Constrained LMS phase noise estimation algorithm for coherent optical M-QAM intradyne receivers

I. Roudas (1,2)
(1) Corning Inc., Corning, NY 14831, USA; E-mail: roudasi@corning.com
(2) Dept. of Electrical & Computer Engineering, University of Patras, Rio 26500, Greece; E-mail: roudas@ece.upatras.gr

Abstract: We analyze and optimize the performance of a single-tap constrained decision-directed least-mean square algorithm for computationally-efficient phase noise estimation in coherent optical intradyne receivers intended for the detection of arbitrary M-QAM constellations.
OCIS codes: (060.2330) Fiber optics communications; (060.1660) Coherent communications

1. Introduction

Several digital carrier phase estimation (CPE) algorithms have been proposed for coherent optical M-ary Quadrature Amplitude Modulation (QAM) intradyne receivers [1]-[5]. Feed-forward CPE algorithms [1], [2] are parallelizable and, therefore, amenable to real-time digital signal processing (DSP) implementation at high symbol rates. In contrast, for computer simulations and off-line processing, recursive CPE algorithms [3]-[5] are, by far, superior, in terms of computational efficiency, and exhibit comparable performance to their feed-forward counterparts, at least of the same order of magnitude, in terms of the maximum allowable 3-dB laser linewidth-symbol period product.

In this paper, we theoretically study the performance of a single-tap constrained decision-directed least-mean square (DD-LMS) CPE algorithm for coherent optical M-QAM receivers [5], [6]. The distinctive feature of this particular DD-LMS algorithm is that its single complex tap is constrained on the unit circle of the complex plane. This yields increased robustness compared to alternative DD-LMS CPE algorithms, e.g., [4]. We derive analytical expressions for the update of the single complex coefficient, the residual phase noise variance, and the optimum step-size parameter, and we calculate, by simulation, the linewidth tolerance of the algorithm for square 16-QAM.

2. Theoretical model of the constrained DD-LMS algorithm

Fig. 1(a) shows the block diagram of the constrained DD-LMS phase noise estimation algorithm. At each discrete time instant $t_k = kT$, where T is the symbol period, the received signal sample x_k is multiplied by a single complex tap of unit magnitude $c_k = e^{-j\hat{\phi}_k}$, where $\hat{\phi}_k$ is the estimate of the phase noise. The corrected signal sample $y_k = x_k e^{-j\hat{\phi}_k}$ is used to make a decision on the received symbol $\hat{d}_k$ and calculate the error $e_k = \hat{d}_k - y_k$. The LMS algorithm [7] minimizes the instantaneous square error magnitude. The minimization is done recursively using the method of steepest descent [7]: The derivative of $\hat{\xi}_k = |e_k|^2$ with respect to $\hat{\phi}$ is calculated at each time instant t_k and $\hat{\phi}_k$ is updated by taking a small step in the opposite direction of the derivative.

More specifically, the derivative of $\hat{\xi}_k$ with respect to $\hat{\phi}$ is $\hat{\xi}_k' = e_k' e_k^* + e_k \left(e_k^*\right)'$, where primes denote derivatives with respect to $\hat{\phi}$ and stars denote the complex conjugates. By substituting $e_k = \hat{d}_k - y_k$ and $y_k = x_k e^{-j\hat{\phi}_k}$, it is straightforward to show that $e_k' = iy_k$, $\left(e_k^*\right)' = -iy_k^*$. Thus, the derivative of $\hat{\xi}$ is expressed as $\hat{\xi}_k' = 2\Im\left\{e_k y_k^*\right\}$, where $\Im\{.\}$ denotes the imaginary part of a complex number. Hence, the LMS recursion is

$$\hat{\phi}_{k+1} = \hat{\phi}_k - \mu \Im\left\{e_k y_k^*\right\} \tag{1}$$

where μ is the algorithm step-size parameter $(\mu \geq 0)$ and the initial guess for the recursion is $\hat{\phi}_0 = 0$.

It is worth mentioning that the aforementioned algorithm was first proposed by [5], although the recursive relationship (1) was given without proof, and the reader was referred to the generic formalism of the stop-and-go equalizers [6]. Above, we provided a much simpler alternative derivation based on LMS equalization theory. The idea of imposing constraints on equalizer coefficients to enhance convergence was explored before in [8].

