The quantity and quality of teachers: Dynamics of the trade-off

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A B S T R A C T

The paper addresses the two-fold rise in teacher–student ratio in the American K-12 school system in the post-World War II period accompanied by the evidence of a decline in the relative quality of teachers. We develop a dynamic general equilibrium framework for analyzing the teacher quantity–quality trade-off and offer an explanation to the observed trends. Our OLG model features two stages of education: basic and advanced (college), the latter required of teachers. The cost of hiring teachers is influenced by the outside opportunities for college graduates in the production sector. We show that the latter factor strengthens the process of endogenous growth and that it affects the optimal trade-off between quantity and quality of teachers such that the number of teachers hired will grow over time while their relative, but not the absolute, human capital attainment will fall. This is accompanied by increasing inequality, among the group of college educated workers in particular. We show that this effect, which we call the rising talent premium, applies whether teacher salaries are determined based on merit pay or, alternatively, by collective bargaining. Moreover, the salary compression characterizing the latter regime exacerbates the loss of the more talented workers by the teaching profession. Further, we analyze a comparative dynamics effect of exogenous skill-biased technological change which raises the college premium. We show that the effect is detrimental to the aggregate quality of teachers and to the quality of basic education. An important insight from this analysis is that in the process of human capital driven economic growth the rise in premium for high ability outpaces that for the average, whereby this effect is accelerated by technological change. This puts a downward pressure on the "real" quality of education inputs and therefore can create a negative feedback effect on human capital development as a factor of economic growth.

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1. Introduction

A reduction of class sizes in K-12 schooling has been one of the main education policy priorities in the United States over the last several decades. This is evidenced by the dynamics of student–teacher ratio which fell from 25.8 in 1960 to 15.7 in 2005. Remarkably, this decline has persisted through the contemporaneous ups and downs of the enrollment dynamics (Digest of Education Statistics, 2007, table 61, reproduced in Table 1 below). Research, however, has shown that students' test scores have not risen despite increased individualized instruction. This has compelled policy makers and researchers to question the role of the quantity of teachers vs. their quality as factors in student performance (see Hanushek et al., 2005), particularly in light of a possible trade-off between the two.

Indeed the changes in education statistics between 1955 and 2005 displayed in our Table 1 suggest the possibility of a quantity–quality trade-off in the supply of teachers over this period: a remarkable growth in the total number of teachers consistently outpacing the growth of student enrollments was accompanied by declining relative teacher salaries, despite essentially steady overall K-12 public education expenditures as a share of GDP since 1970. Another significant trend observed over about the same period is the decline in the aptitude of teachers relative to other educated workers. Hoxby and Leigh (2004) estimate that in 1963 41% of all teachers were of the "middle" aptitude relative to their educated peers, with 17% above and 42% below the average: by comparison, in 2000, 28% of all teachers were of the "middle" aptitude with 5% above and 67% below average. Corcoran et al. (2004) provide similar results.

Thus the dynamics of the K-12 system exhibits a trade-off between the quantity of teachers and their quality. The presence of the trade-off between the two variables creates the well-known endogeneity challenge for the empirical efforts to estimate one of
the variables while controlling for the other.  These empirical results do not thus provide a conclusive basis to explain the observed policy bias in favor of the quantity of teachers (smaller classes) on the grounds of benefits to student performance. This underscores the need for a broader theoretical framework capturing the dynamic interaction between inputs in education as it is influenced by the labor market in the production economy.

Our paper develops a theoretical framework for analyzing the teacher quantity–quality trade-off and offers an explanation to the observed trend biased in favor of quantity. We present a model which incorporates the factors of education quality discussed above, in a dynamic general equilibrium framework where education policy decisions affect and are affected by individual education and employment decisions, whereas the dynamics of human capital accumulation and labor productivity has a feedback effect on both. In our model, a public education agency wishes to maximize the quality of basic education per student and, due to a budget constraint, faces a trade-off between the quality and quantity of teachers to be hired. We explore the following factors that may affect this trade-off: (i) the opportunity cost of teachers in terms of potential earnings in the alternative occupations in the production economy, and its evolution in the process of economic growth; (ii) collective bargaining driven by teacher unions as an institutional factor leading to wage compression among teachers; (iii) the rise of skill premium resulting from technological change, which affects the opportunity cost of teachers.

In our model, basic education is publicly funded through an income tax, whose rate is set exogenously, and is administered by an education agency which allocates these funds while deciding on the levels of teacher salaries and the number of teachers to be hired. The model features two stages of education: basic and advanced (college), the latter being required of teachers. College graduates can also take jobs in the skilled labor force of the production sector and get paid a competitive wage according to their human capital attainment. This opportunity cost implies that the levels of teacher salaries set by the education agency will determine the quality (human capital levels) of teachers who can be hired. Thus the total cost of hiring teachers is affected in our model by the outside opportunities available to skilled individuals in the production sector. We show, moreover, that in the process of endogenous growth this effect strengthens and that it pushes the optimal trade-off between quantity and quality of teachers in the direction of the former. Namely, in the face of rising over time cost of highly able skilled workers, the education agency will find it optimal to opt for increasing the number of teachers hired while reducing the overall relative quality of the pool of teachers. (The absolute human capital attainment of teachers, however, will rise along with the overall human capital accumulation, while sliding toward the lower tail of the distribution of college educated population.) Furthermore, we show that this human capital dynamics is characterized by increasing inequality within the group of college educated workers as well as between it and the unskilled.

In order to assess the role of an institutional factor in the teacher quantity–quality trade-off, we consider two alternative teacher

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**Table 1**

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment</th>
<th>Teachers</th>
<th>Pupil/teacher ratio</th>
<th>Real expenditures per pupil</th>
<th>Expenditures per pupil</th>
<th>Expenditures to GDP</th>
<th>Relative teacher salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>30,680</td>
<td>1,141</td>
<td>26.9</td>
<td>234.2</td>
<td>294.2</td>
<td>3.3%</td>
<td>43.0</td>
</tr>
<tr>
<td>1960</td>
<td>36,281</td>
<td>1,408</td>
<td>25.8</td>
<td>2606.3</td>
<td>375.1</td>
<td>3.6%</td>
<td>44.0</td>
</tr>
<tr>
<td>1965</td>
<td>42,173</td>
<td>1,710</td>
<td>24.7</td>
<td>3440.0</td>
<td>537.7</td>
<td>3.9%</td>
<td>42.0</td>
</tr>
<tr>
<td>1970</td>
<td>45,894</td>
<td>2,059</td>
<td>22.3</td>
<td>4682.1</td>
<td>911.1</td>
<td>4.6%</td>
<td>44.0</td>
</tr>
<tr>
<td>1975</td>
<td>48,819</td>
<td>2,198</td>
<td>20.4</td>
<td>5534.6</td>
<td>1503.6</td>
<td>4.6%</td>
<td>41.0</td>
</tr>
<tr>
<td>1980</td>
<td>40,877</td>
<td>2,184</td>
<td>18.7</td>
<td>5855.3</td>
<td>2501.6</td>
<td>4.0%</td>
<td>41.0</td>
</tr>
<tr>
<td>1985</td>
<td>39,422</td>
<td>2,206</td>
<td>17.5</td>
<td>7045.4</td>
<td>3755.6</td>
<td>3.8%</td>
<td>36.5</td>
</tr>
<tr>
<td>1990</td>
<td>41,217</td>
<td>2,398</td>
<td>17.2</td>
<td>8014.7</td>
<td>5257.8</td>
<td>4.3%</td>
<td>35.0</td>
</tr>
<tr>
<td>1995</td>
<td>44,840</td>
<td>2,598</td>
<td>17.3</td>
<td>8121.8</td>
<td>6146.9</td>
<td>4.3%</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>47,204</td>
<td>2,941</td>
<td>16.0</td>
<td>9214.8</td>
<td>7903.7</td>
<td>4.5%</td>
<td>36.5</td>
</tr>
<tr>
<td>2005</td>
<td>49,113</td>
<td>3,137</td>
<td>15.7</td>
<td>10,041.5</td>
<td>9788.4</td>
<td>4.6%</td>
<td></td>
</tr>
</tbody>
</table>

Notes:

5. In thousands.
6. In percent.
7. 1959 data.
8. College educated females, age 20–29, earning less than average female teacher, age 20–29.
10. In 2005 dollars.
11. College educated males, age 20–29, earning more than average male teacher, age 20–29.
12. Deflated by CPI and in 2006 dollars.

