward comparison of the rate of market response to different event types. At the same time, the DERM allows individual events to affect prices to different degrees. Our applications of the model explore maintained hypotheses that the responses of the market rate of return to new information follow shapes described by the normal, uniform, and beta distributions.

Our study requires empirical measures of the three types of market events. Demand events are assumed to coincide with the monthly release of housing starts estimates. Trade policy events are assumed to take place on the date of quasi-legislative agreements negotiated by the U.S. International Trade Administration (ITA) and the U.S. International Trade Commission (ITC). To define event dates for possible supply restrictions from judicial endangered species rulings, we compile a legal history of the spotted owl controversy and mark the dates of significant rulings as event days.[3] Previous analyses of the impacts of such regulations include Gorte (1993), Montgomery, Brown, and Adams (1994), and Yoder (1994).

There is a large body of research using annual, quarterly, and monthly price series from timber, lumber and wood product markets. (See, for example, Adams, McCarl, and Hamayounfarrokh; Boyd and Krutilla; Lewandrowski, Wohlgenant, and Grennes; and Uri and Boyd.) Notable in connection with the issues we examine are the studies by Murray and Wear (1998) and Wear and Murray (2001). Using monthly lumber prices, they investigate two issues. The first is the impact of logging restrictions due to the Endangered Species Act on the time series properties of regional lumber prices. They find that such restrictions had the effect of more tightly integrating the two major U.S. production regions. Second, they estimate the welfare effects of the restrictions in a market model. Aggregate welfare costs of the spotted owl-related bans, mainly borne by forest product consumers, are estimated to be near $3 billion per year over the 1989-1995 period. They conclude that Canadian trade restrictions—events also studied in the present article—had a much smaller effect. While Murray and Wear focus on the aggregate sizes of these effects, we focus on their timing and price impacts by analyzing daily time series of futures market prices.[4]

The paper proceeds as follows. Predictions about market response differences across event types are developed in Section II. In Section III we describe the data. In Section IV, we

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[3] Related information releases may also occur at times surrounding the events that we identify. What we identify as a price response to information distributed over time is also, in part, a response to these related information events.

[4] Readers interested in state or regional economies may be concerned about the relevance of a single national lumber price series. Previous research by Uri and Boyd (1990), Jung and Doroodian (1994), Yoder (1994), and Murray and Wear (1998) suggests that lumber price movements are highly correlated across regions and that the U.S. lumber market can usefully be analyzed as a national market.

present the results of our empirical analysis using standard F-tests for differences in variances as well as our new method of estimating the impacts of events (the DERM). Section IV also summarizes the relationship between previous event studies and our analysis and methodology. Section V concludes.

II. Variation in Price Response to Different Event Types

Numerous previous studies have used daily data to examine the impact of particular types of events on futures prices in various markets. Sumner and Mueller (1989), for example, examine the information content of USDA harvest forecasts by analyzing movements in corn and soybean futures prices. Robenstein and Thurman (1996) investigate the impacts of the release of reports concerning the adverse health effects of red meat on cattle futures market prices.[5] To our knowledge, however, there has been no comparison—either theoretical or empirical—of the impacts of different types of information-revealing events. The analysis presented below estimates and contrasts the impacts of three different types of events: regular, periodic announcements concerning conditions in a particular segment of the wood products market; irregular releases of potentially important information in the form of court decisions; and sporadic releases of information through the passage of legislation and through quasi-legislative rulings by government agencies and bureaucracies.

We hypothesize that the price response to these events will be systematically different. Consider the first type of event listed above. In the context of the wood products industry, an example is the monthly release of U.S. housing starts estimates. These estimates are released at a time known in advance to market participants and measures are taken to ensure that private

[5] Also see Mann and Dowen (1996), Ederington and Lee (1993), Colling and Irwin (1990), Fortenbery and Sumner (1993), Carter and Galopin (1993), and Miller (1979).

market participants do not know the content of the announcements prior to their release.[6] To the extent that the housing starts estimates are different from the market's expectations, market participants know from experience how to quickly interpret and respond to the information. We predict, therefore, that insofar as the release of a housing starts estimate affects lumber futures prices (because it is different from the market's expectation) the lumber futures price will adjust rapidly. Because market participants' expectations are not observable, we are not able to predict the direction of the impact of an announcement. For example, an announcement that indicates a substantial increase in housing starts can cause lumber futures prices to fall if the announced increase is less than expected.

Consider next the release of potentially important information in the form of court decisions. The court decisions considered below all relate to the Endangered Species Act which, between 1989 and 1996, closed certain private and public lands to logging, thus affecting lumber markets. Court decisions have at least three features that are important in understanding market responses. First, the timing of court decisions cannot be predicted far in advance. Second, there is substantial uncertainty concerning the content of the court ruling in advance. Third, whereas housing starts estimates are a one-dimensional piece of information, court decisions typically are multidimensional and can contain considerable ambiguity. Thus, we predict that compared to the release of housing starts estimates, it will take longer for the ultimate impact of a court decision on prices to be revealed. We have no prediction concerning the relative magnitudes of the market responses to the release of a housing starts estimate relative to a court decision. A

[6] A "housing start" is defined to occur at the time when excavation begins for the footing or foundation of a building. Housing start estimates, based on nationwide sampling by the U.S. Census Bureau and issued monthly, are released at 8:30 a.m. on the twelfth working day of each month. The release date is delayed by one day if the twelfth workday is the first workday of the week. The actual release date is always listed in the prior release. Security is tight, with lockdowns from the time the data are all on one computer (data are collected by numerous individuals in the field) until release time.

given court decision may be very different from the market's expectations and therefore have a relatively large impact. On the other hand, even if a court decision is quite different than expected, it may have little impact because the decision directly affects only a small segment of the total wood products market.

Finally, consider what we believe to be an intermediate case: the release of information through the passage of legislation and through quasi-legislative rulings by government agencies and bureaucracies. Events in this category included in our empirical analysis relate to legislation, rulings, and agreements affecting U.S.-Canada trade. The legislative and rule making processes are quite different from the other two categories of events in at least one dimension. There likely is considerably less uncertainty concerning the nature of the information to be revealed. Public hearings and meetings are involved and, by the time a decision is made and announced, it may contain little news for the market. The information released is likely to be less novel than judicial rulings on environmental supply restrictions, but more difficult to assess by market participants than are housing starts. Thus, we predict that the market's response to a trade event will be more diffuse than its response to a housing start announcement but less diffuse than the response to a court ruling.

III. Data

CME Lumber Data

The Chicago Mercantile Exchange Lumber Futures price data we use for our analysis span the period February 1982 to April 1998. Contracts mature every two months, in January, March, May, July, September, and November. The last trading day for any contract is the 15th of the month in which the contract matures.

All results presented below apply to the two-to-four-month-out contract. The rollover date we use for contracts two to four months out is the last trading day of the month, two months before delivery. For example, the price used for the first trading day in March through the last trading day in May 1991 is the price of the July 1991 contract. The price used for the first day of June through the last day of July is the price of the September 1991 contract, and so on. The price series constructed in this manner is displayed in the top portion of figure 1. When we calculate daily price changes and rates of return on closing prices we discard the price changes and rates of return associated with rollover dates (such as the rate of return between the last day of May and the first day of June) so that each price change and rate of return is calculated using a single contract.

The lumber futures price series is subject to an upper limit on daily price changes. On March 8, 1993, the limit on price changes was changed from a fixed limit of ±$5 to a limit of ±$10 expandable to ±$15 after two days of limit moves. Various authors have addressed price limits, treating limit moves as censored, non-equilibrium observations on price (Yang and Brorsen 1995, Sutrick 1993, Mann and Dowen 1996). Limit moves, which comprise 10.5 percent of the observations in our data set, have potentially important impacts on our results, and we consider these impacts below. The rate of return on the futures contract, which is displayed in the bottom portion of figure 1, is defined as $R_t = \ln(P_t/P_{t-1})$, where P_t is the closing price on trading day t.[7] The two-to-four-month-out contracts we use for our empirical analysis have relatively heavy volumes, which justifies our focus on them. Moreover, preliminary analysis revealed no important differences in our conclusions for contracts with different times to maturity.

[7] For additional information on price changes and various issues associated with trading volumes, see Rucker, Thurman, and Yoder (2000).

Event Descriptions

We consider three sets of events: housing starts releases, Endangered Species Act (ESA) events, and trade policy events. Housing starts releases occur between the 15th and the 20th of each month. Dates and descriptions of the ESA and trade policy events are presented in tables 1 and 2. The middle portion of figure 1 shows the distribution of each set of events over time.[8]

The ESA events we study were chosen from a larger set of related litigation events based on two primary criteria: the court ruling related to a specific set of timber sales or harvests, and the sales and harvests accounted for relatively large volumes of harvestable timber. [9] The trade events we study include all the primary administrative actions from a series of administrative rulings relating to countervailing duty disputes between Canada and the United States during the period covered by our data. An additional trade agreement between Japan and the United States was included because the agreement had the potential to affect trade in wood products between these two countries. Trade issues were limited to U.S.-Canada and U.S.-Japan relations because these two countries are the primary trading partners with the United States for wood products.

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[8] In addition to ESA and Trade events and housing start announcements, we also examined the impacts of three hurricanes that struck during the period of our analysis—Hugo (September 22, 1989), Andrew (August 24, 1992), and Fran (September 5, 1996). Because we found no significant effects on futures prices, we do not discuss them further.

[9] A large number of ESA related cases were not included because they did not relate directly to a specific set of sales or harvests. For example, in *Portland Audubon Society versus Hodel* (decided May 18 1989, 9th Circuit Court of Appeals), the Portland Audubon Society argued that Bureau of Land Management was required to update their Environmental Impact Statements (upon which they base their management decisions) to include new information relating to Spotted Owl habitats. The case did not relate to specific timber sales or harvests (the court of appeals found the case to be not reviewable because of this fact).