To analytically study the properties of the constrained DD-LMS algorithm, we next consider the case of negligible intermediate frequency offset, intersymbol and cross-polarization interference, since these impairments are largely removed by separate DSP modules prior to CPE. At the entrance of the CPE module, the samples of each polarization tributary, can be written, in complex representation, as $x_k = d_k \exp\left(j\phi_k\right) + n_k$, where d_k is the $k-$th symbol, ϕ_k is the laser phase noise, and n_k is a circularly-symmetric additive Gaussian noise, with zero mean and variance $2\sigma_n^2$, due to amplified spontaneous emission (ASE), shot and thermal noises.

Typically, the phase noise is modeled as a discrete Wiener process [9] using the recursion $\phi_k = \phi_{k-1} + \Delta\phi_k$, where ϕ_0 is arbitrary. In the latter expression, the successive phase increments $\Delta\phi_k$ are independent, identically-distributed Gaussian random variables with zero mean and variance $\sigma_{\Delta\phi}^2 = 2\pi\Delta\nu T$, where $\Delta\nu$ denotes the total 3-dB linewidth, i.e., the sum of the 3-dB linewidths of the transmitter laser and the local oscillator.

The magnitude of μ in (1) affects both the feedback loop speed and the residual phase noise $\phi_{r,k}$. We can analytically calculate the residual phase noise variance from (1) assuming that there are almost no symbol errors $\hat{d}_k \cong d_k$ and that $\phi_k - \hat{\phi}_k << 1$. Then, $e_k y_k^* \cong -i|d_k|^2\left(\phi_k - \hat{\phi}_k\right) + d_k n_k^*$ and (1) can be rewritten as $\hat{\phi}_{k+1} = \hat{\phi}_k + \mu|d_k|^2\left(\phi_k - \hat{\phi}_k\right) - \mu\Im\left\{d_k n_k^*\right\}$. We subtract ϕ_k from both sides and after some algebra we obtain

$$\sigma_{\phi_r}^2 = \left[\left(1 - 2m_2\mu + m_4\mu^2\right)\sigma_{\Delta\phi}^2 + 2m_{2,r}\mu^2\sigma_n^2\right] / \left[\mu\left(2m_2 - m_4\mu\right)\right] \tag{2}$$

In (2), we defined the moments $m_\ell = E\left\{|d_k|^\ell\right\}$ and $m_{\ell,r} = E\left\{d_{k,r}{}^\ell\right\}$, where $d_{k,r} = \Re\{d_k\}$ and $\Re\{.\}$ denotes real part.

The optimal value of the step-size parameter, that minimizes the residual phase noise variance, is calculated by taking the derivative of (2) with respect to μ and finding its positive root

$$\mu_{opt} = \left[-m_4 + \sqrt{8m_2{}^2 m_{2,r}\left(\sigma_n^2 / \sigma_{\Delta\phi}^2\right) + m_4{}^2}\right] / \left[4m_2 m_{2,r}\left(\sigma_n^2 / \sigma_{\Delta\phi}^2\right)\right] \tag{3}$$

The results of (3) are accurate in the low noise regime, where the assumptions $\hat{d}_k \cong d_k$ and $\phi_k - \hat{\phi}_k << 1$ are strictly satisfied. In the opposite case, one can numerically optimize the step-size parameter, in order to minimize the symbol error probability, by using the approximate optimal value given by (3) as a starting point.

In the absence of residual phase noise, the bit error probability, for square M-QAM, is given by [10]

$$P_{e|b} = 2(\sqrt{M} - 1)\ \mathrm{erfc}\left(\sqrt{3\rho_s / 2 / (M-1)}\right) / \left(\sqrt{M}\log_2 M\right) \tag{4}$$

where $\rho_s = m_2 / (2\sigma_n^2)$ is the average electronic symbol signal-to-noise ratio (SNR) and erfc(.) denotes the complementary error function [10]. Expression (4) will be used as a reference, in order to evaluate the performance of the constrained DD-LMS algorithm under various operating conditions.