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2. Indeed, Jepsen and Rikvin (2002) argue that using mandated class size reduction programs as natural experiments for estimating the class size effect is problematic when these changes are accompanied by a trade-off between the quantity and quality of teachers. They point out that California’s class size reduction program came at the expense of hiring lower quality teachers to staff additional classrooms, which offset the benefits of smaller classes. Similarly, Hoxby (1996) finds that in unionized public school higher measured inputs may produce no gains to student achievement: the teachers’ unions engage in rent-seeking which leads to lower productivity, via teacher quality or effort, enough to offset any gains from the additional resources, smaller student–teacher ratios in particular.

3. It is indeed well documented that teacher unions significantly contribute to the compression of seniority adjusted wages: unions provide tenure to teachers and tie their salary primarily to experience rather than performance; administrators wishing to hire higher quality teachers are forced by the unions to provide matching raises to teachers across the board. The wage compression in public schools imposes similar labor market distortions on the private school teacher market (Lakdawalla, 2006). It should be noted that unionization is not the sole factor responsible for the compression of teacher salaries. It is also due in part to the difficulty of measuring teacher productivity, especially in terms of educational value added given the unobservable student characteristics as well as student sorting into classes.

4. Recent literature offers estimates of the effects of outside job market opportunities on the quality of teachers. Flyer and Rosen (1997) report that the three-fold increase in direct costs of education per student is attributable to the growing market opportunities for women. Hanushek and Rivkin (1997) document the decline in the earnings of women teachers relative to women in other occupations and suggest that the expansion of alternative opportunities reduced teacher quality. Hanushek and Rivkin (2004) estimate that in 1955, 50% (48%) of all educated male (female) workers earned less than male (female) teachers, compared to 36% (29%) in 2000. Similar analyses concerning the effect of the outside opportunities on teacher quality are offered by Goldhaber and Liu (2003), Stoddard (2003), and Bacolod (2007). Lakdawalla (2006) demonstrates that a rising skill premium of educated workers due to faster technological change coupled with low productivity growth of skilled teachers, has led to the substitution toward a larger quantity of unskilled teachers, away from the increasingly expensive skilled teachers.
employment regimes: the “merit pay” regime where each teacher’s salary is based on his human capital attainment, as well as the “collective bargaining” regime where the education agency is bound by salary contracts with teacher unions which compress teacher salaries. Our results show that the salary compression due to the collective bargaining further exacerbates the loss of the more talented workers by the teaching profession and thus additionally contributes to the decline of teacher quality.

We then build on these results to further analyze the impact of exogenous technological change biased toward skill, i.e., augmenting productivity of skilled workers and thereby the college premium. We show that such technological change will exacerbate the negative trends in the quality of basic education in relation to GDP growth. Specifically, the comparative dynamics effect (relative to the benchmark trajectory) will be detrimental to the aggregate quality of teachers as well as to the quality of basic education, due to the upward shocks to the cost of skilled labor. This is consistent with the aforementioned observation by Lakdawalla (2006) as well as the discussion by Rangazas (2002) of Baumol’s Disease in education sector as a result of technological change driving up the sector’s real labor costs, which leads him to conclude that school expenditures must be deflated by the rate of technological change in addition to the rate of inflation. Our model adds a dimension to this understanding: we show that in the process of human capital driven economic growth the rise in premium for high ability will outpace that for the average even besides the effect of technological change, hence an additional downward pressure on the “real” quality of education inputs.

Thus this paper offers a theory explaining the trend in education policy in favor of lower student–teacher ratios (i.e., higher quantity of teachers) combined with deteriorating teacher quality, despite growing per student schooling expenditures.

The paper is organized as follows. Section 2 develops a dynamic general equilibrium model of production economy and two-stage formation of human capital. In Section 3, we introduce the benchmark model of the administration of basic education with merit pay for teachers and then define and prove the existence of the recursive dynamic equilibrium. Section 4 extends the analysis to the collective bargaining regime for teachers; Section 5 delivers the main results on the dynamics of the quantity and quality of teachers for this case. Section 6 compares the teacher quantity–quality dynamics under the collective bargaining and merit pay regimes. Section 7 concludes. Appendix 1 presents a glossary of notation. Appendix 2 containing most of the proofs is posted at the Journal’s website.

2. The model

We develop a general equilibrium growth model of an economy populated by overlapping generations of individuals whose life-span consists of two periods: childhood and adulthood. We identify a generation with the period when its members are adults, thus the individuals born in period \( t - 1 \) form a generation \( G_t \). We assume that population size is constant in each generation \( G_t \) and that it forms a continuum on the interval \([0, 1]\). Let \( \mu(.) \) be the induced Lebesgue measure on this set, so that \( \mu([0, 1]) = 1 \) for all \( t \).

Children make no decisions of their own and receive basic (or first stage) education which is provided publicly. Young adults are endowed with a unit of time and face an option of devoting a fixed fraction \( n \) of it to acquire advanced education (which we will also refer to as college or second stage education); the balance of time not spent on education is inelastically devoted to work. Specifically, the individuals without college education will work for the full unit of time in the “unskilled” production workforce. Those with college education either work for the remaining fraction of time \( 1 - n \) in the “skilled” production workforce or, if qualified by the education authority, can work as public school teachers. Individuals derive income from work. They spend part of it on consumption when young and invest the rest to use the returns to finance their consumption in retirement, the last period of life.

2.1. Production

The production sector of the economy consists of perfectly competitive firms producing a homogeneous consumption good by means of a technology which uses two factors of production — unskilled and skilled human capital. The aggregate production function is given by:

\[
Y_t = w(L_t + \theta_t H^p_t),
\]

where \( w > 0 \), while \( L_t, H^p_t \) stand, respectively, for the aggregate supply of unskilled human capital and skilled human capital employed in the production sector in period \( t \). The coefficient \( w \) is thus a fixed productivity parameter per unit of unskilled labor. The coefficient \( \theta_t \) characterizes the net productivity augmentation of skilled human capital (adjusted for the shorter employment duration due to the time spent in college) which is imbedded in technology. The sequence of \( \theta_t \) characterizing the evolution (or stationarity, as a special case) of the skill premium in the process of technological change is assumed to be exogenously given.

2.2. Households

The economy is populated by overlapping generations \( G_t, t = 0, 1, \ldots \) of agents whose lives consist of two periods, childhood and adulthood, where the time index \( t \) of a generation refers to the time of its adulthood. While everyone’s childhood is devoted to receiving universal basic education, all adult individuals \( \omega \) of each generation \( G_t \) work and consume their post-tax income \( (1 - \tau_t)L_t(\omega) \), where \( L_t(\omega) \) is the individual’s wage income derived from human capital and \( \tau_t \) is the uniform rate of labor income tax collected by the government whose only outlay is funding basic public education. We leave the process by which the tax rates are determined outside the scope of this study and assume that \( \tau_t \) are exogenous and may either be constant or vary over time (see a discussion of a rationale for the latter possibility in the concluding Section 7).

According to production function (1) individuals working in the production of goods receive the wage at competitive rates \( w \) and \( \theta_t w \), respectively, per unit of their unskilled or skilled human capital, whichever applies. Thus the income of individual \( \omega \) who receives only basic education and attains the level of unskilled human capital \( L_t(\omega) \) will be:

\[
I_t(\omega) = wL_t(\omega).
\]

The individual \( \omega \) who obtains college education, attains the level of skilled human capital \( h_t(\omega) \) and is employed in the production sector, will receive income:

\[
I_t(\omega) = \theta_t w h_t(\omega). \tag{3}
\]

The incomes of college educated individuals who become teachers will be specified later.

According to the above description, the only decisions faced by the households concern (i) the level of educational attainment: basic or college, and (ii) in the latter case, a career choice between production of goods and teaching. We assume that income maximization is the households’ sole motive in making these decisions.
2.3. Human capital formation

The human capital received by each child \( \omega \) of generation \( G_t \) at the first (basic) stage of his education is produced in period \( t-1 \) by combining children’s random innate ability with public education according to:

\[
I_t(\omega) = C a(\omega) E_{t-1}
\]  

(4)

where \( C \) is a positive constant, \( E_{t-1} \) is a uniform quality of public schooling received by each child in period \( t-1 \) while \( a(\omega) \) is the child’s innate ability.

Assumption 1. Innate ability \( a(\omega) \) is distributed independently and identically in each generation (the time indexation is thus omitted); the distribution is uniform in the interval [0, A].

The uniform distribution assumption, which we make following Galor and Moav (2000), is aimed at obtaining similarly tractable analytical results but is not essential for the overall intuition.

