Comparison of analytical models for the nonlinear noise in dispersive coherent optical communications systems

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Abstract: We compare the accuracy of recently-published closed-form expressions for the nonlinear noise in coherent PDM-QPSK and PDM-16QAM optical systems without in-line chromatic dispersion compensation against the numerical solution of the vector nonlinear Schrödinger equation and show their limitations.

1. Introduction

Several computationally-efficient models for the calculation of the impact of nonlinear effects on the performance of long-haul coherent optical communications systems have been recently proposed, e.g., [1]-[5]. Models [1]-[3] essentially share the same formalism: They assume that signal distortion arising from fiber Kerr nonlinearity in dispersive coherent optical communications systems can be represented by an effective, zero-mean, additive Gaussian noise. This assumption allows one to calculate the nonlinear noise variance by solving the Manakov equation in the frequency domain using the undepleted pump approximation, while neglecting polarization mode dispersion (PMD), polarization-dependent loss (PDL), and signal-noise interactions.

Despite their similarities, [1]-[3] derive different expressions for the nonlinear noise variance, due to different simplifications used for mathematical convenience. A comprehensive comparative study of the models [1]-[3] was never performed in the literature. Each model was validated separately using diverse assumptions on the modulation format and other system parameters.

In this paper, we compare the models [1]-[3] under the same conditions, i.e., for polarization division multiplexed (PDM) quadrature phase shift keying (QPSK) and 16-ary quadrature amplitude modulation (16-QAM) coherent optical communications systems, with 10-100 km fiber span lengths. We show that model [3] is the closest to numerical simulations, while model [1] overestimates inter-channel nonlinearities, and model [2] coincides with [1] for long span lengths but is unreliable for short ones.

2. Theoretical model

Consider a coherent optical communications system without in-line chromatic dispersion compensation (Fig. 1). The transmission link of total length L is composed of N_s identical spans. In the beginning of each span, there is a fiber of length ℓ_s, with an effective area A_{eff}, a nonlinear index coefficient n_2, a group velocity dispersion (GVD) parameter β_2 (dispersion parameter D) and an attenuation coefficient a. It is followed by an optical amplifier of gain $G = e^{a\ell_s}$ and noise figure F_A.

The WDM signal is composed of N_{ch} WDM channels carrying M-ary quadrature amplitude modulation (M-QAM). Let P be the total average launch power per channel (in both polarizations) and R_s the symbol rate. We evaluate the performance of the center WDM channel at carrier frequency f_0 (carrier wavelength λ). Analytical expressions below assume a matched filter receiver.

Models [1], [2] are used here with slight modifications. More specifically, we combine the formalism of [1], [2] to derive a common expression for the nonlinear noise variance in Nyquist PDM WDM systems

$$\sigma_{NL}^2 = (2/3)^3 \gamma^2 P^3 \varepsilon(N_s) L_{eff,\infty} \ln\left(\pi^2 |\beta_2| L_{eff,\infty} N_{ch}^2 R_s^2\right) / \left(\pi \ |\beta_2| R_s^2\right) \tag{1}$$

where $\gamma = 2\pi n_2 / \left(\lambda A_{eff}\right)$ and $L_{eff,\infty}$ is the effective length for infinitely large spans, defined as $L_{eff,\infty} = 1/a$. The term $\varepsilon(N_s)$ describes the dependence of nonlinear noise on the number of fiber spans and is equal to $\varepsilon(N_s)=N_s$ in [1] and $\varepsilon(N_s)=N_s+2\sum_{k=1}^{N_s-1}(N_s-k)\exp(-ka\ell_s)$ in [2] (i.e., linear vs. superlinear accumulation of the nonlinear noise, respectively).

Expression (1) is strictly valid for symbol-rate-spaced rectangular individual channel spectra without frequency guard bands among WDM channels and long fiber spans. In the following, (1) is used outside of its limits of validity, e.g., for conventional WDM systems with finite guard bands, non-rectangular spectral shaping, and various fiber span lengths.

