Accurate modal dispersion measurements using maximally-orthogonal Stokes vectors

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Abstract: We propose optimal launch modes minimizing the noise error in the estimation of the fiber modal dispersion vector. For a 20-mode fiber, the SNR is improved by 4 dB compared to conventional mode combinations.

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1. Introduction

Research in multimode and multicore optical fibers (jointly abbreviated by the acronym MMFs) has gained popularity in the past few years, owing to their potential use in high-capacity, long-haul, mode-division multiplexed (MDM) transmission systems [1]. To minimize the computational complexity of digital signal processing in coherent optical receivers, MDM MMFs are designed to exhibit short impulse responses, resulting from small uncoupled differential mode group delays (DMGDs) and strong coupling among eigenmodes. Consequently, modal dispersion characterization of MDM MMFs requires sensitive complex instrumentation (see [2] and the references therein).

Recently, the mode-dependent signal delay method [3], [4] was proposed to retrieve the full information about the principal modes (PMs) and their DMGDs by measuring the input modal dispersion (MD) vector in the generalized Stokes space using an inexpensive direct-detection receiver. More specifically, this method records the group delays of a series of optical pulses propagating though the fiber under test, each pulse corresponding to different input mode excitations. By analyzing these pulse group delays one can infer the MD vector components, as explained below. The accuracy of the MD vector calculation, however, critically depends on the selection of the launch modes.

The main contribution of this paper is the optimization of launch modes in order to maximize the SNR and enhance the accuracy of the mode-dependent signal delay method. For example, for a 20-mode MDM MMF, the optimal mode combinations improve the noise performance of the mode-dependent signal delay method by 4 dB compared to [4].

2. Mathematical model

To determine the MD vector in the case of an *N*-mode fiber, the group delay τ_g of an output pulse can be expressed as a function of the dot product of the input MD vector $\vec{\tau}_s(\omega)$ and the launch state vector $\hat{s}$ in Stokes space [5], [6]

$$\tau_g = \tau_0 + [(N-1)/N]\langle\vec{\tau}_s(\omega)\rangle \cdot \hat{s}, \tag{1}$$

where τ_0 is the average group delay and $\langle . \rangle$ denotes spectral-averaging.

At each frequency, we launch N^2-1 pulses corresponding to linearly-independent states $\hat{s}_i$ in the generalized Stokes space and we observe their group delays $\tau_{g,i}$, $i=1,\dots,N^2-1$. From these measurements, we can calculate the components of $\vec{\tau}_s(\omega)$ based on (1) by solving a set of N^2-1 equations with N^2-1 unknowns [5], [6]. Thermal noise at the direct-detection receiver, however, can lead to an error in the estimate of $\vec{\tau}_s(\omega)$.

Our aim is to choose a set of N^2-1 Stokes vectors $\hat{s}_i$ that minimizes the variance $\sigma^2_{\|\delta\vec{\tau}_s\|}$ of the MD vector. In principle, the lowest value of $\sigma^2_{\|\delta\vec{\tau}_s\|}$ could be achieved by selecting orthonormal $\hat{s}_i$ because, then, the errors of the MD vector components would be uncorrelated. Interestingly, for $N>2$, it is impossible to find such an orthonormal set of N^2-1 Stokes vectors $\hat{s}_i$ due to the incomplete coverage of the generalized Poincaré sphere with valid states [5]. In the following, we compute optimal sets of N^2-1 quasi-orthogonal Stokes vectors $\hat{s}_i$ that correspond to feasible combinations of propagating modes using two different gradient descent algorithms to minimize $\sigma^2_{\|\delta\vec{\tau}_s\|}$ [7].

In the first algorithm, we parametrize the *j*-th unit Jones vector $|s_j\rangle$ using $2N-2$ hyperspherical coordinates, i.e., $|s_j\rangle = \left[\cos(\phi_{j1}), \sin(\phi_{j1})\cos(\phi_{j2})\,e^{i\theta_{j1}}, \dots, \sin(\phi_{j1})\cdots\sin(\phi_{jN-2})\sin(\phi_{jN-1})\,e^{i\theta_{jN-1}}\right]^T$. From this expression, we calculate the corresponding Stokes vector $\hat{s}_j$ [5] and its partial derivatives with respect to ϕ_{jv} and θ_{jv}. Furthermore, we define the vector $\mathbf{p}$ that contains the coordinates ϕ_{jv} and θ_{jv} of all N^2-1 Stokes vectors. Then, we perform unconstrained optimization in a $(N^2-1)\times(2N-2)$ real space using the method of gradient descent [7]: Starting from a random point $\mathbf{p}^{(0)}$, we take successive steps $\mathbf{p}^{(k)}$ opposite to the direction of the gradient of the cost function $\nabla\xi$, where $\xi = \sigma^2_{\|\delta\vec{\tau}_s\|}$, until we reach a local minimum [7]. The iteration is written as $\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} - \mu^{(k)}\nabla\xi^{(k)}$, where $\mu^{(k)}$ is a positive scalar (adaptive step size) [7].