We will now introduce the human capital production function for the advanced (college) stage of education. Consistent with Ben-Porath (1967) and Rosen (1976) we assume that the gains from college education depend on one’s prior preparation, which in turn depends on the innate ability. Moreover, we assume that college education has a pre-requisite human capital threshold \( h^* \). Rather than an ad hoc admission requirement (we assume that all individuals are free to choose to go to college but base this decision purely on income considerations) we view this threshold as a set of benchmark skills, such as adequate language and mathematical proficiency whose deficit would preclude any benefit from learning at the advanced stage. Specifically, we postulate that if an individual \( \omega \) of generation \( G_t \) chooses to go to college, he will become a “skilled” agent with the level of human capital given by:

\[
h_t(\omega) = b I_t(\omega) + B [ I_t(\omega) - h^* ]
\]  

(5)

where \( b \in (0,1) \) and \( B > 0 \) are given constants. Thus according to the expression (5) the gains from college education depend on the extent to which the individual’s pre-college human capital attainment \( I_t(\omega) \) exceeds the threshold \( h^* \).

The college education production function (5) also reflects a partial loss of pre-college human capital, according to the coefficient \( b \), for the purposes of skilled human capital. This loss includes the opportunity cost of the time spent in college. While this loss is counteracted by the net productivity augmentation \( \theta_t \) of skilled human capital according to the economy’s production function (1), we impose a condition:

\[
bb_\theta < 1
\]

which indeed implies that individuals whose pre-college human capital \( I_t(\omega) \) is below, at, or even slightly above the threshold \( h^* \), will not gain from attending college and therefore will not choose to do so. It is likewise logical to assume that highly able individuals, particularly those with the highest ability level \( A \), will benefit from attending college.

According to Eqs. (4) and (5) this will be true in generation \( G_t \) if and only if the following inequality holds:

\[
(b+B)CE_{t-1} \geq Bh^*
\]  

(6)

which has a straightforward meaning: an individual will choose to attend college if and only if his wage income derived from skilled human capital given by formula (5) and adjusted for the exogenous productivity augmentation \( \theta_t \) will exceed his wage based on the unskilled human capital obtained at the first stage of education according to its production function (4).

The kinked form of the college human capital production function (5), combined with pre-college preparation given by Eq. (4), implies that individual’s advanced human capital attainment exhibits increasing returns to ability, for which the quality of basic education is a complementary input. This allows us to capture an important and arguably realistic property of the talent premium of college education: the skill upgrade that it provides to a highly able student is disproportionately larger than the one gained by a less able peer. Furthermore, while higher quality public basic education “lifit all boats”, more able students will derive disproportionately greater benefits from it. This rising talent premium argument is used in some of the recent literature to explain the evidence of increasing dispersion of earnings. For example, Huggett et al. (2006) use lifecycle framework with a multi-stage Ben-Porath type model of human capital accumulation, which exhibits increasing returns to ability at higher stages of education, to explain the evolution of wage dispersion in the U.S. Another strand of models represented by Galor and Zeira (1993) is able to explain intergenerational persistence of inequality by the presence of credit constraints. The underlying mechanism, however, is fundamentally similar to ours: the consequence of the borrowing constraints is that investment in education exhibits increasing returns to agents’ endowments (within a certain range). Restuccia and Urrutia (2004) use a calibrated model which includes explicit early and college stages of education to apportion the factors, including individual ability and borrowing constraints, responsible for the intergenerational persistence of income inequality. By contrast, in most models of public education, such as by Glenn and Ravikumar (1992), human capital accumulation exhibits decreasing returns to private inputs, which leads to vanishing relative variation of income.\(^7\)\(^8\)
2.4. Quality of basic education

We shall now introduce the per student basic education quality $E_t$, i.e. the public input in the basic education production function (4) provided in period $t$, as a function of the quality and quantity of teachers chosen by a public education agency. Recall that only college educated individuals are eligible to be employed as teachers. Let $\Sigma_t$ be the set of individuals $\omega$ in generation $t$ employed as teachers. Let $z_t$ be the total number of teachers. Since population size is normalized to 1 in all generations, $z_t$ is also the fraction of teachers in the overall population in generation $t$, as well as the teacher–student ratio for generation $t+1$ students.

We define the aggregate teacher quality as the aggregate human capital of teachers:

$$ q_t = \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega). $$

(7)

Likewise, the average teacher quality is given by $z_t^{-1} \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega) = z_t^{-1} q_t$. The explicit account for the heterogeneity of teachers’ human capital attainment reflected in this formula is obviously an essential element of our model. Eckstein and Zilcha (1994), to our knowledge the first to explicitly incorporate teacher human capital as an input in (compulsory) schooling, assumed that it equals to the average human capital of their generation.\footnote{Hatsor (2008) contrasts such regime, which implies that teachers are selected at random from the population, with the one where the quality of teachers is an optimal policy decision traded off against their quantity. She focuses on comparing the implications of these regimes for growth and welfare in the framework of strategic interaction between the education and budgeting authorities of the government.}

We now define per student quality of basic education as a Cobb-Douglas function of the quantity and aggregate quality (human capital) of teachers:

$$ E_t = z_t^{-\gamma} q_t^{\gamma}, $$

(8)

with positive coefficients $\gamma, \nu$.\footnote{Much of the empirical literature analyzing the factors of student performance focuses on estimating the returns to either quality or quantity of teachers while attempting to control the other variable. For example, Aaronson et al. (2007), Clotfelter et al. (2007), Rivkin et al. (2005) and Goldhaber and Anthony (2007) estimate the effect of teacher characteristics on student achievement while Angrist and Lavy (1999), Krueger and Whitmore (2001), Krueger (1999) and Jepsen and Rivkin (2002) estimate how class size affects student achievement. All find positive effects of teacher quantity or quality on student achievement.}

The essence of this education quality production function is that $\gamma>0$ implies a distinct effect of the class size (equivalently, that of teacher–student ratio) in addition to the aggregate human capital of teachers on student achievement, and therefore recognizes a trade-off between the quantity and quality of teachers in education production.\footnote{The tax rate is taken by the education administrators as given. While it is plausible that the tax rate may be set by a different, fiscal, authority based on a political or normative decision process, we treat those as exogenous to our analysis. Thus the tax rates are considered exogenous in our model. We also do allow, but do not require, the tax rate to change exogenously over time. While this is not essential for our results, we explicitly allow for this possibility so as to (a) reflect the fluctuations in per student public education funding in relation to GDP growth seen in Table 1 and (b) make the results applicable to a further discussion in the paper of government policies which may indeed involve such rise in education funding.}

Indeed, it can be shown that if $\gamma=0$ was the case, then under teacher collective bargaining (which is a predominant feature of the U.S. teacher labor market) the inherent deadweight loss would be minimized and the education quality maximized (subject to a budget constraint) by hiring fewer but more skilled teachers, i.e., from the top segment of the ability distribution. Overwhelming evidence to the contrary is presented by the empirical literature we discussed in the Introduction, including Hoxby and Leigh (2004) who estimate that 67% of public school teachers are below the average aptitude of their college cohort.

Since the human capital of teachers is the only potentially unbounded educational input in our model, which is in this respect similar to human capital production in Lucas (1988), we must likewise require non-decreasing returns to this factor alone, i.e. $\nu \geq 1$, in order to also ensure that the model can exhibit aggregate growth of human capital across generations.

The special case of Eq. (8) with $\gamma = 1$, i.e., $E_t = z_t \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega)$ has a particularly straightforward interpretation. Assume that all teachers are perfectly sorted across classes, each of equal size $z_t^{-1}$, such that each student is exposed through his classes to a cross-section of teachers perfectly representing their distribution of quality. Then the above expression, which is equivalent to $E_t = \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega)$, can be interpreted as per student average teacher quality.

2.5. Administration of basic education

The basic level of schooling is administered by a public education agency with the goal of maximizing its quality $E_t$, as defined by expression (8) above, subject to the budget constraint given by the revenue from a uniform labor income tax at a flat rate $\tau$.\footnote{Since one’s work career is summarily represented in our model by a single time period, we do not model the wage dynamics over the course of a worker’s or teacher’s career as he accumulates seniority and experience. The income variables in this framework therefore represent the aggregates of earnings over the entire career, such as the present values at a career’s outset.} To this end in each period $t$, the agency must decide on the number of teachers to be hired $z_t$ and must set teacher salaries. The latter will also determine the quality of the teachers hired, i.e., the segment $\Sigma_t$ of the distribution of college-educated individuals who will become teachers, but this relationship depends on the institutional features of the labor market for teachers.

Specifically, as outlined in the Introduction, we distinguish between two alternative institutional regimes with respect to teacher compensation: the benchmark “merit pay regime”, where teachers are paid competitive salaries (as are the production sector workers) according to their individual human capital attainments, and the “collective bargaining regime”, the main focus of our analysis, where the union contract sets a uniform wage for all teachers. In the next two sections we will derive the optimal education policy decisions under each regime marking where necessary the corresponding decision variables with superscripts $\text{MP}$ and $\text{CB}$, respectively. We will then define, under each regime, the economy’s dynamic general equilibrium.