Other cases were not included because the volume of timber in question was relatively small. For example, In the case *Portland Audubon Society Versus Hodel* (decided April 20 1988, 9th Circuit Court of Appeals) the Portland Audubon Society requested an injunction against logging 200-year-old timber within a 2.1 mile radius of 289 known Spotted Owl nesting sites (the injunction was not imposed). This judicial decision was not included in our event set because the volume of timber in question was small relative to the timber market and would likely have little or no impact on the price regardless of the outcome of the case.

IV. Empirical Analysis

We model the lumber futures price as a function of several exogenous variables and a set of event dummies. Following common practice in the literature, and to reduce serial correlation, we use rates of return on the lumber futures price series as the dependent variable, rather than price levels. In portions of the following analysis, we include four variables to control for non-event factors that we expect to affect lumber prices. These variables are the rates of return on the Commodity Research Bureau (CRB) futures market index, interest rates on thirty year treasury securities, Japan/U.S. exchange rates, and Canada/U.S. exchange rates. Daily data are used for all of these variables.

Various methods have been developed in the literature to test the effects of information events on futures markets. Below, we briefly discuss results from one such test.[10] We then develop and report results from a general regression-based approach (the DERM) for measuring and comparing the rates of absorption of new information associated with different types of events. We conclude the section with a discussion of the relationship between our approach and the existing literature on event studies. We also compare the DERM results with the results from two alternative regression-based methods of measuring the effects of information events on futures markets.

A. The Effects of Information Events: F-tests for Larger Variance Surrounding Event Days

A question of fundamental interest is whether the volatility of the rate of return on futures prices increases either on the day of an event or on the days surrounding an event. The ratio of the estimated variances of price movements on event days relative to non-event days,

[10] Details on the results from other previously developed tests are available in Rucker, Thurman, and Yoder (2000).

$\hat{\sigma}^2_{event}/\hat{\sigma}^2_{nonevent}$, is distributed as an F statistic under the null hypothesis of equal variances.[11] The F-tests in each row of table 3 indicate whether the log changes in CME lumber futures prices are large within a symmetric event window relative to the log changes on days not falling within an event window. Instead of analyzing the log changes themselves, we compare the residual variance from event periods with the residual variance from non-event periods, where the residuals are generated from the regression summarized in table 4. The table 4 regression plays the role of a market model for lumber futures prices capturing co-movement with returns on related assets, but also including a quadratic time trend.

Table 3 indicates that, for housing starts events, the F-statistic for the difference in variance between event periods and all other periods is largest for a specification in which an event period consists of the day of the event only (F=1.40, P=0.001). When an event period includes the event day, one day before, and one day after the event (a 3-day window) the F-statistic decreases in value and becomes less statistically significant. For housing starts, broadening the event window has the effect of averaging the impact of the day-of-event effect with the (lesser) impact of the two surrounding days, thus reducing the measured impact and its statistical significance.

In contrast, ESA events are shown to have little impact on the variance of the rate of return on the day of the event (the first row of the middle set of results in table 3). Unlike for housing starts, however, the F-statistic for ESA events increases as the event window widens, becoming more significant until the window is 5 days wide, with a p-value of 0.013. For wider windows, the test statistic diminishes. For trade events, like housing starts, the F-statistic is largest for the day of the event only.

[11] See, for example, Sumner and Mueller (1989) for an application of this test to examining the information content of USDA harvest forecasts.

This exercise shows that selection of window width affects the size and significance of the estimated impact of events on market price volatility. (Similar conclusions were reached, in a different event study context, by Lamdin, 1999.) Other factors that may affect estimated impacts of events include the location of the window (for a given width) relative to an event and the shape of the price-response path. These factors motivate the estimation strategy discussed next.

B. A Comparison of Market Reaction Times: The Distributional Event Response Model (DERM)

We now describe and implement a regression-based estimation method that (1) allows for considerable flexibility in measuring the impacts of events and (2) provides easily interpretable estimates of the speed of the market response to a set of events. We dub the model the Distributional Event Response Model (DERM) because it constrains response patterns to correspond to shapes of specified probability distributions. Our model is a generalization and extension of an empirical model developed by Ellison and Mullin (1995).[12]

The DERM is a nonlinear regression model in which (following Fama, et al. (1969) and numerous others) the rate of return on lumber futures is regressed on one or more market indexes to pick up market and macroeconomic effects not specific to the lumber industry, as well as on

[12] Ellison and Mullin used weekly data to study the effects on sugar refining and related firms from the passage of sugar tariff reform legislation in 1912. They analyzed the market effects of what is essentially a single event: the passage of the legislation, about which they knew several key dates–approvals by the House, by the Senate and by President Taft. In their analysis they imposed the constraints that responses between two of the dates rose according to a normal c.d.f., while responses on other dates were limited to the week that included the date. We generalize their analysis to alternative distributions. We extend their analysis by applying the distributed event response idea to multiple realizations of the same type of event and by estimating and comparing speeds of adjustment in market prices to different types of events. In more recent work (Ellison and Mullin, 2001) the same authors apply isotonic regression to a related problem: inferring the market effects of the gradual release of information–over a 20-month period–of political proposals to reform health care. Their method is nonparametric and particularly well suited to identifying gradual, but permanent, price responses to a large number of unidentified information releases.

dichotomous variables that represent the set of events of interest. With k events of the same type, the DERM is:

$$y_t = \alpha + \delta' X_t + \sum_{i=1}^{k} \beta_i f(d_t^i; \theta) + \varepsilon_t, \tag{1}$$

where y_t is the rate of return on lumber futures prices, X_t is a vector of continuous explanatory variables, δ is its associated coefficient vector, k is the number of events, and d_t^i is the number of days from event i. The function $f(d_t^i; \theta)$ is a density function for d_t^i with parameter vector θ; its form must be chosen to fully specify the model. We alternately specify f(.) as a uniform, a normal, and a three-parameter beta probability density function. The coefficients $\beta_1, ..., \beta_k$ are scaling coefficients for the k events.

Figure 2 illustrates the DERM where f(.) is chosen to be the normal p.d.f. There are three events, on dates t_1, t_2, and t_3. Specifying a mean (μ) and variance (τ^2) determines the spread of the response patterns and their locations relative to the announcement days, as shown on the top line of the figure. Given μ and τ, each event has its own scaling parameter, β_i, which allows the magnitude and sign of the event's effect to be independent of other events, as shown on the second line of the figure.

We assume that the regression errors ε_t are i.i.d. $n(0, \sigma^2)$, which implies that nonlinear least squares is maximum likelihood. This implies that, conditional on θ, the model is a linear regression with the $f(d_t^i;\theta)$ playing the roles of regressors. The estimation problem can be solved, then, by minimizing the sum of squared errors over choices of the intrinsically nonlinear parameters θ, with the other model parameters estimated by Ordinary Least Squares for each θ. This reduction in the parameter space results in a large increase in computational efficiency and makes estimation of the DERM feasible for large data sets, such as our own, with large numbers of events. In our application, the large number of housing start events results in 184 model parameters. Under the normal and uniform versions of the DERM, only six parameters (two for each type of event) are intrinsically nonlinear. This concentration of the likelihood function also suggests a computationally efficient procedure to estimate the covariance matrix of the parameter estimates, which is described in an appendix.

The model as described in equation (1) posits a single type of event, of which there are k individual instances. Because we analyze three types of events: trade policy events, housing starts announcements, and ESA litigation rulings, we add two more terms to the DERM illustrated by equation (1) to capture separately each of the three types. In the results reported below, each of the three event types has its own response function f(.) with its own parameters, to describe the expected response of futures prices to a specific event type.

The Uniform Event Response Model (UERM)

Consider first the uniform variant of the DERM, the Uniform Event Response Model (UERM). It implies that the price response for event i is uniformly distributed from a beginning number of days relative to the event, θ_B, to an ending number of days relative to the event, θ_E. The magnitude of the ith event's response is scaled by β_i. For event *i*, $f(d_t^i;\theta) = d_t^i(\theta_B, \theta_E)$, where d_t^i is a dummy variable that equals one for days in the interval $(\text{eventday}_i - \theta_B,\ \text{eventday}_i + \theta_E)$ and zero otherwise. This variant is closely related to a traditional event study structure in which a dichotomous variable is specified to represent a given event, as illustrated by the Individual Event variables in table 7. In either case, the response function is flat, and the magnitude of the impact of an event for each day in the event window is β_i. The usual event study practice is to impose window widths and window location as implicit maintained hypotheses. The UERM, on the other hand, entails choosing the window width and location that maximizes the likelihood of the model parameter estimates, including θ_B and θ_E, subject to the data and UERM functional form.

The results of the UERM, estimated by nonlinear least squares in GAUSS are presented in table 5. With our assumption of normality for the stochastic errors, ε_t, the estimates also are maximum likelihood. In our context, the X_t variables in equation (1) include four daily non-event market indexes included to pick up market and macroeconomic effects as well as linear and quadratic time trend variables. The significance and signs of the estimated coefficients on the daily market variables provide support for our general approach. The coefficient on the rate of return on the CBOT futures market index is positive and significant, consistent with a standard market model and a risky asset. The coefficient on the rate of return on the 30-year treasury securities (T-bill rate), which we include as a proxy for mortgage rates, is negative and significant. Our interpretation of this result is that an increase in mortgage rates reduces lumber demand (through the housing markets) leading to lower lumber prices. The percentage change in the Japan-U.S. exchange rate is estimated to have little impact on lumber futures price changes, but the percentage change in the Canada-U.S. exchange rates has a negative and marginally significant effect. We interpret the latter result as follows: a decrease in the exchange rate ($ Canadian per $ U.S.) makes Canadian dollars more expensive, causing U.S. demand for Canadian lumber to decrease and U.S. demand for U.S. lumber to increase, thereby causing U.S. lumber prices to rise. Given that the United States is a net importer of softwood lumber from Canada, these demand effects outweigh the offsetting supply shifts caused by the exchange rate decline. While the daily-observed variables are not of primary interest here, it should be noted that our interpretations of their effects rest on changes in these variables being unexpected to market participants (that is, a maintained hypothesis is that the daily market indexes follow random walks).