In addition, we adopt a more generic formula for the amplified spontaneous emission (ASE) noise variance than the one in [1]-[3], which is accurate both for high- and low-gain amplifiers [6], i.e., $\sigma_{ASE}^2 = hf_0 N_s (GF_A - 1) R_s$, where h is Planck's constant.

Combining the above expressions for the nonlinear and ASE noise variances, for the case $\varepsilon(N_s) = N_s$, we derive an approximation for the optimal fiber span length that maximizes the optimum Q-factor

$$\ell_{s,opt} = \left[3/(2a)\right]\left[1 - F_A^{-1}\mathrm{e}^{-3/2} - (3/2) F_A^{-2}\mathrm{e}^{-3} - (27/8) F_A^{-3}\mathrm{e}^{-9/2} \ldots\right] \tag{2}$$

It is worth noting that the optimal fiber span length depends, to first-order, on the attenuation coefficient exclusively, while higher-order corrections depend on the attenuation coefficient and the amplifier noise figure.

We compare the accuracy of models [1]-[3] against the numerical solution of the vector nonlinear Schrödinger equation using the split-step Fourier method. We study the following cases: (i) an eight-channel, 50-GHz, 9,000-km, 25-GBd, NRZ PDM-QPSK coherent optical system, using a representative commercially-available large effective area pure silica core fiber (PSCF); and (ii) an eight-channel, 50-GHz spacing, 28-GBd, NRZ, PDM-16-QAM coherent optical system, at various transmission distances of standard single-mode fiber (SSMF). In the former case, a =0.038 1/km, γ=0.76 1/(W km), β_2=26.09 ps^2/km, F=4.5 dB, whereas, in the latter case, a=0.05 1/km, γ=1.17 1/(W km), β_2=21.14 ps^2/km, F=5 dB, at carrier frequency f_0. The coherent receiver's optical

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and electrical filters are represented by 3-rd order Butterworth and 5-th order Bessel filters with 50-GHz and 15-GHz 3-dB bandwidth, respectively. PMD, PDL, phase and ADC noises are neglected (referred to as ideal case), unless otherwise stated.

3. Results and discussion

Fig. 2(a),(b) show plots of the maximum Q-factor and the optimal launch power, respectively, as a function of span length, for the 8-channel, WDM NRZ PDM-QPSK coherent optical system under study. From Fig. 2 (a), we observe that the model based on [3] is in excellent agreement with the maximum Q-factor and optimal launch power predicted by simulation. Moreover, the analytical model [1] in the proposed form (1) closely follows the shape of the average maximum Q-factor as a function of the span length (despite a multiplicative constant mismatch). For instance, the optimum span length predicted by simulation is 35.68 km. Expression (2), which is derived from model [1], predicts 36.14 km using the second order approximation. The relative error is 1.27%. A comparison for different commercially-available fibers (not shown here) indicates that (2) always yields a small relative error (i.e., <14%). In contrast, Fig. 2(b) reveals that the results of the analytical model of [1] for the optimal launch power differ by ~1 dB from the results of numerical simulation at the optimum span length. This is due to the assumption of symbol-rate channel spacing and equal strength of spectral components in (1), which tends to overestimate inter-channel nonlinearities and leads to lower launch powers. Finally, model [2] is identical to model [1] for large span lengths $(a\ell_s \to \infty)$ but fails for short fiber spans.

To further illustrate the differences between models, we plot the Q-factor vs. launch power for the NRZ PDM-16-QAM coherent optical system with 100-km and 50-km spans of SSMF (Fig. 3). The maximum Q-factor predicted by model [3] (green dashed curve) differs by 0.2-0.5 dB from the one given by Monte Carlo simulation (open circles) for the ideal case. This small discrepancy is largely due to the omission of the impact of adjacent channel crosstalk, which is non-negligible for 50-GHz spaced, 28-GBd, NRZ, PDM-16-QAM channels [3]. As before, the results of the analytical models [1], [2] (red and blue dashed curves, respectively) are conservative, both regarding the optimal Q-factor (by >0.5 dB) and the optimal launch power (by ~1.25 dB). Moreover, the difference between models [1], [2] increases for shorter span lengths.