3. The benchmark case: the teacher merit pay regime

We assume throughout that an individual’s human capital attainment is a universal measure of his capacity in any occupation, production or teaching,\footnote{We note that the empirical education literature also, understandably, treats educational attainment as a proxy for teacher quality, due to the obvious difficulty to measure teaching ability directly.} thus under the merit pay regime a college educated individual $\omega$ employed as a teacher will be paid the same salary of $\theta_\omega h_\omega(\omega)$ he would get, if he chose the employment in the production economy, as given by the expression (3), i.e., his opportunity cost of working as a teacher.\footnote{Since our work career is summarily represented in our model by a single time period, we do not model the wage dynamics over the course of a worker’s or teacher’s career as he accumulates seniority and experience. The income variables in this framework therefore represent the aggregates of earnings over the entire career, such as the present values at a career’s outset.} We assume for simplicity that given identical wages, an individual will always choose teaching over production occupation, based, for example, on hedonic preference which, however, plays only a marginal role and does not result in a wage differential.\footnote{The essence of this education quality production function is that $\gamma>0$ implies a distinct effect of the class size (equivalently, that of teacher–student ratio) in addition to the aggregate human capital of teachers on student achievement, and therefore recognizes a trade-off between the quantity and quality of teachers in education production. Indeed, it can be shown that if $\gamma=0$ was the case, then under teacher collective bargaining (which is a predominant feature of the U.S. teacher labor market) the inherent deadweight loss would be minimized and the education quality maximized (subject to a budget constraint) by hiring fewer but more skilled teachers, i.e., from the top segment of the ability distribution. Overwhelming evidence to the contrary is presented by the empirical literature we discussed in the Introduction, including Hoxby and Leigh (2004) who estimate that 67% of public school teachers are below the average aptitude of their college cohort.}

We assume therefore represent the aggregates of earnings over the entire career, such as the present values at a career’s outset.\footnote{The special case of Eq. (8) with $\gamma = 1$, i.e., $E_t = z_t \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega)$ has a particularly straightforward interpretation. Assume that all teachers are perfectly sorted across classes, each of equal size $z_t^{-1}$, such that each student is exposed through his classes to a cross-section of teachers perfectly representing their distribution of quality. Then the above expression, which is equivalent to $E_t = \int_{\omega \in \Sigma_t} h_\omega(\omega) d\mu_t(\omega)$, can be interpreted as per student average teacher quality.}
The budget constraint on the aggregate teacher wage bill is thus given by:

\[ \theta_t W \int_{a_t \leq \omega \leq A} h_t(\omega) d\mu_t(\omega) = T_t^{MP} \]

(9)

where \( T_t^{MP} \) is the tax revenue collected by the government in period \( t \).

Since all workers' wages under this regime are proportionate to their human capital levels, the tax revenue is given by:

\[ T_t^{MP} = T_t W\left(1 + \theta_t H_t\right) \]

(10)

where

\[ H_t = \int_{a_t \leq \omega \leq A} h_t(\omega) d\mu_t(\omega) \]

is the aggregate human capital of all college graduates working in the goods production and education sectors. We note that according to expression (7) the budget constraint in Eq. (9) uniquely determines the aggregate quality of teachers:

\[ q_t^{MP} = \int_{a_t \leq \omega \leq A} h_t(\omega) d\mu_t(\omega) = \frac{T_t^{MP}}{\theta_t W}. \]

(11)

Therefore, the objective of the maximum per student quality of basic education given by formula (8) is attained by maximizing the number of teachers \( z_{t}^{MP} \) subject to keeping their aggregate quality as given by Eq. (11). This implies that the optimal stock of teachers is the bottom segment of college graduates, i.e., the optimal set \( z_{t}^{MP} \) of teachers to be hired consists of all individuals whose levels of human capital \( h_t(\omega) \) attained in college fall into the interval \( [h_t(a_t^{MP}), h_t(\omega)] \) where \( h_t(\omega) \) is the human capital of the most able teacher hired under this scenario while \( h_t(a_t^{MP}) \) is the human capital attainment of the lowest ability college graduate in generation \( G_t \), the cut-off ability level \( a_t^{MP} \) being defined by the expression (6). Therefore, the number of teachers to be hired is the measure of this set, \( z_t^{MP} = \mu\left(\omega | h_t(a_t^{MP}) \leq h_t(\omega) \leq h_t(\omega)\right) \), while their aggregate absolute quality is given by the expression:

\[ q_t^{MP} = \int_{h_t(a_t^{MP})}^{h_t(\omega)} h_t(\omega) d\mu_t(\omega). \]

(12)

Let \( a_t^{MP} \) denote the innate ability level which corresponds to the top teacher human capital \( h_t^{MP} \) as defined above. Then according to the production functions of basic and advanced education given by the expressions (6) and (7), we can write:

\[ a_t^{MP} = \frac{h_t^{MP} + Bh_t^{MP}}{(b + B)C_{t+1}^{MP}} \]

(13)

and rewrite the expression (12) as

\[ q_t^{MP} = \int_{a_t \leq \omega \leq A} h_t(\omega) d\mu_t(\omega). \]

(14)

Since innate ability was assumed to be uniformly distributed on \([0, A]\), the number of teachers to be hired can be expressed as:

\[ z_t^{MP} = \frac{q_t^{MP}}{a_t^{MP} - q_t^{MP}} \]

(15)

Further in this section we will show that the education agency's optimization problem, which determines the optimal aggregate absolute quality of teachers \( q_t^{MP} \) and their quantity \( z_t^{MP} \), can be reduced to solving Eq. (11) in terms of \( z_t^{MP} \). This according to Eq. (15) will set the optimal upper ability cut-off \( a_t^{MP} \), i.e., a characteristic of the relative teacher quality, which in turn according to Eq. (14) will determine the optimal aggregate teacher quality \( q_t^{MP} \).

We can now summarize the fundamental elements of the model, under the benchmark teacher merit pay regime, and their relationships in a general equilibrium framework. We will first define the dynamic general equilibrium for given education agency's policy parameters under the merit pay regime and then incorporate the optimal education policy into the recursive dynamic general equilibrium framework.

Definition. Assuming the merit pay regime for teachers and given the initial per student basic education quality \( E_{t-1}^{B} \) provided to generation \( G_t \) individuals as children in period \( t = 0 \), a sequence of tax rates \( \{\tau_t\}_{t=0}^{\infty} \) and a sequence of education policy decisions, i.e., the numbers of teachers hired \( \{z_t^{MP}\}_{t=0}^{\infty} \), which under this regime determine the set of teachers at the bottom of the human capital distribution of the respective generation's college graduates and thereby uniquely determine aggregate teacher quality levels \( q_t^{MP} \). We define the dynamic general equilibrium as a collection of sequences of the aggregate variables \( \{Y_t, L_t, H_t, T_t^{MP}, E_t^{MP}, a_t^{MP}\}_{t=0}^{\infty} \), the distributions of human capital attainment \( [h_t(\omega), h_t(\omega)]_{t=0}^{\infty} \) resulting from the individual education decisions, as well as the employment decisions by all college graduates such that (i) each individual \( \omega \) in generation \( G_t \) makes a decision about attending college and the type of employment while taking as given teacher employment opportunities set by the education agency; the quality of basic education \( E_t^{MP} \) provided to the next generation \( G_{t+1} \) individuals as children is determined by the expression \( (8) \) and the education policy variables \( q_t^{MP} \) and \( z_t^{MP} \); (ii) the markets for skilled and unskilled labor in the production sector clear in each period; (iii) the aggregate tax revenue is composed of labor income taxes collected from all categories of employees, so the relationship in Eq. (10) holds.

We can now define the education agency's optimal policy, for the teacher merit pay regime, in period \( t \) recursively, based on the above general equilibrium construct. Namely, the education agency chooses the upper ability cut-off \( a_t^{MP} \) for teachers in period \( t \) by solving the education quality maximization problem, which will also determine the number of teachers hired \( z_t^{MP} \) while taking as given the aggregate tax revenue \( T_t^{MP} \) and the distribution of skilled human capital attainment \( h_t(\omega) \) by generation \( G_t \) individuals. Noting the interdependence between the general equilibrium variables in period \( t \) and the optimal education policy, we define the Education-Economy recursive dynamic equilibrium (RDE for brevity) as a fixed point of this relationship, recursively determined for each period \( t \).

