Modal dispersion characterization of multimode fibers

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***Abstract*— The mode-dependent signal delay method can be used for the characterization of modal dispersion of multimode fibers. We revise the formalism used by this method and quantify measurement errors due to receiver thermal noise.**

Keywords—Modal dispersion; multimode fiber characterization.

I. INTRODUCTION

Data traffic is expected to grow exponentially in the near future due to cloud computing, the internet of things, and 5G wireless [1]. This traffic increase can cause severe congestion in the fiber-optic internet backbone network, an event dubbed [2] 'capacity crunch.' It is possible to mitigate the capacity crunch by using new optical fiber types, e.g., multimode and multicore optical fibers (referred to collectively in the following as 'multimode fibers' (MMFs)) [3].

Modal dispersion (MD) due to the difference in the propagation constants of various modes in MMFs and mode coupling at random points along the signal path [4] induce pulse distortion and intersymbol interference (ISI) at the decision instant. In long-haul optical communications systems, ISI can be largely compensated using adaptive electronic equalization after coherent detection [1]. To reduce the computational complexity of these equalizers, the ISI duration must be minimized [1]. To this end, MMFs with negligible uncoupled differential mode group delay (DMGD) compared to the average group delay and strong mode coupling are beneficial [1]. Due to the quasi-degeneracy of modes in these fibers, MD measurement techniques initially conceived for conventional 850-nm MMFs, typically exhibiting high MD, are inadequate. Instead, since MD can be viewed as a generalization of polarization-mode dispersion (PMD) in single-mode fibers (SMFs) [5]-[9], variants of PMD measurement techniques can be used for MD characterization [10], [11].

The recently-proposed mode-dependent signal delay method (MD-SDM) [10] consists in launching short pulses corresponding to different combinations of modes at the fiber input and measuring the corresponding group delays at the fiber output. From these measurements, it is possible to estimate the components of the MD vector. The latter encapsulates both the DMGDs and the principal modes (PMs) of long MMFs [12].

In its initial formulation [10], the MD-SDM was based on an analytical relationship that used unconventional definitions for the MD and generalized Stokes vectors and overlooked the spectral averaging of the MD vector due to its variability across the pulse spectrum. In this paper, we correct the aforementioned deficiencies of the analytical expression of [10] and quantify errors in the estimation of the input MD vector due to receiver thermal noise.

II. MATHEMATICAL MODEL AND MEASUREMENTS

The experimental setup used by the MD-SDM is shown in Fig. 1 (a). Indicative cartoons of the input and output pulses are shown in Fig. 1(b) (orange and blue lines, respectively). We want to link the group delay τ_g to the MD vector and the unit Stokes vector representing the launch combination of modes.

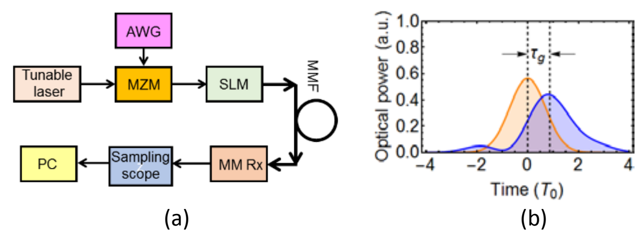

Fig. 1 (a) Experimental setup for MD characterization of MMFs using the MD-SDM (Abbreviations: AWG: arbitrary waveform generator, MZM: Mach-Zehnder modulator, SLM: spatial-light modulator, MMF: multimode fiber, MM Rx: multimode receiver, PC: computer); (b) Input Gaussian pulse with unit energy (orange line) and output pulse (blue line) (T_0=half-width at 1/e power point).

The electric field of a monochromatic optical wave at a given time instant and position in a N-mode MMF can be expressed as the vector sum of individual modes with complex coefficients $c_k, k = 1, \dots, N$ representing the mode excitations [7]. We define the generalized unit Jones vectors as $|s\rangle \triangleq [c_1, \dots, c_N]^T$, where T denotes transpose. Combinations of propagation modes are described by such vectors. Linear optical devices are represented by $N \times N$ complex matrices called generalized Jones matrices, similar to the two-dimensional case. Their action results in a simple multiplication of the input Jones vector by the corresponding Jones matrix.