3. Results and discussion

Fig. 1(b),(c) show contours of optimal step, given by (3), and the corresponding normalized minimum residual phase noise standard deviation, given by (2) after substituting (3), respectively, as a function of the total 3-dB linewidth-symbol period product and the average electronic symbol SNR for square 16-QAM. Fig. 1(b) indicates that the optimal step size decreases as the 3-dB linewidth and the average electronic symbol SNR decrease. This is similar to the dependence of the M-th power law algorithm [9], typically used in quadrature phase shift keying (QPSK) receivers, on the averaging block size. The fixed step-size parameter proposed in [5] is not optimal. In Fig. 1(c), we observe that the minimum residual phase noise standard deviation is always larger than $\sigma_{\Delta\phi}^2$.

Fig. 2(a)-(c) show representative phase traces, residual phase noise histograms, and constellation diagrams, respectively, for an average electronic symbol SNR ρ_s=19 dB and for $\Delta\nu T$=10^{-4}. Fig. 2(a) indicates that the algorithm closely tracks the phase noise for 500,000 symbol periods. However, there are significant residual phase errors, although this is not visible on the scale of the graph. The histogram of Fig. 2(b) reveals that residual phase errors extend up to 0.9 rad (~52 deg), exceeding the minimal azimuthal separation of ~37 deg between neighboring points in the square 16-QAM constellation [10]. In other simulation runs for the same $\Delta\nu T$ and lower ρ_s, the algorithm becomes increasingly unstable until, eventually, its estimate completely diverges from the actual phase noise. Track loss due to cycle slips can be mitigated to some extent by differential encoding [9].

Finally, Fig. 3(a) shows plots of the average bit error rate (BER), obtained by Monte Carlo simulation based on 2×10^6 bits, as a function of the average electronic symbol SNR for various $\Delta\nu T$. From Fig. 3(a) we calculate the average electronic symbol SNR penalty at BER=10^{-3}, compared to the ideal case. Fig. 3(b) shows a plot of the resulting penalty vs. $\Delta\nu T$. We obtain a 1-dB penalty for $\Delta\nu T$=1.1×10^{-4}. This value is more conservative than the one reported previously [5] and slightly inferior to the linewidth tolerance of the considerably more complex feed-forward blind phase search algorithm ($\Delta\nu T$=1.4×10^{-4}) [1], which also requires mandatory differential encoding.

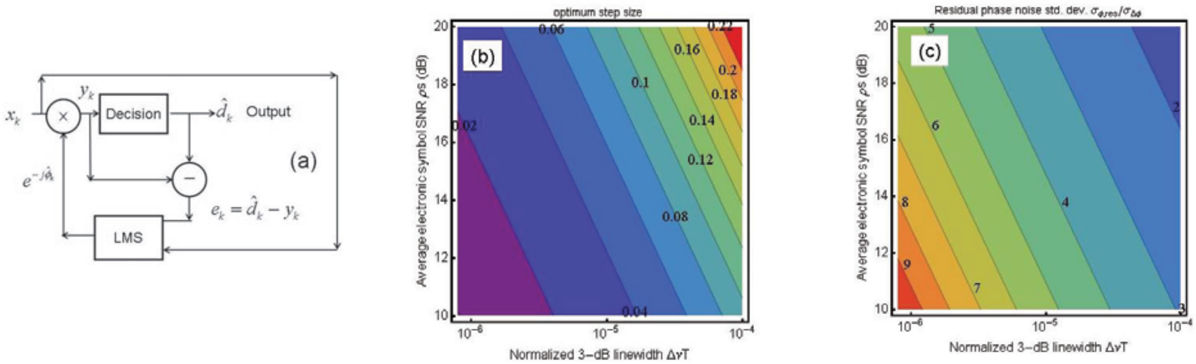

Fig. 1 (a) Block diagram of the constrained DD-LMS CPE algorithm; (b) Contours of optimal step size; (c) Contours of normalized minimum residual phase noise standard deviation for square 16-QAM (Conditions: m_4=1.63, m_2=1.11, $m_{2,r}$=0.55).