It can be shown that the education agency's optimization problem, which maximizes the objective in Eq. (8) subject to constraint in Eq. (11), can be reduced to a quadratic equation, which uniquely defines the optimal quantity of teachers \( z_t^{MP} \) as a function of the prior period's education quality \( E_{t-1}^{B} \). The quantity of teachers \( z_t^{MP} \) along with their quality \( q_t^{MP} \) will then determine the current period's quality of basic education \( E_t^{MP} \) according to the formula \( (8) \). This produces an iteration of the recursion uniquely defining the recursive dynamic equilibrium (RDE) in our model under the merit pay regime for teachers. This argument, presented in detail in Appendix 2, leads to the following result:

**Proposition 1.** (Existence and uniqueness of RDE in the merit pay regime.) In each period \( t = 0, 1, \ldots \), given the prior period's education quality \( E_{t-1}^{MP} \) and college attendance decisions, the maximization of basic education quality in Eq. (8) under the teacher merit pay regime has a unique solution for the optimal quantity and quality of teachers...
(whereby the optimal set of teachers locates at the bottom of the human capital distribution of their generation’s college graduates), with the aggregate human capital of teachers characterized by the expression:

\[ q_{t^0} = \frac{\tau_1(b + B)ACE_{t-1}^{t^0}}{2} \left( 1 - \frac{2Bh_t}{(b + B)ACE_{t-1}^{t^0}} + \frac{\theta_i(Bh_t)^2}{(b + B)(ACE_{t-1}^{t^0})^2} \right) \]

This solution in turn uniquely determines for period t basic education quality \( E_t^{t^0} \), and as a consequence the college attendance and employment decisions by generation t individuals. This recursion defines the unique recursive dynamic equilibrium in the model under the teacher merit pay regime for all t = 0, 1, … given the initial generation’s basic education quality \( E_{-1} \).

4. The teacher collective bargaining regime: recursive dynamic equilibrium

As discussed in the Introduction, we postulate that under the collective bargaining between the teachers’ unions and the education agency all teachers in generation \( G_t \), receive equal salary, which we denote by \( E_t^{CB} \). Given the alternative occupation available to the college educated individuals in the production sector, the teacher salary figure \( E_t^{CB} \) will uniquely determine the highest level of human capital attainment \( h_t^{CB} \) among the individuals who will choose to become teachers. Indeed, according to the expression (3) it should satisfy the equation\(^{16}\):

\[ \theta_iwh_t^{CB} = I_t^{CB}. \tag{16} \]

Thus all the college graduates with human capital level \( h_t(o) \) at or below \( h_t^{CB} \) will be obviously motivated to accept the employment as a teacher rather than to work in the production sector. However, the education agency’s goal to maximize the overall education quality may dictate a minimum qualification requirement \( h_t^{CB} \) for the human capital of teachers, such that the optimal set \( \Sigma_t^{CB} \) of teachers to be hired will consist of all individuals whose level of human capital \( h_t(o) \) attained in college falls into the interval \( [h_t^{CB}, h_t^{BC}] \). This implies that the number of teachers to be hired can be expressed as the measure of the above interval:

\[ z_t^{CB} = \mu\left( h_t^{CB} \leq h_t(o) \leq h_t^{BC} \right). \tag{17} \]

Thus, according to Eqs. (16) and (17), the agency’s choice of teacher salary \( E_t^{CB} \) (equivalently, the choice of the top human capital cut-off \( h_t^{CB} \)) and the number of teachers \( z_t^{CB} \) will jointly determine the lower teacher qualification cut-off \( h_t^{BC} \).

Recalling the production functions of basic and advanced education given, respectively, by the expressions (4) and (5), we define the cut-off innate ability levels \( a_t^{CB} \) and \( a_t^{BC} \) which characterize the teachers who possess, respectively, the cut-off levels of human capital \( h_t^{CB} \) and \( h_t^{BC} \) induced by the education agency’s decisions described above. In other words (we again refer the reader to the glossary of notation available in Appendix 1),

\[ a_t^{CB} = \frac{h_t^{CB} + Bh_t}{(b + B)CE_{t-1}^{CB}} \quad \text{and} \quad a_t^{BC} = \frac{h_t^{BC} + Bh_t}{(b + B)CE_{t-1}^{BC}}. \tag{18} \]

\(^{16}\) While the teachers’ union collective bargaining agreements stipulate wage differentials based on seniority, the income variables in this framework stand for the aggregates over one’s career.

Then the number of teachers to be hired, as defined by the formula (17), can be expressed as:

\[ z_t^{CB} = \frac{a_t^{CB} - a_t^{CB} - \theta_i}{A}. \tag{19} \]

For the education authority’s policy choice of \( E_t^{CB}, z_t^{CB} \), to be feasible, the minimum teacher qualification threshold \( h_t^{CB} \) defined by Eq. (17) must belong to the range of human capital levels attained by college graduates. In other words, the corresponding ability level \( a_t^{CB} \) must exceed the college attendance cut-off level \( a_t^{CB} \).

Thus according to Eq. (8) the basic education quality optimization problem can be stated as:

\[ \max E_t \]

subject to (17),

\[ z_t^{CB} = T_t^{CB} \quad \text{and} \quad a_t^{CB} \geq a_t^{CB} \]

where \( T_t^{CB} \) is the tax revenue collected by the government in period t.

Fig. 1 below offers an illustration for this problem.

The horizontal axis represents workers’ ability. In the top graph, the vertical axis represents the workers’ wage incomes given by \( w_t \) for the unskilled workers and by \( \theta_iwh_t \) for the skilled production sector workers with the corresponding human capital attainment. The sloped line in the top graph represents the distribution of incomes of production sector workers in generation t as a function of individual ability according to the formulas (4) and (5), where the kink corresponds to the ability cut-off for college attendance \( a_t^{CB} \). The vertical bars illustrate several feasible education policy combinations of the number of teachers to be hired and teacher salaries satisfying the education budget constraint: \( (z_t^{CB}, h_t^{CB}), (z_t^{CB}, h_t^{CB}), (z_t^{CB}, h_t^{CB}) \). The width of a bar corresponds to the number of teachers \( z_t^{CB} \) while its height stands for teacher salary \( E_t^{CB} \), so the bar’s area is the total expenditure on basic education for the particular combination. Policy \( (z_t^{CB}, h_t^{CB}) \) represented by the left-most bar in the figure is characterized by the lowest student-teacher ratio (the largest quantity of teachers) but also the lowest teacher quality. This policy option is highly inefficient in terms of

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Fig. 1. Basic education optimization problem under collective bargaining regime.
education quality derived from the given budget revenue, as shown by the large deadweight loss represented by the triangular portion of the bar located above the sloped line: this is the excess salary paid to teachers above their opportunity cost value. The deadweight loss is the highest here because this policy involves hiring an inefficiently large number of low quality teachers who nevertheless have to be paid at the same rate as their best peer. The policy \( z_t^{CB}, h_t^{CB} \) (the right-most bar in the figure) delivers the highest teacher quality, relative to the other two policy combinations. This education policy has the lowest deadweight loss associated with the collective bargaining regime but it still has low per pupil education quality due to small number of teachers, i.e., high student–teacher ratio. The interior policy option \( (z_t^{B}, h_t^{B}) \) is where the per student education quality \( E_t \) is the highest amongst the three options depicted, providing the balance in terms of student–teacher ratio and the deadweight loss. This fact is illustrated by the graph at the bottom of the figure, where the vertical axis measures the values of \( E_t \) corresponding to different policy combinations uniquely identified by the ability of the top teacher under the policy.

We define the dynamic general equilibrium under the teachers’ collective bargaining regime for given education policy decisions similarly to the benchmark merit pay regime, except that here the education policy decisions include the uniform teacher salaries \( E_t^{CB} \) set for each period, which individuals take as given in their education and employment decisions. Accordingly, in lieu of the expression (10), the aggregate tax revenue composed of labor income taxes collected from all categories of employees is given in this case by:

\[
T_{t}^{CB} = \tau_t \left( w_t L_t + \theta_t w_t H_t' + z_t^{CB} h_t^{CB} \right)
\]

(21)

where \( H_t' \) stands, as before, for the aggregate human capital of the college educated workers in the production sector.

Likewise, we recursively define the optimal education policy in period \( t \) based on the above general equilibrium construct. Namely, the education agency chooses teacher salaries \( E_t^{CB} \) and the number of teachers \( z_t^{CB} \) for period \( t \) by solving the optimization problem in Eq. (20) where the top teacher quality \( h_t^{CB} \) is determined by Eq. (16), while taking as given the economy’s general equilibrium values of the aggregate tax revenue \( T_{t}^{CB} \) and the distribution of skilled human capital attainment \( h_t^{CB}(\omega) \) by generation \( G_t \) individuals. Similar to the benchmark case, this leads to the Education-Economy recursive dynamic equilibrium (RDE) construct.