The linear and quadratic time trend variables are jointly statistically significant at near the .10 level. Our interpretation of them is that there are omitted factors in our specification that vary systematically over time, initially causing the rate of change in lumber prices to decline and then (about three-quarters of the way through our sample) to increase. The net effect from beginning to end of sample is close to zero—the impact of the trend variables to its minimum is to reduce the daily rate of price change by 3.29 percentage points; the impact from the minimum to the end of the sample is an increase of 3.11 percentage points. Inclusion of the trend variables affected little the estimates of other parameters.

Of primary interest in table 5 are the estimated impacts of individual events on the lumber rate of return. The estimated parameters of the event response distributions are listed first in the column labeled "UERM estimate." Given discrete data, the parameter space for θ_B and θ_E is discrete. The ML estimates are $\theta_B = \theta_E = 0$: an event window that is open only on the announcement day. For comparison with the Normal Event Response Model (NERM), table 5 displays the mean of the uniform distribution, μ, and standard deviation, τ, of the uniform distribution: $\mu = (\theta_B + \theta_E)/2$ and $\tau = (\theta_E - \theta_B + 1)/12^{1/2}$, where τ is calculated as though the distribution were uniform over the interval $[\theta_B - .5, \theta_E + .5]$. The estimated value of $\tau_{\text{Housing Starts}}$ is .29, implying a short interval over which the market digests the news of the monthly announcements. The UERM results are consistent with the F-tests in table 4. The information content of a housing starts announcement is reflected in the lumber futures price effectively on the day of the announcement.

The estimated response to trade events is later relative to housing starts and slightly more diffuse. The ML estimates of the event window endpoints are $\theta_B = 0$ and $\theta_E = 2$. This suggests that there is no anticipation of the events and that the full implications of a trade event are reflected in prices over the day of the event and the two days following.

The third event response distribution, that for Endangered Species Act announcements, is quite different from the first two. The ML estimate of θ_B is -5 suggesting that the market anticipates ESA rulings. Further, the full impact is not felt until $\theta_E = 7$ days after the ruling. There no doubt is a flow of information to the market before and after the judicial announcement with implications for lumber supply and demand.[13] The individual event coefficients, the β_i, also are listed in table 5 for housing starts and ESA announcements. The individual event coefficients for the 147 housing starts events are not shown.

The estimates of θ_B and θ_E for the three event types reported in table 5 were found by searching over points in the six-dimensional parameter space for the θ_B's and θ_E's that yield the minimum sum of squared errors. For each point evaluated, the remaining 178 coefficients were calculated by the normal equations for the linear model. The estimates of the 178 parameters other than the θ_B's and θ_E's are conditional on the ML choice of the six nonlinear parameters. This has two undesirable effects. The first is that the standard errors of the 178 linear parameters, calculated conditionally, are understated. The second is that there is no natural way

[13] An important issue arises concerning the interpretation of our results. The interpretation that we adopt for presentation purposes is that we are estimating the speed of the market's response to an event. The fact that the market takes a considerable amount of time to respond to some events (e.g., ESA court rulings) does not suggest that the lumber futures market is inefficient. To some extent, our estimates likely reflect patterns of additional related news releases or other sources of information. Insofar as such additional information is playing an important role, an appropriate interpretation of our estimation strategy is that it provides measures of the shape of "events" as well as of the speed of market response to events.

to derive estimates of the sampling variability of the six nonlinear parameters.[14] An alternative way to estimate directly the event distribution parameters and to estimate the sampling variability of the estimates is to employ a variant of the DERM that has a continuous parameter space.[15]

The Normal Event Response Model (NERM)

One variant of the DERM with a continuous parameter space is the Normal Event Response Model (the NERM). Substituting the normal p.d.f. for $f(d_i^t, \theta)$ in (1) instead of the uniform p.d.f. allows for an intuitively plausible buildup and symmetric tailing off of event responses. Table 5 reports the NERM estimates in parallel fashion to the UERM estimates. The parameter estimates of the NERM can be seen to be quite similar to those of the UERM, although the shape of the event response distribution is different. Notice that the fit of the two models is quite similar as well, with the R^2 from the UERM (0.0987) being higher at the fourth decimal place than the R^2 of the NERM (0.0984.) In the NERM the housing start effects are

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[14] Due to the discrete parameter space of the UERM (due, in turn, to discrete time data) there is no simple way to employ Wald-type confidence intervals for the nonlinear parameters. The sampling variability of the nonlinear parameters in the UERM could, however, be estimated by inverting the likelihood ratio statistic. One could evaluate the likelihood function (a monotonic transformation of the sum-of-squared-errors) over a wide range in the six-dimensional parameter space, evaluating the likelihood at each point. One could then concentrate the likelihood, leaving only the parameters of interest, and find the extreme values of the parameters by the metric of the likelihood ratio statistic. The method is feasible, but presents numerical obstacles.

[15] As indicated above, the number of limit moves in the lumber futures price data is relatively large—about ten percent of the observations involve limit moves. Insofar as limit moves occur frequently on (or around) event days, our estimates of the impacts of events may be affected. To examine the relationship between limit moves and event days, we created a dichotomous variable called "Limit Move" to which we assigned a value of one for each day that there is a limit move in lumber futures prices and a value of zero otherwise. Dichotomous variables, defined as suggested by the UERM results, were used to indicate event days.

The results of Fisher's Exact Tests for independence suggest that housing starts release dates are positively and significantly correlated with limit moves (p-value=.0055), but that neither ESA nor TRADE events are significantly correlated with limit moves (p-values of 0.8919 and 0.7857). We take the correlation between housing start events and limit moves as evidence that in the analysis reported below, we underestimate what would be the response of unrestricted prices to those events. Such systematic underestimation is not an issue for either ESA or TRADE events. Other than recognizing this relationships for housing starts, our analysis does not treat limit moves differently from non-limit moves.

highly statistically significant ($p=1.6 \times 10^{-4}$), the trade effects are modestly significant (p=.118) and the ESA effects are slightly less significant (p=.148).[16]

Our maximum likelihood estimation of the DERM is based on an assumption of normally distributed disturbances. (This is a separate issue from whether or not the normal p.d.f. is a good fit to the event response shape.) Test results support the normal specification. We calculate a Wald statistic reported in Greene (2000, p. 397-8) which jointly tests for skewness and kurtosis measures that depart from normality. The p-value from the Wald test using the residuals from the NERM in table 5 is .85, implying that the null hypothesis of normality cannot be rejected. This supports our approach to estimation and inference.

Consider next economic significance. The sample mean of the absolute value of the dependent variable, the mean size of the log-change in daily prices, is .01199: on an average day, the lumber futures price changes by about 1.2%. This figure can reasonably be used as a benchmark for assessing the economic significance of the individual event coefficients. The only trade event significant at the .05 level is #11, with an estimated β_i coefficient of -.082.

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[16] An aliasing issue arises for the NERM for small values of τ. For such values, the NERM reduces to a restricted version of the UERM due to the use of a continuous distribution to model discretely sampled data. For small values of τ, the value of the normal p.d.f. near the mean is much larger than the value of the p.d.f. slightly farther away–the tails decline exponentially. This implies that for small values of τ, the NERM effectively puts a spike of event effect on the day nearest μ and no effect on any other day, a response pattern essentially the same as for the UERM. As μ changes, the day on which the spike occurs changes and the likelihood function's value is identical to the likelihood value for a UERM with a one-day window for the day nearest μ. As τ increases, the aliasing problem disappears because the normal p.d.f. puts non-trivial and non-uniform event effect on several days, a pattern that the UERM cannot represent.

The aliasing affects our estimation strategy in the following way. As in table 5, we consider separately the UERM and the NERM. For the UERM, we consider all possible values of θ_B and θ_E and report the maximum likelihood estimates. For the NERM, we maximize the likelihood function subject to the restriction that we avoid regions of (μ,τ) space that mimic the UERM. For the Housing Starts and Trade Events models, this restriction has no force: the MLEs that we report are global maxima. But for ESA events, the maximum to the NERM likelihood function occurs in a low-τ region where the NERM aliases for a restricted UERM. We disregard this region because of the aliasing issue and focus on the NERM estimates reported in Table 5. The reported NERM ESA point estimate, in addition to being consistent with the estimation strategy just described, has two desirable properties. First, it is a point in the parameter space that is a relative likelihood maximum over a broad area relative to the aliasing region. Second, as can be seen by comparing the NERM μ and τ with the UERM mean and standard deviation for ESA events, the NERM point we report is consistent with the global maximum (θ_B,θ_E) for the UERM.

Trade Event #11 is therefore estimated to have had a total impact of -8.2% on the futures price, an impact that was distributed over the period before and after the event day. Using the ML estimates of μ_{Trade} and τ_{Trade}, the distributed response peaks on the day after an event, and 47% of the effect (of every trade event) is felt on that day. According to the estimate of τ_{Trade}, the rest of the effect is felt within the two or three days prior to and after the peak day. Comparing -8.2% to the sample average change of 1.2%, the estimated size of the Trade Event #11 effect is quite large. Several other point estimates of Trade event effects are large in this metric as well, but their standard errors are large enough that they cannot reliably be differentiated from no effect.

Three ESA events are statistically significant at conventional levels and their estimated impacts are huge. The largest is ESA Event #10 with a 23.3% reduction in the price of lumber futures. The standard errors of the ESA events are larger than those of the Trade events, implying that the estimated effects need to be quite large to be reliably distinguished from zero.

The 147 individual Housing Start Event coefficients, which are not reported in table 5, range from an estimated positive effect of +6.6% to an estimated negative effect of -6.0%. As implied by the estimate of τ_{Starts}, the effect is concentrated in time with 72% of the effect of each event being felt on the day of the announcement. Further, many of the estimated effects are large. More than half are larger than the average absolute daily percentage change of 1.2%. Table 5 reports that 35 of the 147 effects are statistically significant at the .10 level.