To transition to Stokes space, we first define the $N^2 - 1$ generalized Gell-Mann matrices $\mathbf{\Lambda}_i$ as in [12]-[14]. Then, in analogy to the Pauli spin vector [6], we define the Gell-Mann spin vector $\mathbf{\Lambda} \triangleq [\mathbf{\Lambda}_1, \dots, \mathbf{\Lambda}_{N^2-1}]^T$. Finally, we can compute the unit Stokes vector $\hat{s}$ corresponding to the unit Jones vector $|s\rangle$ from the quadratic form proposed by [8], [9] $\hat{s} \triangleq \sqrt{N/[2(N-1)]}\langle s|\mathbf{\Lambda}|s\rangle$, where the normalization coefficient $\sqrt{N/[2(N-1)]}$ yields $\|\hat{s}\| = 1$ [12].

Consider the ideal case of an N-mode MMF with negligible mode dependent loss. The fiber transfer function can be described by a unitary matrix $\mathbf{U}(\omega)$. The input PMs are the eigenstates of the group-delay operator [12],

$$i\mathbf{U}^\dagger(\omega)\mathbf{U}_\omega(\omega)|p_i(\omega)\rangle = \tau_i(\omega)|p_i(\omega)\rangle, \qquad (1)$$

where the index ω denotes differentiation with respect to the angular frequency, a raised dagger denotes the adjoint matrix, and $\tau_i(\omega)$ are the DMGDs of the input PMs $|p_i(\omega)\rangle$, i=1,…,N.

The input MD vector $\vec{\tau}_s$ is defined here as [12]

$$i\mathbf{U}^{\dagger}(\omega)\mathbf{U}_{\omega}(\omega) \triangleq \sqrt{(N-1)/(2N)}\,\vec{\tau}_s(\omega)\boldsymbol{\Lambda}, \qquad (2)$$

where using $\sqrt{(N-1)/(2N)}$ is a convention adopted for compatibility with the PMD case ($N=2$) [6].

Following [6], [10], we can now prove that the group delay τ_g of an optical pulse corresponding to a given combination of launch modes is related to the spectrally-averaged input MD vector $\langle\vec{\tau}_s(\omega)\rangle$ and the input launch state $\hat{s}$ in Stokes space [12]

$$\tau_g = \tau_0 + [(N-1)/N]\langle\vec{\tau}_s(\omega)\rangle\hat{s}, \qquad (3)$$

where the pulse group delay τ_g is defined as the first moment in time [6] and τ_0 is the average group delay.

Our expression (3) differs from the initial expression (16) of Milione et al. [10] on two important points: the input MD vector $\vec{\tau}_s$ is spectrally-averaged and there is a corrective multiplicative factor of $(N-1)/N$ in its length.

Assume that we use optical pulses with sufficiently narrow spectrum so that $\langle\vec{\tau}_s(\omega)\rangle \simeq \vec{\tau}_s(\omega)$. First, τ_0 can be estimated by launching pulses corresponding to N arbitrary orthogonal states in Jones space and averaging the corresponding group delays τ_g [12]. Subsequently, expression (3) can be used to form a $(N^2-1)\times(N^2-1)$ system of linear equations at each frequency by launching (N^2-1) linearly independent input states in Stokes space and measuring the corresponding group delays τ_g. More specifically, assume that we launch the states $\hat{s}_i$ and we observe the group delays $\tau_{g,i}$ i=1,…, N^2-1. We form $\boldsymbol{S} \triangleq \left[\hat{s}_1, \dots, \hat{s}_{N^2-1}\right]^T$, $\mathbf{T}_{\mathrm{g}} \triangleq \left[\frac{\mathrm{N}}{\mathrm{N}-1}\right]\{\left[\tau_{g,1}, \dots, \tau_{g,N^2-1}\right]^T - \tau_0\}$. Then, $\vec{\tau}_s(\omega)$ is given by $\vec{\tau}_s(\omega) = \mathbf{S}^{-1}\mathbf{T}_{\mathrm{g}}$.