Since the premise of this theoretical analysis is to capture the effect of economic growth on the dynamics of the educational resources, the quantity and quality of teachers, we will now impose some conditions on parameters of basic and college education systems, the level of education tax rates, skill premium in the production sector, as well as on per student basic education quality \( E_t^{B} \) provided to students in the initial generation \( G_0 \), such as to ensure economic growth.

**Assumption 2.** Education funding is sufficiently high and both basic and college education technologies are sufficiently productive to make sustained growth of human capital possible. Specifically, \( v \geq 1 \) (returns to the human capital of teachers are non-decreasing) \(^{17} \) and the productivity coefficients \( C \) and \( (b + B) \) are sufficiently large, while education tax rates \( \tau_t \) are not too low while not exceeding \( 1 - \epsilon \) at all times.

The gist of all of the parameter restrictions imposed by Assumption 2 is ensuring that the funding, determined by the tax rates, and productivity of both basic and college level education systems, determined by respective parameters \( C \) and \( (b + B) \) of the education production functions (4) and (5) under the condition of non-decreasing returns to teacher quality \( v \geq 1 \) discussed in Section 2.4, are sufficient to make sustained growth of human capital attainments across generations possible. The fact that such sustained growth of education quality, particularly at the basic level, actually takes place under these conditions is established in Lemma 1 below. The additional condition \( \tau_t \leq 1 - \frac{\epsilon}{2} \) in Assumption 2 will be certainly met if \( v \geq \gamma \), i.e. if the relative importance of teacher quality in schooling effectiveness is not lower than that of the teacher–student ratio, which will clearly be the case here under the natural requirement that \( \gamma \leq 1 \), i.e. that the there are non-increasing returns to the number of teachers.

More specific technical restrictions on the parameters involved in Assumption 2 are spelled out in Appendix 2 where we then prove the existence and uniqueness of the recursive dynamic equilibrium (RDE) in the model under the teacher collective bargaining regime. We do so by also deriving the explicit solution of the optimal education policy problem in Eq. (20) reformulated in terms of decision variables \( z_t^{CB} \) and \( \phi_t^{CB} \), as well as the optimal lower ability cut-off among those hired as teachers \( g_t^{B} \) and the resulting recursive dynamic equilibrium sequence of these variables. Specifically, we prove the following:

**Proposition 2.** (RDE solutions under the teachers’ collective bargaining regime).

(i) In each period \( t = 0, 1, \ldots \), given the prior period’s education quality \( E_{t-1}^{B} \) and individual college attendance decisions, the basic education quality optimization problem in Eq. (20) with the basic education budget \( T_{t}^{B} \) defined by the relationship in Eq. (21), has a unique solution characterized by the following expressions, respectively, for the optimal quantity and the absolute quality (i.e. the aggregate human capital) of teachers:

\[
z_t^{CB} = \left( \left( \frac{\gamma}{2B + \gamma} \right) \left( \frac{\tau_t}{1 - \tau_t} \right) \right)^{1/2}, \quad (22)
\]

\[
\phi_t^{CB} = v \tau_t (b + B) E_{t-1}^{B} \left( \frac{Bh_t^{B} + \phi_t^{CB}}{2B + \gamma} \right), \quad (23)
\]

where

\[
f_t^{CB} = \left( 1 - \frac{2Bh_t^{B}}{(b + B) E_{t-1}^{B}} \right)^{1/2}, \quad (24)
\]

as well as the upper and lower ability cut-offs for teachers to be hired:

\[
a_t^{CB} = \frac{Bh_t^{B}}{(b + B) E_{t-1}^{B}} + A_z^{CB} \left( \frac{1 - \tau_t}{\gamma} + \frac{1}{\gamma} \right), \quad (24)
\]

\[
a_t^{CB} = \frac{Bh_t^{B}}{(b + B) E_{t-1}^{B}} + A_z^{CB} \left( \frac{1 - \tau_t}{\gamma} - \frac{1}{\gamma} \right)
\]

(ii) This optimal solution for period \( t \) education policy uniquely determines period \( t \) basic education quality \( E_t^{B} \) and as a consequence, the college attendance decisions by generation \( t \) individuals. This recursion defines the unique recursive dynamic equilibrium in the model under the teachers’ collective bargaining regime for all \( t = 0, 1, \ldots \) given the initial generation’s basic education quality \( E_{-1} \).

5. The dynamics of the quantity and quality of teachers under collective bargaining

In this section we will establish the main dynamics results for the teachers’ collective bargaining regime, which is the main focus of our analysis. In Section 6, we will turn to the dynamics under the benchmark merit pay regime and will then characterize and discuss the comparison between the two.
The following important auxiliary facts concerning the dynamics under the collective bargaining regime are proven in Appendix 2:

**Lemma 1.** (Growth of Basic Education Quality). The recursive equilibrium dynamics under the collective bargaining regime for teachers exhibits sustained growth of the quality of per student basic education. Specifically, there is a factor \( g > 1 \) such that \( E_{t+1} > gE_{t-1} \) for all \( t = 0, 1, ... \)

**Lemma 2.** (The Interiority Property under the Collective Bargaining). In the recursive dynamic equilibrium, the ability of the least qualified teacher exceeds the college attendance cut-off ability in all time periods, i.e. \( a^{CB}_t < a^{CB}_t \) is true for \( t = 0, 1, ... \). Thereby the human capital of the least qualified teacher will not be the lowest among his contemporary college graduates.

**Lemma 3.** The ability cut-off for college attendance \( a^*_t^{CB} \) satisfies equality (6), i.e.,

\[
a^*_t^{CB} = \frac{1}{C_{E_{t-1}} - \theta_i(b + B) - 1}
\]

which means that an individual will choose to attend college if and only if his resulting skilled human capital given by formula (5) adjusted for the net productivity augmentation \( \theta_i \) will exceed his unskilled human capital derived from the basic stage of education according to its production function (4).

**Remark.** Since we assumed that individuals make a decision whether to attend college solely on the basis of the income motive, it is clear that the ability cut-off for college attendance \( a^*_t \) should satisfy the inequality:

\[
a^*_t^{CB} \leq \frac{1}{C_{E_{t-1}} - \theta_i(b + B) - 1}
\]  

which shows that the rise of income inequality among skilled workers can be attributed to

(i) the rising basic education quality \( E_{t-1} \) according to Lemma 1, which increases human capital and income of all workers, but disproportionately more at the high end of the ability distribution, as well as

(ii) the falling college attendance ability cut-off \( a^*_t \), according to Lemma 2, which brings lower ability workers into the fold of the skilled, thereby further increasing the intra-group inequality.

We now turn to our main focus on the dynamics of teacher quantity and quality which accompanies and is entailed in the process of growth. The following Lemma proven in Appendix 2 addresses a step in this recursion: it characterizes the effect of rising basic education quality \( E_{t-1} \) in period \( t-1 \) on period \( t \) education decision variables derived in the expressions (6), (22) and (24) above.

**Lemma 4.** The effects of increased quality of basic education \( E_{t-1}^{CB} \) in period \( t-1 \) are expanded college attendance (lower college attendance ability cut-off) in the next generation, larger quantity of teachers, and their lower quality given by their lower ability cut-off values. Specifically, the following is true for all \( t = 0, 1, ... \) in the recursive dynamic equilibrium:

\[
\frac{\partial a^*_t^{CB}}{\partial E_{t-1}^{CB}} > 0, \quad \frac{\partial a^*_t^{CB}}{\partial E_{t-1}^{CB}} < 0, \quad \frac{\partial a^*_t^{CB}}{\partial E_{t-1}^{CB}} = 0.
\]

Combining the last two facts, \( \frac{\partial a^*_t^{CB}}{\partial E_{t-1}^{CB}} < 0 \), \( \frac{\partial a^*_t^{CB}}{\partial E_{t-1}^{CB}} > 0 \) for \( t = 0, 1, ... \), with Lemma 1, which shows that education quality \( E_{t-1}^{CB} \), does in fact grow over time, yields one of our central results.

**Theorem 1.** (Dynamics of the Quantity and Quality of Teachers under Collective Bargaining). The recursive dynamic equilibrium (RDE) exhibits the following evolution of education policy variables:

- the quantity of teachers \( g_t^{CB} \) grows over time;
- the relative quality of teachers characterized by the range of their innate abilities falls: both the upper and the lower cut-offs \( a^*_t^{CB}, a^*_t^{CB} \) decrease over time;
- the college attendance ability cut-off \( a^*_t \) also drops over time (thus the college-bound population expands); \( a^*_t \) remains consistently below the lower ability cut-off for teachers \( a^*_t^{CB} \) (according to Lemma 2).