Table 5 also reports Wald-type standard errors for the estimated μ's and τ's from the NERM, calculated from the inverse of the Hessian of the likelihood function evaluated at the ML estimates. They imply that the parameters of the event response distributions are estimated quite precisely. The distributions for different types of events also are statistically distinguishable from one another. Table 6 reports results from joint hypothesis tests that address this issue. The first test in table 6 addresses the hypothesis that each of the three event-type distributions is centered on the day of the announcement: $\mu_{starts}=\mu_{esa}=\mu_{Trade}=0$. The joint hypothesis cannot be rejected at conventional levels. But the data strongly resist the restriction that the τ's from the three distributions are identical ($p=2.0 \times 10^{-4}$). That is, there is strong evidence of differences in speeds of adjustment to the three types of events. Testing separately the equality of pairs of τ's reveals that the speed of the market's response to trade events cannot easily be differentiated from the speed of response to housing start events (p=0.1908), but the quite slow response to ESA events can easily be distinguished from the response to trade events (p=.00036) and also from the response to housing start events (p=.000079). Figure 3 provides a visual comparison, for the NERM, of the shapes and locations of market responses to the three event types.

The DERM posits, first, a systematic response shape for all the events of a certain type and, second, a response size that is unique to a specific event. With regard to the size of effect, one can examine table 5 and identify which events are large relative to their sampling variability. Among the Trade Events, Event #7 has a positive effect with a t-ratio of 1.63 and Event #11 has a larger negative estimated effect with a t-ratio of –2.52. As indicated in table 2, Event #7 was the 1992 imposition of a quite large (14.48%) countervailing duty on Canadian lumber imports; we infer that it was unanticipated, or at least under predicted, by the market. Event #11 marked the adoption of an agreement between Canadian and U.S. trade negotiators, four years after the imposition of the first countervailing duty, that arguably forestalled a U.S. trade action. Relative to market expectations, the announcement implied lower equilibrium U.S. lumber prices.

Among the ESA events, Event #3 has a t-ratio of 1.90 and Event #10 has a t-ratio of –2.30. The former marked a broad 1991 proscription of timber sales on 17 national forests, an event that caused lumber prices to rise. The latter marked a 1995 decision to release previously prohibited timber sales in Oregon and Washington, which apparently was unanticipated or under anticipated. The estimated effect of ESA Event #11 is hard to interpret. It is statistically significant, large, and positive, implying the effect of ESA-related information that implies a restricted timber supply relative to market expectations. Event #11, however, is an Appeals Court upholding of the ruling announced in Event #10, which released land for logging and which has an estimated negative effect. It is difficult to understand why the upholding of a ruling, whose effect was negative, could itself have a positive effect.

It is interesting to note that in each of the four significant ESA and Trade Events, except for ESA #11 discussed above, the market's response reinforced the natural change in prices. For example, if the event was one that marked a rightward shift in supply then the measured response was a reduction in price, implying that the price-lowering effect of the announcement was not fully incorporated into the lumber futures price prior to the announcement. This reinforcing effect is consistent with an observation made by Milton Friedman in the context of a 1960s devaluation of the Mexican peso: conditional on the occurrence of a price-lowering event, the observed effect in financial markets will appear to be a surprise. The rational market's expectation prior to the event must be a weighted average of the price expectations conditional on the two possible outcomes (event and no event) weighted by the probabilities of the two outcomes.[17]

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[17] See Lewis (1988) and Krasker (1980). Lewis (1988) refers to an earlier discussion of the peso problem in Rogoff's 1977 MIT dissertation as the first. It is characterized as a low-probability event than does not occur in a particular sample, but which influences asset prices. Apparently, in the mid-70's the forward and futures market for the Mexican peso consistently under predicted the dollars-per-peso exchange rate. The reason offered is that a devaluation was expected. She cites, among others, Krasker (1980). Krasker analyzes problems with statistical analyses of the efficiency of forward and futures exchange markets when there are low-probability dramatic events overhanging the market. He suggests alternate tests of market efficiency with more accurate size.

Turn next to the DERM results in table 5 that concern market speeds of adjustment. Housing starts announcements are routinely absorbed and processed by markets and, correspondingly, the standard deviation measure in the DERM for housing starts (τ) is small. We find that the market seems similarly well adapted to processing the information from trade events in that the spread of the response distribution is somewhat larger than, but similar to, that for housing starts. By contrast, the ESA events take a much longer time to be fully reflected in lumber futures prices, as indicated by a much larger measure of temporal influence spread.

**The Beta Event Response Model (BERM)
and a Comparison of Event Response Shapes**

The variants of the DERM thus far considered are symmetric. The beta p.d.f. provides a family of event response shapes that allow for asymmetric responses. The beta p.d.f. is positive over the (0,1) interval and has two parameters. It can be skewed left or right and can be unimodal or bimodal. Because it has finite support, the beta does not nest the normal distribution. It can, however, approximate normal distributions well. We augment the two-parameter beta by introducing a third parameter, C, that gives it positive support over a range (-C/2, C/2). While its domain of support is symmetric, the two shape parameters allow for any degree of skewness.

The BERM estimates are reported in table 5, where we calculate the means (μ) and standard deviations (τ) of the beta distributions to make them comparable with the NERM estimates. The NERM and BERM shapes can be seen to be quite similar in their first two moments. Further, the maximum likelihood estimates of the BERM reveal no marked asymmetry for any of the event classes. Figures 4a - 4c compare the NERM, UERM, and BERM shapes for each event class.

The visual similarity of the estimated BERM and NERM carries over to the statistical comparison of the models' parameters in table 5, with the exception of the standard errors for τ for the BERM versions of Trade and Housing Starts events. Those standard errors are quite large and substantially larger than the standard errors for the NERM estimates of τ, despite the fact that the estimates of τ themselves are similar between the NERM and the BERM. This is due to the fact that the Trade and Housing Starts events are positive over a small interval, meaning that the model is being fit to a pattern over only two or three days. The BERM, with three parameters, can fit a three-day pattern quite well but the three parameters are almost unidentified. The situation is different for ESA events, which take place over a wider window. The BERM is not overparameterized for the ESA events, as reflected in the similarity of the calculated μ and τ from the BERM with those from the NERM, and also as reflected in the similarities of the standard errors for those values between the two models.

The fits of the three models (NERM, UERM, and BERM) are quite similar, the standard errors of the regression differing across the three by no more than two hundredths of one percent. But statistical significance and goodness of fit are different issues. On the issue of statistical significance, Vuong (1989) develops a method for modifying the likelihood ratio statistic to compare non-nested models. In our context, the normal, beta, and uniform versions of the DERM are non-nested: none can be derived from another by parametric restriction. The null hypothesis in Vuong's test is that two estimated models are equally close to the unknown true model in the Kulback-Leibler metric. The alternative is that one dominates the other.

The Vuong statistics confirm the similarity of the models in terms of statistical significance. In a comparison of the three-parameter beta (BERM) with the normal (NERM) the asymptotic Z-statistic takes a value of .44 suggesting only slight evidence that the BERM is preferred to the NERM. This is unsurprising given the visual similarity between the two estimated models shown in figures 4a-4c. The other two pairwise comparisons provide further evidence that the statistical differences among the models are small. The Z-statistic in a comparison of the BERM with the UERM is .05; the Z-statistic in a comparison of the UERM with the NERM is .09.

C. The DERM Compared to Previous Event Study Approaches

Event study methodology was developed by financial economists to assess the performance of securities markets. The seminal paper in this literature is Fama, Fisher, Jensen, and Roll (FFJR, 1969), who studied the effects of stock splits on firm returns. Reviews of the voluminous literature that followed can be found in Binder (1998) and MacKinlay (2000). A later application of the methodology was to studies of regulation, which maintained the informational efficiency of financial markets and used event analysis to estimate the impacts of regulations on firms or industries. An early proponent of the application was Schwert (1981). Applications include Maloney and McCormick (1982), Binder (1988), and Ellison and Mullin (1995).

A typical and important feature of financial economics event studies is the availability of time series for a large number of firms. Researchers estimate for each security a market model relating firm returns to overall market returns and calculate abnormal returns as the residuals from the fitted models. If an event affects a number of firms, then the estimated effect of the event is the average across firms of the abnormal returns for that day. Following FFJR, the average across firms of abnormal returns on the day of a stock split announcement, which are different calendar days for different firms, would be the event study measure of a stock-split announcement effect. The one-day-after effect would be the average abnormal return across firms on the day following a stock-split announcement.[18] Because there are a large number of firms in the typical study, one can nonparametrically estimate the time pattern of market response to an event by taking simple averages across firms for the relevant days relative to events. FFJR use monthly time series on 622 securities to estimate the average abnormal returns for a 60-month period surrounding stock split announcement dates. Due to the large number of firms involved they need not impose any structure to the time pattern of estimated response to the announcements.

Those studying the effects of regulation typically do not have a large number of securities that are affected by the regulatory change in question. The typical regulatory change affects a small number of firms and the problem of finding "pure plays" among diversified firms usually reduces the number of securities available for analysis to just a handful. The usual event study methodology can be and is applied in these situations, but as pointed out by Binder (1998) the power of the tests to reject a null hypothesis of no regulatory effect is low.

Our analysis of lumber futures prices is similar to previous studies of the impacts of regulations, but our measurement goals are different. To concentrate specifically on the effects on transactions prices for lumber, we study lumber futures prices instead of the returns to lumber-related firms.[19] Our data set is more limited than most used in financial economics because we only have a single time series. On the other hand, our data set contains over 3,000 observations and it is quite rich in that it contains multiple realizations of similar events (in the

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[18] See Binder (1998) for a detailed discussion of issues involved in applying the methodology.

[19] Other event studies of futures market prices include Sumner and Mueller (1989), Fortenbery and Sumner (1993), Miller (1979), and Robenstein and Thurman (1996).

case of housing starts announcements, it contains 147 monthly realizations) and it contains three distinct types of events.