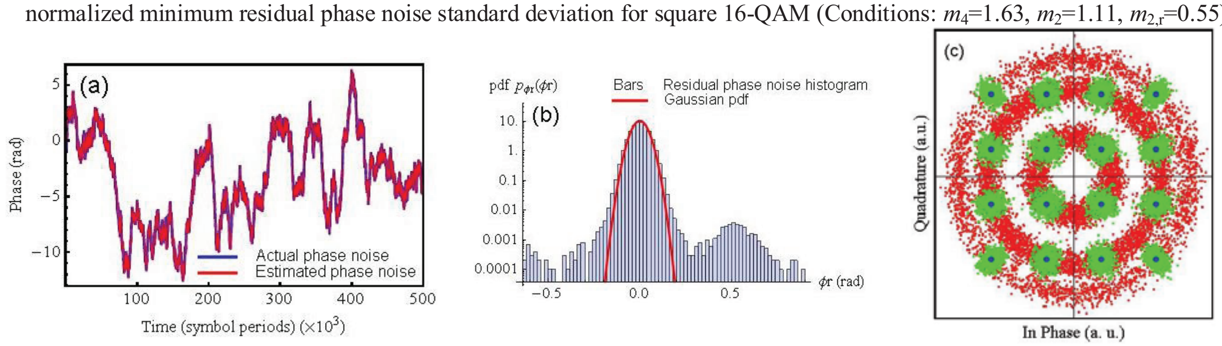

Fig. 2(a) Representative phase noise trace and its estimation by the constrained DD-LMS algorithm;(b) Residual phase noise histogram and Gaussian fit; (c) Constellation diagrams: Ideal (blue points), before (red points) and after carrier phase noise estimation (green points). (Conditions: 16-QAM, ρ_s=19 dB, normalized 3-dB linewidth $\Delta\nu T$=10^{-4}, optimum step size μ=0.23).

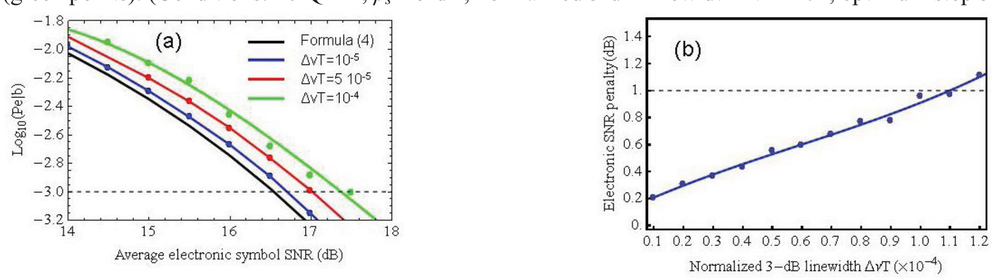

Fig. 3 (a) BER vs. average electronic symbol SNR; (b) Electronic symbol SNR penalty vs. 3-dB linewidth at BER= 10^{-3}.

4. Conclusions

We theoretically studied the performance of a fast single-tap constrained DD-LMS CPE algorithm for coherent optical M-QAM receivers. We derived analytical expressions for the update of the single complex coefficient, the residual phase noise variance, and the optimum step-size parameter and we thoroughly calculated, by simulation, the linewidth tolerance of the algorithm. For square 16-QAM, the maximum $\Delta\nu T$ that can be mitigated is 1.1×10^{-4}, compared to $\Delta\nu T=1.4\times10^{-4}$ for the much more computationally intensive blind phase search algorithm [1].

5. References

[1] T. Pfau, S. Hoffmann, and R. Noe, *J. Lightwave Technology*, vol. 27, no. 8, pp. 989–999, Apr. 2009.
[2] I. Fatadin, D. Ives, and S. J. Savory, *J. Lightw. Technol*., vol. 27, no. 15, pp. 3042–3049, Aug. 2009.
[3] I. Fatadin, D. Ives, and S. J. Savory, *IEEE Photonics Technology Letters*, vol. 22, no. 9, pp. 631–633, May 2010.
[4] Y. Mori, C. Zhang, K. Igarashi, K. Katoh, and K. Kikuchi, *Opt. Expr*., Vol. 17, pp. 1435–1441, Feb. 2009.
[5] I. Fatadin, D. Ives, and S. J. Savory, *J. Lightw. Technol*., vol. 27, no.15, pp. 3042–3049, Aug.1, 2009.
[6] G. Picchi and G. Prati, *IEEE Trans. Commun*., vol. 35, no. 9, pp. 877–887, Sep. 1987.
[7] B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications*. Wiley, 1998.
[8] I. Roudas et al., *J. Lightwave Tech*., Vol. 28, No. 7, pp. 1121–1134, Apr. 2010.
[9] M. G. Taylor, *J. Lightw. Technol*., vol. 27, no. 7, pp. 901–914, Apr. 2009.
[10] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.