The decline of the relative quality of teachers measured against the contemporaneous college educated cohort does not imply the fall in their absolute levels of human capital. As shown by the following fact proven in Appendix 2 the absolute human capital levels of teachers will, in fact, grow. This is due to the overall growth in education established in Lemma 1, which permits relatively less-able teachers in the new generations to attain higher human capital level than did relatively more able teachers of previous generations.

**Corollary.** While the relative quality of teachers falls over time in the RDE under the collective bargaining regime (according to Theorem 1), the absolute quality of teachers characterized by their human capital attainment grows: both the human capital of the top teacher and the least qualified one, \( h^*_t^{CB}, h^*_t^{CB} \), rise over time.

**Discussion**

The intuition for the above results derives from the mechanics of economic growth in our model. Rising per student quality of basic education (Lemma 1) opens up the opportunity to pursue higher education for an expanding group of students. Namely, college attendance becomes worthwhile for an ever broader population, adding on students with relatively low ability. At the same time, the human capital attainment of highly able students increases disproportionately.
relative to their less able peers due to increasing returns to ability exhibited by the college education production function (5). Thus economic growth drives the rise of income inequality within the group of college graduates, as was spelled out in Proposition 3 above. As a result, the opportunity cost of highly able college graduates measured by their increasing relative earnings in the production sector rises over time. Therefore hiring high ability individuals as teachers is becoming a relatively more expensive option (we have characterized this as an effect of the rising talent premium), which pushes the quality-quantity trade-off for the education policy-makers in favor of the latter. In other words, hiring a larger number of teachers of relatively lower quality is becoming an increasingly more cost-effective policy.18

Fig. 2 below illustrates the evolution of the optimal education policy choices and the corresponding per-student education quality levels, along with the evolution of income distribution in successive time periods, t and t + 1.

The parts of the graph pertaining to period t correspond to the illustrations given in Fig. 1 of Section 4: the dashed sloped line in the top graph represents the income distribution of production sector workers in period t, while the right bar depicts the optimal education policy (the bar’s width \(Z_{t+1}^{CB}\) is the number of teachers; its height is teacher salary in period t); the dashed parabola in the bottom graph tracks per student education quality in period t corresponding to education policy options similar to Fig. 1. The solid sloped line in the top graph represents income distribution in period t + 1. It exhibits a kink further to the left, i.e., a lower college attendance ability cut-off than in period t, consistent with Theorem 1. Furthermore, the comparison of the solid and dashed lines illustrates the fact expressed by Proposition 3 that the benefit derived by a skilled individual from the quality of basic education grows disproportionately with ability. This implies the rising relative cost of highly able teachers and results in the shift of the quantity-quality trade-off depicted by the position of the bar corresponding to the optimal education policy choice in period t + 1 to the left of the bar corresponding to period t, as stated in Theorem 1. The position of the solid parabola relative to the dashed one reflects lower optimal relative quality of teachers in period t + 1, consistent with Theorem 1, and higher per student education quality than in the previous period as per Lemma 1.

Much of the recent growth literature which presents the evidence of increasing dispersion of incomes of skilled workers over the last four decades of the XX century attributes this to the skill biased nature of technological change (see, e.g., Acemoglu, 1998, 2002 and Galor and Moav, 2000). A competing view, supported, e.g., by the findings of Eckstein and Nagypal (2004), gives more credence to the evidence of increasing dispersion in educational attainments. This view is consistent with our Proposition 3 where the rising income inequality was derived exclusively from the growing provision of basic education, which has unequal impact on individuals across the distribution of abilities because of the complementary relationship between education, which has unequal impact on individuals across the distribution of abilities because of the complementary relationship between education, and its implications for inequality was documented for the case of Britain by LeGrand (1982).

We will next verify this conjecture, that there is an additional skill-biased technological change (SBTC) effect on the cost of talent and thereby on the optimal quality and quantity of teachers, distinct from the rising talent premium effect.

We will now introduce the exogenous skill biased technological change into the model given by positive shocks to the skill premium coefficients \(\theta_t\), and will explore its effects on the education policy variables and the quality of education. Specifically, we consider the recursive dynamic equilibrium (RDE) corresponding to the original exogenously given sequence \(\{\theta_t\}_{t=0}^\infty\) (the benchmark sequence, which may, in particular, be stationary) and assume that the productivity augmentation of skilled labor receives a positive shock from time \(t_0\) on, i.e., that for \(t = t_0, t_1, \ldots\) the values \(\theta_t\) are replaced with some \(\theta_t > \theta\). We will characterize the effect of this exogenous change on the RDE, particularly on the education policy variables. We obtain the following comparative dynamics result (see Appendix 2 for the proof based on differentiating the expressions (22) and (24) and applying an induction argument).

**Theorem 2. (The Effect of Skill Biased Technological Change).** Consider the comparative dynamics experiment described above, where skill premium coefficients \(\theta_t\) receive an exogenous positive shock from period \(t_0\) on. The corresponding recursive dynamic equilibrium, relative to the original RDE, will be characterized, for \(t \geq t_0\) by

- lower quantity of teachers \(q_{t+1}^{CB}\);
- lower aggregate quality of teachers \(q_t^{CB}\) and therefore
- lower quality of basic education \(E_t^{CB}\).

Note the negative effect on both the number and aggregate quality of teachers which is due to an upward shock to the cost of skilled labor. The Theorem thus shows that the technological change biased toward skilled labor will have a detrimental (SBTC) effect on the absolute quality of basic education, exacerbating the negative rising talent premium effect of a secular downward trend in the relative quality of teachers stated in Theorem 1. These results will apply, in particular, when education tax rate \(\tau\) stays constant, which means that education budget grows at the rate of GDP growth.

This leads to an important implication of our analysis: given the negative impact of rising skill premium on the quality of education, a policy aimed at neutralizing this effect would require an increase

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18 The effect of rising opportunity cost of teaching is particularly strong among college educated women, as documented by Lakdawalla (2006) among others, due to expanding non-teaching job market opportunities for women. This fact can be captured in our model, for example, by introducing a gender wage gap in non-teaching occupations. As the gap exogenously declines over time, the opportunity cost of teaching increases especially fast for women, which adds impetus to the trend in favor of hiring larger quantity of lower quality teachers described above.

19 A somewhat similar argument for the magnifying effect that greater public education funding may have on income inequality is advanced by Glenn and Kaganovich (2003) in the presence of complementarity between public and parental private inputs, imperfect altruism, and borrowing constraints. The fact of such complementary relationship and its implications for inequality was documented for the case of Britain by LeGrand (1982).
in funding of education (assuming no change in the institutional setup of the school system and teachers’ labor market) at a rate faster than GDP growth, i.e. raising the fraction of GDP devoted to education. Note that Rangazas (2002), in his accounting of sustained growth of the U.S. economy since 1870, while attributing its biggest share to human capital accumulation, distinguishes between an unsustainable (“transitional”) component due to the rise of education expenditures as a share of GDP, and the sustainable one caused by their balanced growth proportionately to that of GDP, as well as the other two “sustainable” factors: intergenerational human capital externality and a secular trend of technological change. Our results, however, suggest that these factors are intertwined. Indeed, the rising “talent premium” and technological progress lead to increasing relative cost of maintaining a given teacher quality standard, thereby if a steady growth of human capital were to be sustained, educational expenditures would have to rise faster than GDP. While this may call into question the separation between the “transitional” and “sustainable” factors in human capital accumulation, the distinction is limited to the current education technology, which is itself plausibly undergoing transition to a new paradigm.