For comparison with previous work, it is instructive to compare the DERM procedure with two alternative linear regression-based approaches to estimating the impacts of events from single time series. They involve two different definitions of event dummies: Individual Event dummies and Event Period Relative dummies. The differences in the definitions are illustrated in table 7, where it is assumed that two events occur, the first in period #3, the second in period #7. For the Individual Event dummy variables, a variable is defined for each event, with a fixed response window. In the table, the Event #1 and Event #2 variables are assigned a value of one in the period of the event and in surrounding periods. The event window in this example arbitrarily spans the interval from one period before an event to one period after. The determination of the actual window width is an important practical issue in estimating event impacts. The Individual Event formulation imposes the constraint that all periods in the window for any given event have equal impact. Studies that have used this approach include Maloney and McCormick (1982) on the effects of cotton dust regulation on a portfolio of textile companies; Binder (1988) on the impacts of two Supreme Court decisions on railroad stocks; Bonser-Neal, Roley, and Sellon (1998) on the effects of monetary shocks on exchange rates; and Brown and Hartzell (forthcoming) on the impacts of events on the value of shares in the Boston Celtics basketball team.

The second formulation uses Event Period Relative dummies. In table 7, a Period (-1) variable is assigned a value of one during the periods before each event. Similarly, Period (0) and Period (+1) variables represent the periods on and after the events. Additional variables could be defined for other periods before or after the event. This specification imposes the constraint that the impact of all events is the same, for example, during the period prior to the events. The Event Period Relative dummy variables pick up effects in event time just as the alignment of multiple return series in event time does in FFJR. By including Event Period Relative dummies in a linear regression, with rates of return as the dependent variable, one would estimate the time path of event effect similarly to the way that FFJR and subsequent studies use residuals averaged across firms.

The Individual Event and Event Period Relative dummy variables impose different constraints on the structure of the time path of market reactions. Neither model is nested within the other. The uniform variant of the DERM, the UERM, is similar to the Individual Event model but with an important difference. In the UERM, the width and location of the event windows are estimated rather than imposed. The non-uniform variants of the DERM relax the assumption of constant event effect within the event window. The DERM differs from the Event Day Relative model in two ways. It imposes smoothness on the time path of price response and it allows events to vary in the size and sign of their effects.[20]

V. Summary and Conclusion

The evidence presented in this paper suggests that housing start releases contain information that significantly affects lumber futures prices. Moreover, the impacts of these releases are absorbed quickly by the market, approximately within a day of the release. Evidence concerning the impacts of ESA events is more mixed. The Distributional Event Response Model applied to rates of return imply weak joint significance for the ESA events, but statistically discernible effects for three court decision announcements. The influence that we measure is spread out over a fairly wide interval surrounding the ESA-related announcements—one that encompasses about an 8-day interval.

[20] Results using Individual Event and Event Period Relative variables to examine the impacts of the three different types of events of interest in this study are presented and discussed in Rucker, Thurman and Yoder (2000).

The statistical significance of the trade events, summarized across models, is stronger than that for the ESA events. Two individual events appear to have had an impact on lumber futures prices. Further, and unlike the slow absorption of ESA-related news, the trade events appear to be digested quickly. Most of the impact is impounded in futures prices within a three-day period. While the estimated effects of a number of the ESA and trade events are large, the standard errors are large suggesting that the data do not have much power to reject hypotheses of no effect. This state of affairs is similar to that reported by Binder, who concludes that the event study methodology often has little power to statistically discern the effects of regulatory changes.

A methodological contribution of the paper is our comparison of the impacts of different types of events in the Distributional Event Response Model. The model allows a market's typical response to different event types to be compared, while allowing individual events to have their own signs and size of impacts. The DERM is a more flexible alternative to standard dummy variable models for measuring the impacts of events that affect markets.

Beyond measuring the size of the market's response to specific events, the DERM parameterizes the speed of market response, which fact we use to compare market response across event types. We find that the speeds of market response vary predictably across event types and can clearly be distinguished among the three classes of events we study. The point estimates of the response speeds are as we expect: housing starts information is absorbed more quickly than trade events information, and trade events information more quickly than ESA event information. The market's speed of absorption of ESA events is much slower than and clearly different statistically from the speed of absorption of the other two.

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Table 1. ESA Events: Dates and Descriptions.

Event #	Event Description
1	**March 24, 1989**. U.S. District Court Judge William Dwyer prohibits timber sales from Spotted Owl Habitat on U.S. Forest Service land until a final judicial hearing. Dwyer argues that the 1988 Forest Service management plan likely violated the National Forest Management Act and the National Environmental Policy Act. (*Seattle Audubon Society v. Robertson,* Nos. C89-160WD and C89-99(T)WD).
2	**May 11, 1990**. Judge Dwyer prohibits a timber sale in an Oregon National Forest because it violates the Department of Interior and Related Agencies Act's (1990) requirement to minimize fragmentation of ecologically significant old growth forest (*Seattle Audubon Society v. Robertson*; Nos. C89-160WD and C89-99(T)WD).
3	**May 23, 1991**. Judge Dwyer prohibits timber sales on 17 National Forests until the Forest Service complies with requirements of the National Forest Management Act, finding that "the Forest Service and the Fish and Wildlife Service deliberately and systematically refused to comply with the laws protecting wildlife" (*Seattle Audubon Society v. Evans*, No. C89-160WD).
4	**December 23, 1991**. The 9th Circuit Court of Appeals upholds Judge Dwyer's May 23, 1991 ruling (*Seattle Audubon Society v. Evans*, No. 91-35528).
5	**February 19, 1992**. U.S. District Court Judge Helen Frye prohibits 26 timber sale awards and 23 timber sale offers by the Bureau of Land Management in order to allow further review by the court. The plaintiff (Portland Audubon Society) claims that the BLM is in violation of the National Environmental Policy Act (NEPA) by not preparing an environmental impact statement regarding new Spotted Owl related information . The court rules that "the destruction of owl habitat without compliance of the law is significant and irreparable injury," and that the plaintiff is likely to prevail in its contentions that the BLM awards and offers are illegal under NEPA (*Portland Audubon Society v. Lujan*, No. 87-1160-FR).
6	**June 8, 1992**. Judge Frye halts Bureau of Land Management timber sales (*Portland Audubon Society v. Lujan*, No. 87-1160-FR).
7	**June 6, 1994**. Judge Dwyer lifts injunction on timber sales in Spotted Owl habitat (*Seattle Audubon Society v. Lyons*, No. C92-479WD).
8	**August 24, 1995**. U.S. District Court Judge Carl Muecke issues an injunction against all timber harvests on New Mexico and Arizona national forests until the federal agencies involved study the effects of harvest on Mexican Spotted Owl populations (*Silver v. Babbitt*, Nos. CIV 94-337 PHX CAM and CIV 94-1610 PHX CAM).

Table 1. ESA Events: Dates and Descriptions (continued).

Event #	Event Description
9	**September 6, 1995**. District Judge Michael Hogan allows a fire salvage sale, citing the timber salvage rider (*Northwest Forest Resources Council v. Glickman*, No. 95-6244-HO).
10	**October 17, 1995**. Judge Hogan releases previously prohibited timber sales on National Forest land in Oregon and Washington and Bureau of Land Management land in Western Oregon, based on the language of the timber salvage rider (*Northwest Forest Resources Council v. Glickman*, Nos. 95-6244-HO and 95-6267-HO).
11	**October 25, 1995**. Judge Hogan's October 17 ruling is upheld by the 9th Circuit Court of Appeals (*Northwest Forest Resources Council v. Glickman*, No. 95-36042).
12	**May 27, 1996**. The Supreme Court upholds U.S. Department of Interior Secretary Bruce Babbitt's designation of 6.8 million acres as Spotted Owl Habitat. The case was brought because Babbitt failed to file environmental impact statements required by the National Environmental Policy Act, but the Supreme Court upheld the 9th Circuit court's finding of an exemption for critical-habitat decisions. (*Douglas County, Oregon v. Babbitt*, No. 95-371).

Table 2. Trade Events and Description.

Event #	Event Description
1	**October 21, 1986**. The Department of Commerce International Trade Administration (ITA) announces that Canadian producers of rough, dressed or worked softwood lumber, siding, and flooring were effectively receiving fifteen percent *ad valorem* subsidy under U.S. countervailing duty laws. The ITA therefore requires the U.S. Customs Service to immediately suspend the sale of all relevant merchandise and require a cash deposit or bond (a countervailing duty) equal to the 15% subsidy (U.S. Dept. of Commerce 1986).
2	**December 30, 1986**. United States and Canada agree upon a Memorandum of Understanding (MOU) that requires Canada to impose a 15% export tax (revenues to be retained by Canada) instead of the 15% countervailing duty imposed by the ITA in October of 1986 (Joshi 1997).
3	**June 15, 1990**. US/Japan Super 301 agreement is announced. This agreement provides more market access for U.S. products into the Japanese market.
4	**August 20, 1990**. Forest Resource Conservation and Shortage Relief Act (104 Stat 714) is enacted, reinforcing the existing ban on logs from federal timber lands, and extending the ban to the export of unprocessed logs from state lands.
5	**September 3, 1991**. Canada terminates the MOU claiming the agreement is no longer needed because most of the Canadian provincial policies in question had been changed. (Joshi 1997).
6	**October 4, 1991**. In response to Canada's termination of the MOU, the U.S. Department of Commerce International Trade Administration imposes an interim bonding measure on all Canadian exports entering the United States (Joshi 1997).
7	**March 13, 1992**. The ITA imposes an interim 14.48 percent Countervailing duty on Canadian lumber.
8	**May 15, 1992**. The ITA imposes a final countervailing duty of 6.51 percent (Joshi 1997).
9	**July 21, 1992**. The International Trade Commission (ITC) affirms injury to the United States, and supports the ITA countervailing duty ruling.[21]

[21] The International trade Commission (ITC) is an independent quasi-judicial federal agency. It makes a preliminary determination in title 7 (Tariff Act of 1930) investigations such as this as to whether a U.S. industry is threatened or injured by alleged dumping or subsidies on the part of a U.S. trade partner. If their finding is affirmative, the International Trade Administration (ITA) of the Department of Commerce determines whether and to what degree a U.S. trade partner is practicing dumping or providing subsidies to one of its industries. If the ITA finds in the affirmative, then the ITC makes a final determination as to whether this dumping or subsidy imposes injury on a U.S. industry. If and only if both the ITC and the ITA rule in the affirmative, the ITA officially issues policy orders (with regard to countervailing duties, for example).