Finally, Fig. 4 shows contour plots of the maximum reach at 11.5 dBQ (i.e., to account for typical 7%-overhead forward error correction (FEC) code limit and allowing for 3-dB margin) vs. attenuation coefficient and fiber effective area for an 8-channel, 50-GHz spaced, 28-GBd, NRZ PDM-16-QAM coherent optical system in the ideal case. Only families of contours for 1500/2600 km are depicted to avoid clutter. Similarly to the trend observed in Fig. 3, the results of Monte Carlo simulation (in black) lie in between those based on analytical expressions [1], [2] (in red, blue), and the ones based on [3].

In conclusion, we compared the accuracy of the models [1]-[3] under the same conditions, for the performance evaluation of WDM NRZ PDM-QPSK and PDM-16-QAM long-haul coherent optical communications systems without in-line chromatic dispersion compensation. Although used here outside of its limit of validity, model [1] is still adequate for coarse system performance predictions. Model [2] coincides with model [1] for large span lengths but is unreliable for short span lengths. Finally, model [3] is fairly close in accuracy to the full numerical solution of the vector nonlinear Schrödinger equation for QPSK and slightly optimistic for 16-QAM. It can be used either for the performance evaluation of idealized coherent optical systems or as a precursor to realistic numerical simulations, in order to set the launch optical power per channel to its optimal level.

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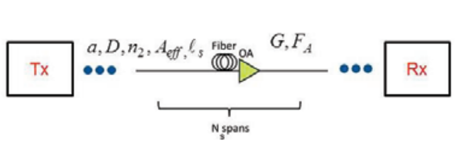

Fig. 1 Representative long-haul coherent optical communications system without in-line chromatic dispersion compensation (Symbols: Tx: transmitter, Rx: receiver, OA: optical amplifier).

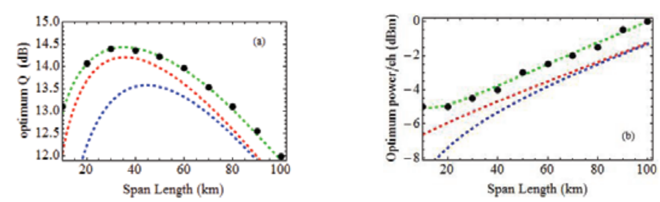

Fig. 2 (a) Average maximum Q-factor of inner channels (#2-#7) and (b) Optimum launch power per channel vs. span length for an 8-channel, 50-GHz, 25-GBd, NRZ PDM-QPSK coherent optical system, with 9,000 km link of PSCF (Curves: red: [1], blue: [2], green: [3], points: simulation/Gaussian semi-analytical method).

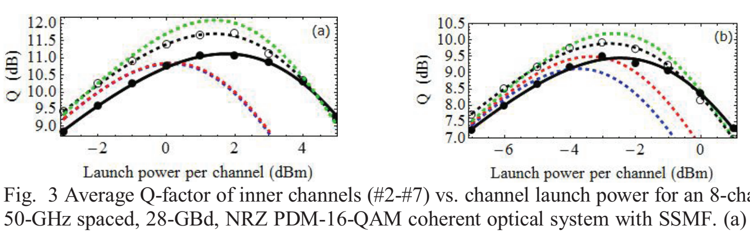

Fig. 3 Average Q-factor of inner channels (#2-#7) vs. channel launch power for an 8-channel, 50-GHz spaced, 28-GBd, NRZ PDM-16-QAM coherent optical system with SSMF. (a) Total link length: 600 km, 100-km spans; (b) 2400 km, 50-km spans; (Symbols: Curves: red: [1], blue: [2], green: [3], points: Monte Carlo simulation (open: ideal case without PMD, PDL, phase noise, and ADC noise, filled: non-ideal case with the above effects turned on)).

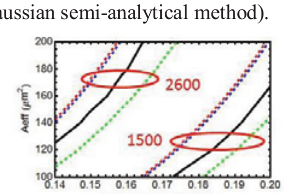

Fig. 4 Contour plots of the maximum signal reach (in km) for the system of Fig. 3 with 100-km fiber spans (Contours for 1500-km/2600-km link lengths: red: [1], blue: [2], green: [3], black: Monte Carlo simulation).