6. Dynamics under the merit pay regime: comparative results

The uniform pay requirement for teachers imposed by the collective bargaining obviously ensures that most teachers are paid above their marginal product. This clearly creates inefficiency in the allocation of funding of basic education. Therefore one can expect that in the absence of the distortion, under the merit pay regime, a higher quality of basic education could be produced given the same fundamentals at the outset. We will indeed first of all establish such static and dynamic comparisons and will also provide a comparative characterization of the allocation of educational resources in terms of the quantity and quality of teachers under the two regimes. Specifically, we establish the following fact\(^{20}\):

**Theorem 3.** Compare the dynamics of educational and economic outcomes of the two economies starting from the same initial per student basic education quality \(E_{t-1}\) provided to generation \(G_0\) at \(t = 0\) and given the same sequence of tax rates \((\tau_t)_{t=0}^\infty\) whose only difference is the teacher pay arrangement: one with collective bargaining resulting in the uniform teacher wage, and the other with merit pay for teachers. Then

(i) the merit pay arrangement yields relatively higher quality of education at all times, i.e., \(E_t^{MP} > E_t^{CB}\) is true at \(t = 0, 1, \ldots;\)

(ii) the higher quality of basic education under the merit pay regime is attained through the allocation of educational resources characterized by the higher quantity of teachers as well as their higher absolute quality (i.e., their aggregate human capital) under the merit pay regime, i.e., the inequalities \(z_t^{MP} > z_t^{CB}\) and \(q_t^{MP} > q_t^{CB}\) are true at \(t = 0, 1, \ldots;\)

(iii) the relative quality at the top of teacher pool is also higher under the merit pay regime subject to an additional condition that the contribution (the elasticity in the basic education production function (8)) \(\gamma\) of the teacher–student ratio is not too small relative to that of the teacher quality given by parameter \(v\). Specifically, under the condition that \(\frac{v}{z_t} > \frac{1}{1-\sqrt{1-\tau_t}}\), the inequality \(\bar{a}_t^{MP} > \bar{a}_t^{CB}\) is true at \(t = 0, 1, \ldots;\)

Remarks

1. The parametric condition imposed in part (iii) of the Theorem can be best understood when combined with the conditions \(v \geq 1\) and \(\frac{v}{z_t} \leq 1 - \tau_t\) imposed by Assumption 2 in the previous section. While the latter imposes a natural upper bound on the relative contribution of teacher–student ratio (see the discussion following Assumption 2), the new condition sets a plausible lower bound on it, consistent with the upper one.

2. The basic education quality dominance result in part (i), when applied to formula (6), implies that the college-attendance ability cut-off \(a^*_n\) will be always relatively lower under the teachers’ merit pay regime. Combined with the result of Lemma 2, the interiority of the teacher cohort under the collective bargaining regime, this means that the relative quality at the bottom of the distribution of teachers will always be inferior to that under the collective bargaining. Coupled with the part (iii) result, and consistent with the fact \(z^{MP} > z^{CB}\) in part (ii), this offers a compelling and realistic comparison of the distributions of teacher quality across the regimes: the merit pay arrangement is characterized by a strictly broader range of teacher quality along with smaller average class size, resulting in the overall higher per student education quality.

3. The result similar to that of Theorem 2 on the negative effect of skill biased technological change on the quantity and quality of teachers and on the quality of basic education is also true in the merit pay case. We state this here without proof, which can be obtained along the lines of the proof of Theorem 3 using the expressions (11) and (20).

Finally, based on Theorem 3, we obtain the following characterization of the dynamics of educational variables under the merit pay regime analogous to those established in Theorem 1 for the collective bargaining regime. (The proof, also placed in Appendix 2, uses the line of argument similar to that of Theorem 3.)

**Theorem 4.** (Dynamics of the Quantity and Quality of Teachers under Merit Pay). The recursive dynamic equilibrium (RDE) exhibits the following evolution of inputs in basic education:

- the quantity \(z_t^{MP}\) of teachers grows over time, so do the absolute aggregate quality of teachers \(q_t^{MP}\), as well as the quality of basic education \(E_t^{MP}\); and
- the relative quality of teachers characterized by the range of their innate abilities falls: both the upper and the lower cut-offs \(\bar{a}_t^{MP}\), \(a_t^{MP}\) decrease over time.

**Theorem 4** shows that the phenomenon of the quantity–quality trade-off in the dynamics of teacher staffing is also in effect under the merit pay regime. This is indeed due to the same factor, the rising talent premium (see in the discussion following Theorem 1) which is at work under either of the teacher pay regimes. However, according to part (iii) of Theorem 3, the quantity–quality trade-off in the merit pay case is characterized by a less significant relative quality decline (and stronger absolute quality gains, as per part (ii) of Theorem 3) than occurring under collective bargaining. More precisely, the collective bargaining regime exacerbates the teacher quantity–quality trade-off, particularly in terms of the loss of the most able teachers.

Thus, the combination of our main results obtained in Theorems 1–4 offers a taxonomy of factors contributing to the dynamics of quantity–quality trade-off in teacher staffing: (i) human capital attainments rise over time, disproportionately so for highly able individuals (“the rising talent premium”), which leads to increasing opportunity cost of teaching for such individuals and pushes the trade-off in favor of quantity under both teacher pay regimes according to Theorems 1 and 4; (ii) Theorem 3 shows that the wage

\(^{20}\) Its proof placed in Appendix 2 is based on comparing the magnitudes of the quantity and quality of teachers under the two regimes as well as the rates of their growth obtained in Propositions 1 and 2 as well as Lemma 4, built into a chain of induction arguments.
compression under the collective bargaining regime exacerbates the loss in teacher quality; (iii) the rise of skill premium, resulting from technological change, is an additional, independent factor, which strengthens the quantity–quality trade-off in favor of the former, according to Theorem 2.

7. Conclusions

Over the last forty years, education policy in the U.S. has changed significantly, focusing in particular on lowering the student–teacher ratio. We have developed a model which offers an insight into this evolution by relating it to the changes in the US economy characterized by rising skill premium and overall income inequality. Our model predicts that as incomes of college educated individuals rise and become more dispersed (the phenomenon we call the “rising talent premium”), education policy-makers are forced to adjust relative teacher salaries and thereby the quality standards. Education quality is optimized by lowering relative quality of teachers while increasing their numbers. This causes the higher ability college graduates to choose private sector employment which offers higher reward to skilled workers. We show moreover that this loss of the more talented workers by the teaching profession is exacerbated by the teacher wage compression resulting from the teacher union imposed collective bargaining agreements.

Further, we argue that a rise in skill premium caused, in particular, by skill biased technological change will deepen the negative trends in the relative quality of education. Indeed, the labor of college graduates will further appreciate relative to the average wage and hence relative to the tax revenue. Countering this trend would therefore require an increase in the share of GDP spent on basic education, assuming that the institutional setup of the school system remains unchanged.

Our finding that skill biased technological change can have a negative effect on the quality of education is an interesting case of negative feedback, since SBTC literature points to the rise in the supply of skill due to growing availability of education as its underlying cause. Furthermore, this leads to an issue which appears important for future research on the aggregate long term effects of SBTC: as technical change brings about productivity gains, one needs to factor in its effects on the cost and quality of education and the corresponding policy responses in order to assess the full long-term impact.

Appendix 1

Glossary of notation

- $n$: Fraction of time devoted to acquired advanced education
- $Y_t$: Total output in the goods sector in period $t$
- $w$: Total Factor Productivity in goods production sector
- $l_t$: Aggregate unskilled human capital in goods production in period $t$
- $τ_t$: Labor income tax rate in period $t$
- $MP$: Merit pay regime
- $CB$: Collective bargaining regime
- $τ^*_t$: Total government revenue in period $t$ for $j=MP, CB$
- $A$: Upper bound on innate ability
- $a_*(o)$: The innate ability of individual $o$
- $a^*_t$: Ability cut-off level for attending college in period $t$ for $j=MP, CB$
- $h^*_t$: Human capital attainment of the lowest ability college graduate in period $t$ for $j=MP, CB$
- $l_j(o)$: Unskilled individual’s level of human capital in period $t$ for $j=MP, CB$
- $h_j(o)$: Skilled individual’s level of human capital in period $t$
- $H_t$: Aggregate supply of skilled human capital in period $t$
- $H^*_t$: Aggregate supply of skilled human capital in goods production in period $t$
- $θ_t$: Goods production sector productivity augmentation of skilled human capital (skill premium) in period $t$
- $l_j(o)$: Wage income of individual $o$
- $C$: Productivity coefficient of compulsory basic education
- $e_j$: Public first-stage education quality in period $t$ for $j=MP, CB$
- $b$: Coefficient of in-college depreciation of pre-college human capital.
- $B$: Productivity coefficient of higher education
- $h^*$: Human capital threshold for admission to college
- $Σ_j$: Set of individuals employed as teachers in period $t$ for $j=MP, CB$
- $q_j$: Aggregate teacher quality in period $t$ for $j=MP, CB$
- $γ$: Total number of teachers (fraction of teachers in overall population) in period $t$ for $j=MP, CB$
- $ν$: Elasticity of the quantity of teachers in the production of basic education quality
- $ε$: Elasticity of the quantity of teachers in the production of basic education quality
- $l^*_t$: Teacher salary in period $t$ under the collective bargaining regime
- $γ_j$: The lowest ability level among teachers in period $t$ for $j=MP, CB$
- $h_j^*$: The lowest human capital level among teachers in period $t$ for $j=MP, CB$
- $a_j^*$: The highest ability level among teachers in period $t$ for $j=MP, CB$
- $h_j^*$: The highest human capital level among teachers in period $t$ for $j=MP, CB$

Appendix 2. Supplementary data

Supplementary data to this article can be found online at doi:10.1016/j.jpubeco.2011.10.003.

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