Table 2. Trade Events and Description (continued).

Event #	Event Description
10	**December 17, 1993**. A binational panel (3 Canadians, 2 U.S. representatives) nullifies ITA findings of injury. The vote was split along country lines.
11	**February 16, 1996**. An “Agreement in Principle” between the United States and Canada is announced in which the U.S. agrees not to pursue anti-dumping or countervailing duty actions. In exchange, Canada imposes fees on British Columbia lumber exports to the United States exceeding nine billion board feet per year. The fees are to be US$50 per thousand board feet on the first 250 million board feet (above 9 billion) and US$100 per thousand board feet for higher quantities. These fees are to be collected and remitted to the involved Canadian provinces. The three other affected provinces (Quebec, Ontario, and Albert) agree to pursue other measures to avert U.S. trade action, such as increasing stumpage fees and timber license fees (Office of the United States Trade Representative 1996, Canadian Department of Foreign Affairs and International Trade 1996, Press releases).
12	**April 2, 1996**. A Final Canada-U.S. Softwood Lumber Agreement is announced that results in export fees similar to the British Columbia export agreement of February 16, 1996, for all four affected provinces. Combined volumes of lumber originating from the four provinces in excess of 14.7 billion board feet are to be assessed US$50 per thousand board feet for the first 650 million board feet, and US$100 per thousand board feet for higher quantities. Based on 1995 exports from these four provinces to the U.S. of 16.2 billion board feet, this fee would have been applied to about 9 percent of the total trading volume (Office of the United States Trade Representative 1996, Canadian Department of Foreign Affairs and International Trade 1996, press releases.).

Table 3.
F-tests for differences in the variance of lumber futures price residuals between event days and non-event days for various symmetric window widths. F= $\hat{\sigma}^2_{event} / \hat{\sigma}^2_{nonevent}$

	Starts				ESA				Trade			
	f	df(n)	df(d)	p	f	df(n)	df(d)	p	f	df(n)	df(d)	p
Day of Event	1.40	146	2973	0.001	1.31	11	3108	0.210	1.48	11	3108	0.133
3-day window	1.11	440	2679	0.072	1.34	35	3084	0.086	0.99	35	3084	0.485
5-day window	1.09	734	2385	0.075	1.46	59	3060	0.013	0.87	59	3060	0.755
7-day window	1.10	1028	2091	0.035	1.25	82	3037	0.065	0.77	83	3036	0.943
9-day window	1.08	1322	1797	0.056	1.10	103	3016	0.235	0.70	107	3012	0.991
11-day window	1.07	1616	1503	0.090	1.08	123	2996	0.264	0.72	131	2988	0.992

Table 4.
F-tests in table 3 are based on the residuals from the following regression. Dependent variable is the rate of return on CME lumber futures closing price.

Variable (N=3121, R^2=0.0211)	Estimate	Standard Error	t-Statistic	Pr>|t|
Intercept	0.013	0.025	0.52	0.602
time	-0.188	0.425	-0.44	0.659
time^2	0.665	1.808	0.37	0.713
CBOT futures index (rate of return)	0.350	0.050	6.97	<.0001
Japan-US exchange rate (Rate of return)	0. 016	0.045	0.36	0.721
Canada-US ex. rate (Rate of Return)	-0.179	0.109	-1.65	0.099
T-bill rate	-0.173	0.039	-4.42	<.0001

Table 5. Estimates of the Distributional Event Response Model (DERM): Uniform (UERM) Normal (NERM) and Beta (BERM) Response Variants.

Dependent variable: $\ln(P_t/P_{t-1})$
n = 3121

Variable	**UERM estimate**	**NERM estimate**	**BERM estimate**
Housing Starts			
μ	$0.00^{\dagger}$	0.142	$0.119^{\dagger\dagger}$
	$(\theta_B=0,\theta_E=0)$	(0.140)	(0.238)
τ	$0.29^{\dagger}$	0.538	$0.502^{\dagger\dagger}$
		(0.076)	(3.57)
Trade Events			
μ	$1.00^{\dagger}$	0.989	$1.098^{\dagger\dagger}$
	$(\theta_B=0, \theta_E=2)$	(0.533)	(0.543)
τ	$0.87^{\dagger}$	0.840	$0.801^{\dagger\dagger}$
		(0.217)	(3.76)
ESA			
μ	$1.00^{\dagger}$	0.121	$-0.137^{\dagger\dagger}$
	$(\theta_B=-5, \theta_E=7)$	(1.229)	(1.232)
τ	$3.75^{\dagger}$	4.418	$4.607^{\dagger\dagger}$
		(0.969)	(1.340)
Intercept	0.0252	0.0305	0.0300
	(0.0248)	(0.0255)	(0.0256)
Time	-0.3927	-0.4817	-0.4727
	(0.4257)	(0.4381)	(0.4403)
Time^2	1.5161	1.8842	1.8467
	(1.8907)	(1.8620)	(1.8711)
CBOT futures index	0.3462	0.3433	0.3421
(Rate of return)	(0.0495)	(0.0494)	(0.0494)
Japan-US ex. rate	0.0075	0.0088	0.0078
(Rate of return)	(0.0448)	(0.0449)	(0.0449)
Canada-US ex. rate	-0.1637	-0.1794	-0.1824
(Rate of Return)	(0.1083)	(0.1090)	(0.1089)
T-bill rate	-0.1993	-0.2036	-0.2028
	(0.0394)	(0.0397)	(.0398)

Table 5 (continued). Estimates of the Distributional Event Response Model (DERM): Uniform (UERM) Normal (NERM) and Beta (BERM) Response Variants.

Variable	UERM estimate	NERM estimate	BERM estimate
Trade Event 1	-0.014	-0.024	-0.015
Trade Event 2	-0.025	-0.029	-0.024
Trade Event 3	-0.010	-0.015	-0.022
Trade Event 4	-0.012	-0.014	-0.012
Trade Event 5	-0.019	-0.004	-0.002
Trade Event 6	-0.011	-0.011	-0.007
Trade Event 7	0.058*	0.053	0.060
Trade Event 8	-0.030	-0.031	-0.037
Trade Event 9	-0.010	-0.000	-0.009
Trade Event 10	-0.033	-0.053	-0.045
Trade Event 11	-0.084***	-0.082**	-0.082***
Trade Event 12	-0.018	-0.010	-0.002
ESA Event 1	0.016	0.012	0.005
ESA Event 2	0.009	0.023	0.031
ESA Event 3	0.116**	0.125*	0.145**
ESA Event 4	0.044	0.057	0.053
ESA Event 5	0.058	0.071	0.058
ESA Event 6	0.023	0.028	0.030
ESA Event 7	-0.054	-0.034	-0.030
ESA Event 8	-0.002	0.001	-0.034
ESA Event 9	-0.005	0.006	0.047
ESA Event 10	-0.2033	-0.233**	-0.276***
ESA Event 11	0.184	0.183*	0.225**
ESA Event 12	.0102	0.103	0.106
Housing Start Events (147) Number significant at α=.10	35	35	28
R^2 - unadjusted	.0987	.0984	.0989
Joint tests of significance			
Housing Starts (149 d.f.)	p = 3.3 x 10^{-4}	p = 1.6 x 10^{-4}	p = 1.4 x 10^{-4}
Trade Events (14 d.f.)	p = 0.146	p = 0.118	p = 0.101
ESA Events (14 d.f.)	p = 0.022	p = 0.148	p = 0.114

Standard errors are reported in parentheses.
* denotes significant at α=.10
** denotes significant at α=.05
*** denotes significant at α=.01
† Values for μ and τ for the UERM are calculated as the mean and standard deviation of a uniform distribution over (θ_B-.5, θ_E+.5), where θ_B and θ_E are the maximum likelihood estimates of the UERM parameters.
†† Values for μ and τ for the BERM are calculated as the mean and standard deviation of a three-parameter beta distribution with parameters equal to the maximum likelihood estimates of the BERM parameters, which are not reported. The standard errors of the μ and τ estimates are calculated via the delta method from the asymptotic covariance matrix of the BERM parameters.

Table 6. Comparing Speeds of Market Response: Joint Hypothesis Tests from the NERM.

H_o: $\mu_{Starts} = \mu_{ESA} = \mu_{Trade} = 0$ (All event response distributions are centered on event days)	p = 0.264
H_o: $\tau_{Starts} = \tau_{ESA} = \tau_{Trade}$ (Speed of market response is the same for the three event types)	$p = 2.0 \times 10^{-4}$
H_o: $\tau_{Starts} = \tau_{ESA}$	$p = 7.9 \times 10^{-5}$
H_o: $\tau_{Starts} = \tau_{Trade}$	p = .19
H_o: $\tau_{ESA} = \tau_{Trade}$	$p = 3.6 \times 10^{-4}$

Table 7. Two Representations of Event Effects: Individual Event Dummy Variables and Event Period Relative Variables: Events Occur in Periods 3 and 7.

Period	Individual Event		Event Day Relative		
#	Event 1	Event 2	Pre-Event Day (-1)	Event Day Day (0)	Post-Event Day (+1)
1	0	0	0	0	0
2	1	0	1	0	0
3	***1***	0	0	***1***	0
4	1	0	0	0	1
5	0	0	0	0	0
6	0	1	1	0	0
7	0	***1***	0	***1***	0
8	0	1	0	0	1

Figure 1. Lumber Futures Prices, Event Dates, and Lumber Futures Rate of Return, 1986-1999.