The presence of thermal noise at the receiver can lead to a random offset $\boldsymbol{\delta}\mathbf{T}_{\mathrm{g}}$ in the estimation of the DMGD matrix $\mathbf{T}_{\mathrm{g}}$. Consequently, there is an error in the estimate of $\vec{\tau}_s(\omega)$, namely $\delta\vec{\tau}_s = \mathbf{S}^{-1}\boldsymbol{\delta}\mathbf{T}_{\mathrm{g}}$. As a figure of merit of the measurement error, we can compute the variance in the estimation of the length of the MD vector from the following relationship

$$\sigma^2_{\|\delta\vec{\tau}_s\|} = \sigma^2_{\delta T_g}\sum_{k=0}^{N^2-1}\lambda_\kappa^{-2}, \qquad (4)$$

where λ_κ are the singular values obtained from the singular value decomposition of the matrix $\mathbf{S}$ and $\sigma^2_{\delta T_g}$ is the variance of each of the components of the column vector $\boldsymbol{\delta}\mathbf{T}_{\mathrm{g}}$.

Fig. 2 illustrates the steps of the MD-SDM. We simulate a concatenation of two MMF segments supporting the LP_{01} and LP_{11} modes. We assume that the states of polarization within each mode are completely degenerate and set τ_0=0. Then, the fiber can be characterized using only eight measurements (N=3). Fig. 2(a) shows the electric field and intensity profiles of a linear combination of LP_{01} and LP_{11} modes at the fiber input. It is impossible to find mode combinations in Jones space that correspond to eight orthogonal vectors in Stokes space [13]-[15]. Fig. 2(b)-(c) show pulse output waveforms and calculated PMs, respectively, using a set of eight linearly independent Stokes states selected in pairs from four mutually unbiased bases [15] to reduce the variance in (4) compared to previously proposed arbitrary launch states [16]. Further accuracy improvement can be achieved using launch Stokes states computed by numerical minimization of (4).

III. Summary

In this article, we revised the MD-SDM formalism for the characterization of MMFs. We provided a rigorous analytical expression linking the group delay of an optical pulse to the spectrally-averaged input MD vector. Finally, we analytically calculated the variance in the estimation of the length of the input MD vector due to receiver thermal noise.

The author would like to thank T. A. Nguyen, D. Nolan, W. A. Wood, and J. Yang of Corning for fruitful discussions.

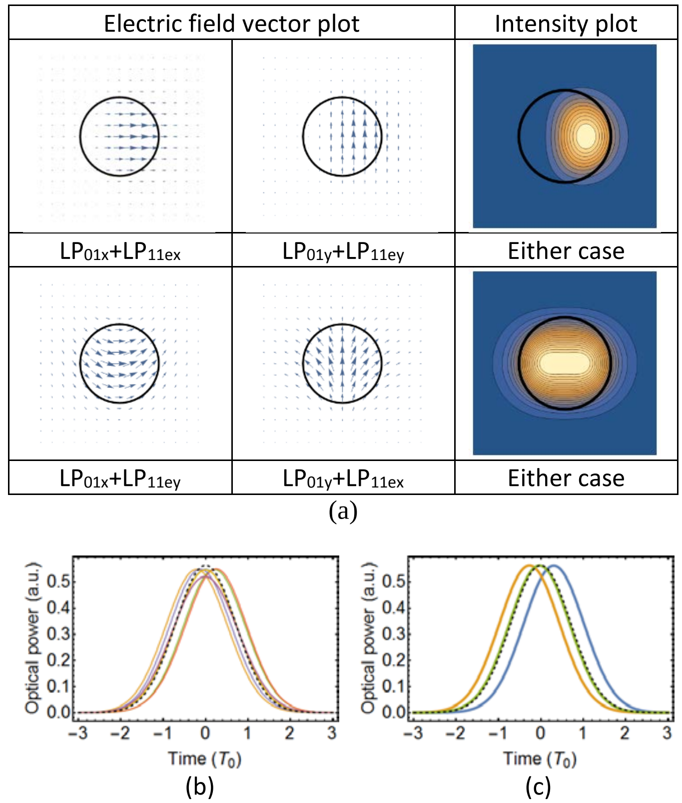

Fig. 2 Characterization of a trimodal fiber using the MD-SDM: (a) Example of a specific launch mode; (b) Output pulses corresponding to 8 such modes selected from 4 mutually unbiased bases [15]; (c) Output pulses corresponding to the PMs for N=3 (solid lines). The launched Gaussian pulse with unit energy is shown by a dashed line.

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