In the second algorithm, we parametrize the j-th Jones vector $|s_j\rangle = (s_{jv})_{v=1}^{N}$ using $2N$ real parameters $x_{jv} = \Re(s_{jv})$ and $y_{jv} = \Im(s_{jv})$. Now, the parameter vector $\mathbf{p}$ contains the coordinates x_{jv} and y_{jv} of all $N^2 - 1$ Stokes vectors. The optimization takes place in a $(N^2 - 1) \times 2N$ real space, where we impose $N^2 - 1$ unit length constraints $\langle s_j | s_j \rangle = 1, j = 1, \ldots, N^2 - 1$. We use the modified update rule $\mathbf{p}^{(\mathrm{k}+1)} = \mathrm{proj}[\mathbf{p}^{(\mathrm{k})} - \mu^{(\mathrm{k})} \nabla \xi^{(\mathrm{k})}]$, where $\mathbf{p}^{(0)}$ is chosen at random and proj(.) means that only the component of the gradient tangential to the constraints is employed (projected gradient descent) [7].

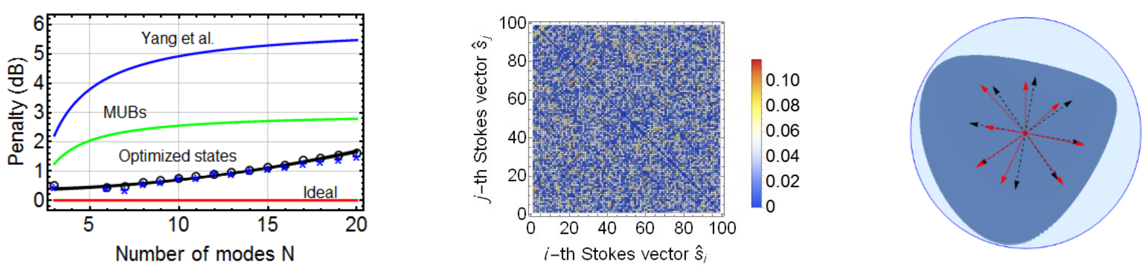
3. Results and discussion

Fig. 1(a) shows plots of the SNR penalty vs the number of modes for four different vector sets. The SNR penalty is calculated using the ideal albeit infeasible case of orthonormal Stokes vectors $\hat{s}_j$ as baseline (in red). For N=20, we observe that there is a 4 dB noise reduction by using the optimal vector set (circles and stars) instead of the launch states proposed by Yang et al. [4] (in blue). In waveguides where the number of propagating modes N is a power of a prime, selecting launch states from mutually unbiased bases (MUBs) [6], [8] (in green) yields better performance than using the launch states proposed by Yang et al. (in blue) [4]. Still, for N=19, there is a 1.26 dB noise reduction by using the optimal vector set instead of launch states from MUBs [8]. Finally, it is worth noting that, for N>2, orthonormal vector sets do not exist, as already mentioned in Sec. 2 [5]. For instance, for N=20, we observe that there is a 1.44 dB penalty for using the optimized states compared to the ideal case of orthonormal vectors.

To exemplify the deviation of the optimal vectors from orthogonality, we compute the correlation matrix (after subtracting the identity matrix) for the vectors in the optimal set for N=10. Each pixel of the density plot shown in Fig. 1(b) depicts the absolute value of the corresponding dot product of two vectors of the optimal set. Notice that the absolute value of the off-diagonal elements spans a range reaching 0.12. This result implies that the optimum vectors deviate from orthogonality by up to ±6.7 deg.

Finally, we can catch a glimpse of the optimum set of Stokes vectors using a 2D projection (e.g., see Fig. 1(c) for $N = 3$). We know that the $N^2 - 1$ Stokes vectors should be ideally orthonormal. It is possible to project orthonormal vectors onto a plane so that their projections have equal angular separations (dashed black vectors). Now we can superimpose on the same plane the projections of the actual optimal vectors given by the numerical optimization procedure (red vectors), as well as the projection of the manifold of allowed states on the surface of the Poincaré sphere (dark blue area) [5]. All vectors are bounded by the projection of the Poincaré sphere onto the plane (shaded circular disk with unit radius).

In conclusion, we optimized the launch states used in the mode-dependent signal delay method for the measurement of modal dispersion in MDM MMFs. Since both numerical optimization algorithms presented here find local minima, the SNR penalty obtained by using the proposed launch states should be considered as an upper bound of the globally optimal solution.



(a) (b) (c)

Fig. 1 (a) SNR penalty compared to the ideal case vs the number of modes for four different vector sets (Symbols: Blue line: Yang's vectors [4]; Green line: launch states selected from $N + 1$ mutually unbiased bases (MUBs) in groups of $N - 1$ vectors ($N + 1$ MUBs are known to exist only when the number of modes N is a power of a prime) [6], [8]; Red line: Orthonormal Stokes vectors; Circles: unconstrained optimization algorithm; Stars: projected gradient descent algorithm; (b) Density plot of the optimum vector set correlation matrix minus the identity matrix for N=10; (c) 2D projections of various vector sets for N=3 (Symbols: Dashed black vectors: ideal orthonormal vectors; Red vectors: actual optimal vectors given by the numerical optimization of Sec. 2; Dark blue area: projection of the manifold of allowed states on the surface of the Poincaré sphere).

5. References

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