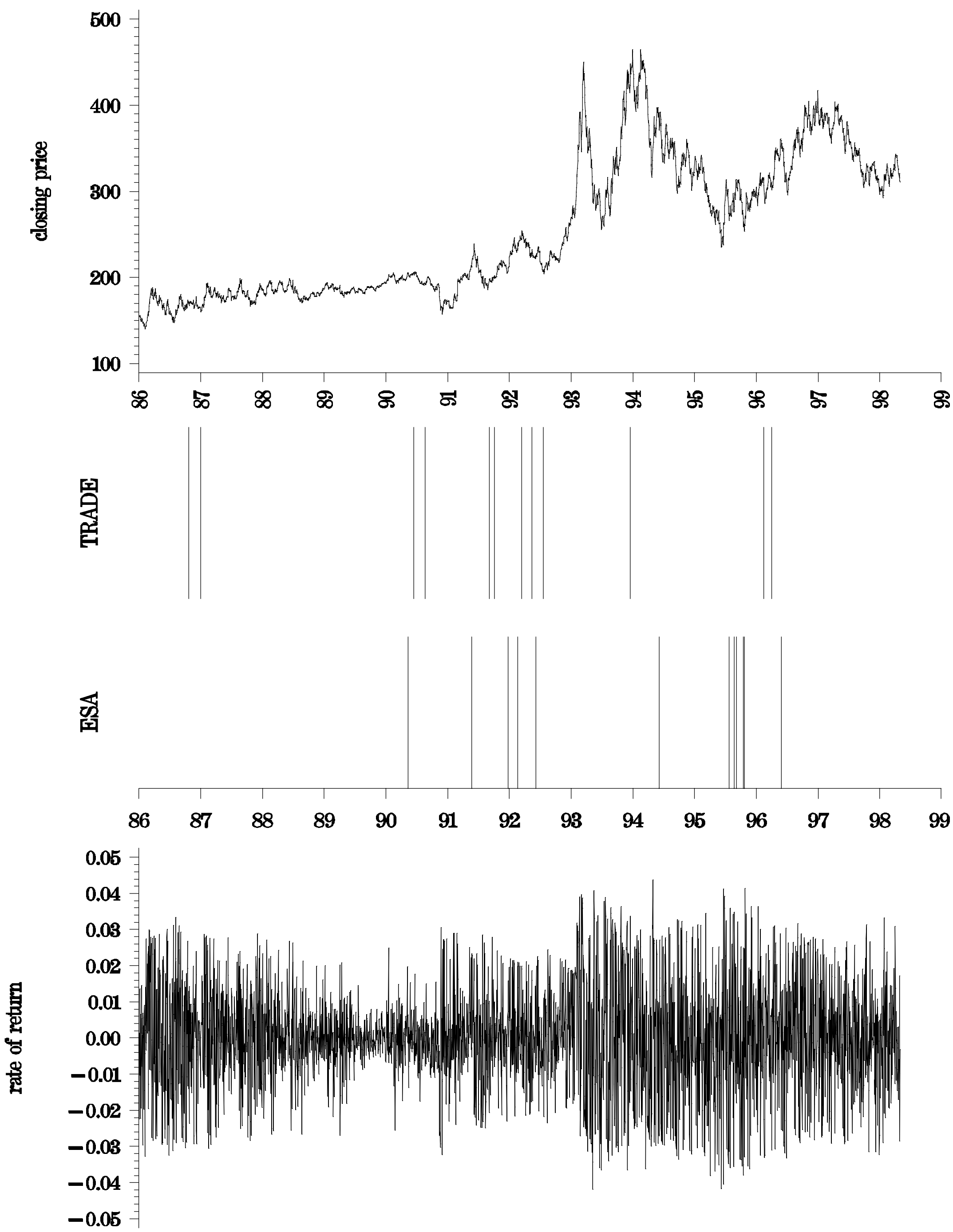


Figure 2. The Distributional Event Response Model

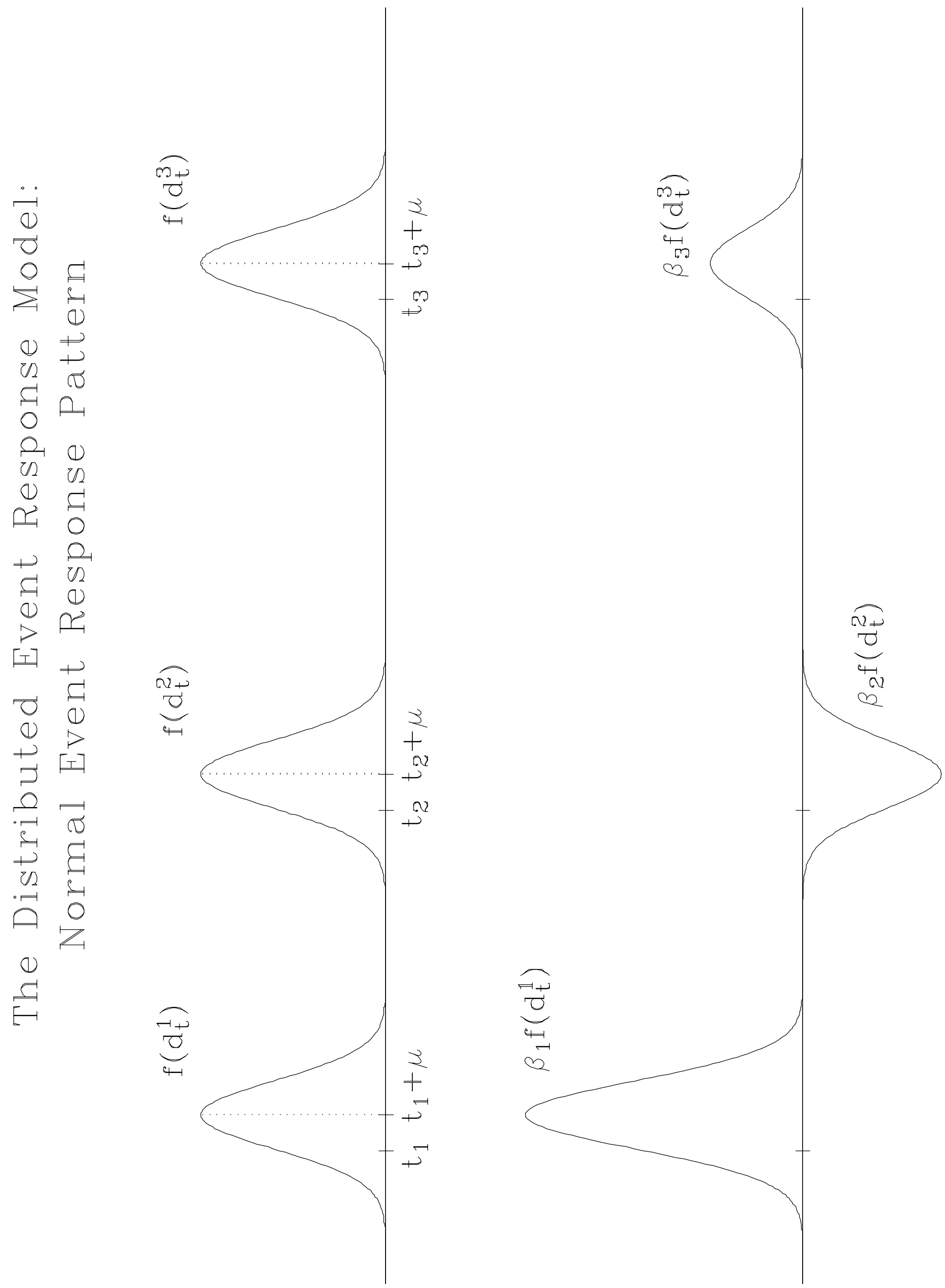
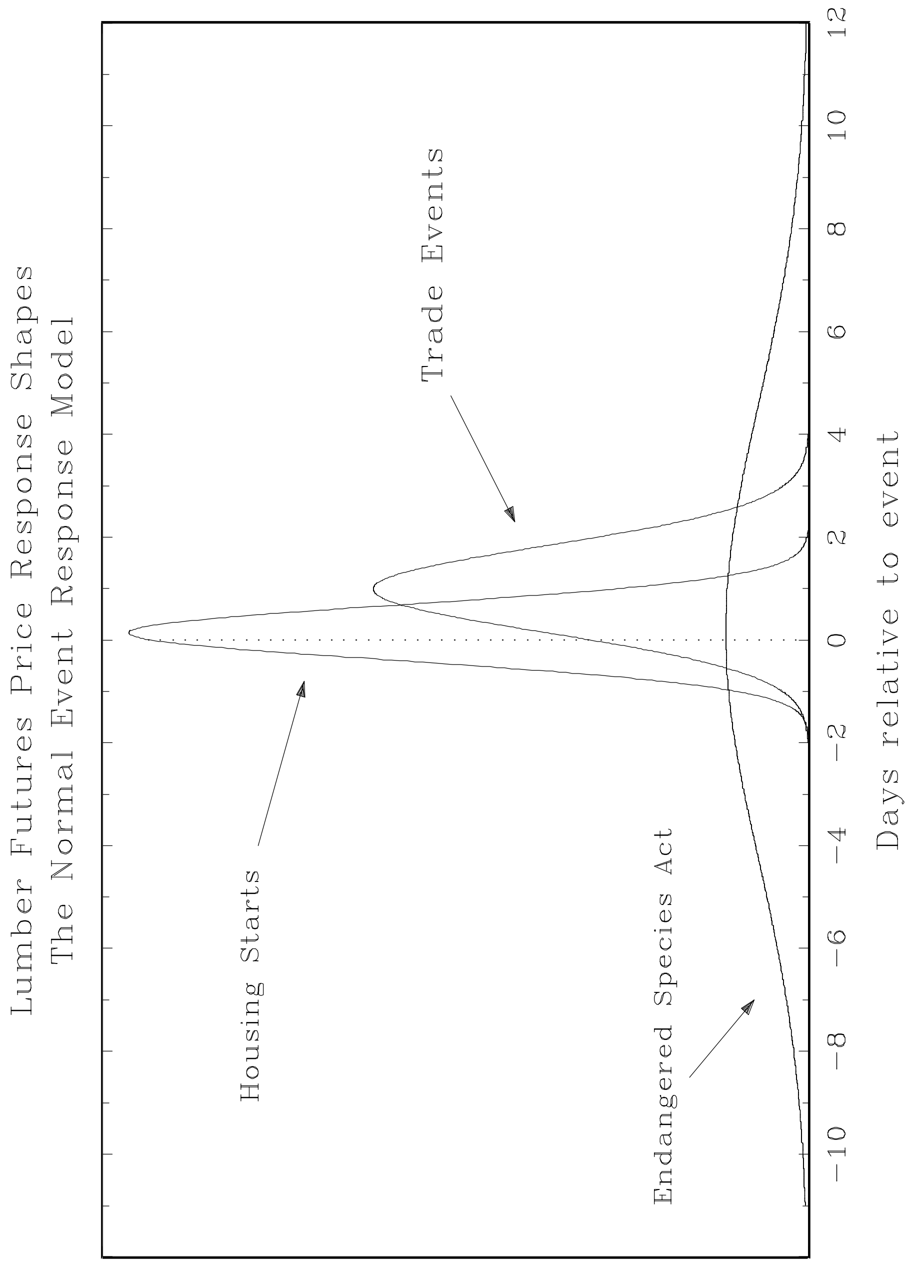
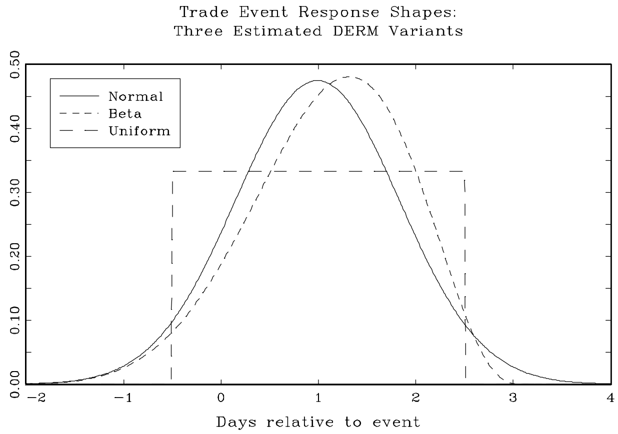
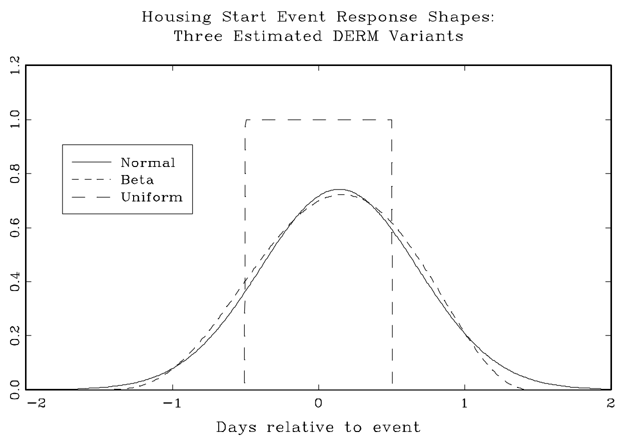


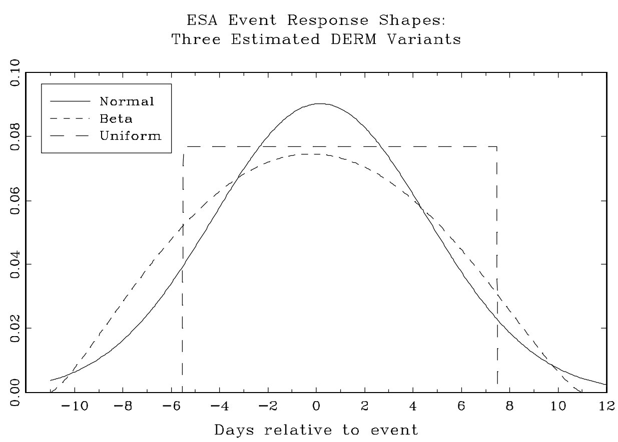
Figure 3. A Comparison of the Estimated NERM for Three Event Types



Figures 4a-c. A Comparison of Estimated DERM Variants for Three Event Types







Appendix: An Efficient Procedure for Estimating the Covariance Matrix of the DERM Parameters

The DERM is nonlinear in the parameters of the event response shape (μ and τ in the case of the NERM), but is linear in the β_i parameters (the coefficients of individual events) and the parameters associated with the continuous daily-observed variables. This structure is also seen in other models such as the Box-Cox, which is a linear model conditional on the transformation parameter. This fact suggests an efficient way to calculate nonlinear least squares estimates of the DERM: optimize over the nonlinear parameters, calculating explicitly the ordinary least-squares solution for the linear parameters for each choice of the nonlinear ones. In our application, this reduces a 184-dimensional search to either a six or nine-dimensional search, depending on the variant of the DERM that is estimated. If the model errors are normal so that the nonlinear least squares estimates also are maximum likelihood, then the estimation strategy can be extended to calculate efficiently the asymptotic covariance matrix of the estimates, both linear and nonlinear.

Denote the log-likelihood as $L(\theta_1,\theta_2)$, where the score function is nonlinear in the vector θ_1 and linear in the vector θ_2. The likelihood function can be maximized by maximizing

(A1) $\tilde{L}(\theta_1) \equiv L[\theta_1, \hat{\theta}_2(\theta_1)]$,

where $\hat{\theta}_2(\theta_1)$ is the choice of θ_2, conditional on θ_1, that maximizes L(.,.). The optimal choice

$\hat{\theta}_2(\theta_1)$ is characterized by:

(A2) $L_2[\theta_1, \hat{\theta}_2(\theta_1)] \equiv 0$,

where L_2(.,.) denotes $\partial L(\theta_1, \theta_2)/\partial\theta_2$.

Totally differentiating both sides of (A2) yields:

(A3) $d\ln L_2 = L_{21} d\theta_1 + L_{22} d\theta_2 = 0$,

where $L_{21} = \dfrac{\partial^2 L}{\partial\theta_2 \partial\theta_1'}$ and $L_{22} = \dfrac{\partial^2 L}{\partial\theta_2 \partial\theta_2'}$.

Equation (A3) implies that, at an optimum,

(A4) $\dfrac{\partial\hat{\theta}_2(\theta_1^*)}{\partial\theta_1} = -L_{22}^{-1}(\theta_1^*, \theta_2^*)\, L_{21}(\theta_1^*, \theta_2^*)$, where (θ_1^*, θ_2^*) maximizes $L(\theta_1,\theta_2)$ and $\partial\theta_2(\theta_1^*)/\partial\theta_1$ is the gradient of the maximum-likelihood dependence of θ_2 on θ_1, evaluated at the optimal choice for θ_1. Finally, (A4) can be used to calculate L_{21} as:

$$\text{(A5)} \quad L_{21}(\theta_1^*, \theta_2^*) = -L_{22}^{-1}(\theta_1^*, \theta_2^*)\frac{\partial\hat{\theta}_2(\theta_1)}{\partial\theta_1} = \frac{1}{\sigma^2}X'X\frac{\partial\hat{\theta}_2(\theta_1)}{\partial\theta_1},$$

with the right-hand side derivatives evaluated at (θ_1^*, θ_2^*).

In calculating the asymptotic covariance matrix of the ML estimators, we use minus the sample Hessian of the log-likelihood to approximate the information matrix. Applying result (A5) to the structure of the DERM, we obtain:

$$\text{(A6)} \quad V(\hat{\theta}_1, \hat{\theta}_2) = \left(-\frac{\partial^2 L}{\partial\theta\,\partial\theta'}\right)^{-1} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}^{-1} = \hat{\sigma}^2\begin{bmatrix} \frac{1}{2}H & -G'X'X \\ -X'XG & X'X \end{bmatrix}^{-1},$$

where $H \equiv \dfrac{\partial^2 SSE(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_1'}$, $G \equiv \dfrac{\partial\hat{\theta}_2(\theta_1)}{\partial\theta_1}$,
$SSE(\theta_1,\theta_2)$ is the sum of squared errors function, and X is the matrix of "regressors," which are known conditional on θ_1.

Using the partitioned inversion formula to obtain an explicit solution for the last term in (A6), we obtain the asymptotic covariance estimators:

$$\text{(A7)} \quad V_{11} = V(\hat{\theta}_1) = \hat{\sigma}^2\left(\frac{1}{2}H - G'X'XG\right)^{-1}$$

$$V_{22} = V(\hat{\theta}_2) = \hat{\sigma}^2(X'X)^{-1} + GV(\hat{\theta}_1)G'$$

$$V_{12} = V(\hat{\theta}_1)G'.$$

The expressions in (A7) are both useful and informative. Note in particular how the expression for V_{22} modifies the usual estimator of the covariance matrix of the linear regression coefficients, $\hat{\sigma}^2(X'X)^{-1}$, which is conditional on the X's, with the term $GV(\hat{\theta}_1)G'$, reflecting sampling variability in the estimation of the nonlinear parameters, θ_1. The latter term can be seen to be a positive semi-definite matrix, which implies that the estimate of the conditional variance of the linear parameters will necessarily be less than (or equal to) the consistent estimate for the unconditional variance.

The expressions in (A7) require a Hessian calculation, H, and a gradient calculation, G. Closed form expressions for them are cumbersome in the case of the DERM. We implement the covariance calculations with numerical calculations of H and G, carried out in GAUSS. Two advantages ensue from this procedure over direct calculation of a numerical Hessian with respect to all the parameters of the model. The first is that there is a large gain in computational efficiency–on the order of 200:1. The direct solely numerical approach took several hours to calculate the covariance matrix of a single DERM variant. The second advantage is the reduction in numerical error that accompanies the implementation of (A7).