Asymmetric Probability Density Function of a Signal with Interferometric Crosstalk

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***Abstract*—An asymmetric probability density function due to the beating of interferometric crosstalk and amplifier spontaneous emission noise in a system with optical preamplifier is observed experimentally and explained theoretically. An accurate derivation of the probability density function of the photocurrent in the presence of N incoherent homodyne interferers is presented, for the first time, to explain the asymmetry. The model is compared with the experiment and is shown to be in excellent agreement.**

***Index Terms*—Amplifier noise, cochannel interference, networks, optical crosstalk.**

I. Introduction

INTERFEROMETRIC crosstalk is one of the major impairments in transparent wavelength-division-multiplexing (WDM) optical communication systems and networks and can cause severe performance degradation [1]–[5]. A number of models with various degrees of accuracy were proposed in the literature for the evaluation of the probability density function (PDF) of the direct-detection receiver photocurrent in optically amplified communication systems in the presence of N incoherent homodyne interferers, e.g., most recently [5]. These analyses usually neglect the crosstalk-crosstalk and amplified spontaneous emission (ASE) noise-crosstalk beatings, as well as the direct detection of the crosstalk and the ASE noise. The omission of the aforementioned terms results in a symmetric photocurrent PDF. However, in a realistic system with optical amplifiers, the interplay between interferometric crosstalk and ASE noise in the receiver cannot be neglected when crosstalk is not small. Obvious asymmetric PDF has been observed in the experiment [6].

In this letter, we present a theoretical and experimental study of the PDF of the direct-detection receiver photocurrent in the presence of multiple interferometric crosstalk. An accurate derivation of the photocurrent PDF, taking into account the aforementioned neglected terms, is used to explain the asymmetry. The theoretical PDF is compared with the experiment and is found to be in excellent agreement.

II. Experiments and Results

The experimental setup is described in [6]. The measurements are taken by combining one to eight unmodulated optical signals, which have the same wavelength and polarization state, with an unmodulated signal using a coupler. An optical amplifier is placed in front of a broad-band direct-detection receiver. No electronic low-pass filter is used [6].

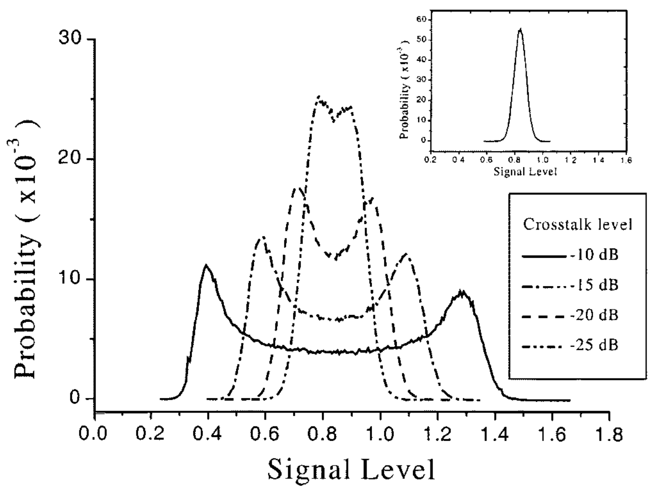

Fig. 1. Histograms of optical power with one crosstalk term when a preamplifier is used. The inset plot is the histogram of the optical power in the absence of crosstalk.

Fig. 1 shows histograms of the optical power when one crosstalk term is present with crosstalk levels in the range -10 dB to -25 dB. The inset plot shows the histogram of the optical power of the signal without any crosstalk. The observed PDF with one crosstalk term resembles the arc-sine statistics of interferometric crosstalk predicted by the previous models, e.g., [1]; however, the two peaks are obviously asymmetric even though the phases of the signal and crosstalk are uncorrelated. The asymmetric broadening of the optical PDF is due to the ASE noise. For an optically preamplified receiver, the dominant signal-ASE noise beating, whose variance increases with the signal power, leads to a larger broadening of the part of the PDF associated with the constructive interference, i.e., higher signal levels, than the one associated with the destructive interference. Fig. 2 shows the PDF of the signal with one crosstalk term in the absence of optical preamplifier in front of the receiver. Without ASE contribution, the PDF is symmetric.

The asymmetries of the PDFs with multiple crosstalk channels are shown in Fig. 3.

An accurate model which takes into account both interferometric crosstalk and ASE noise is given below to provide a detailed explanation for the asymmetric behavior.

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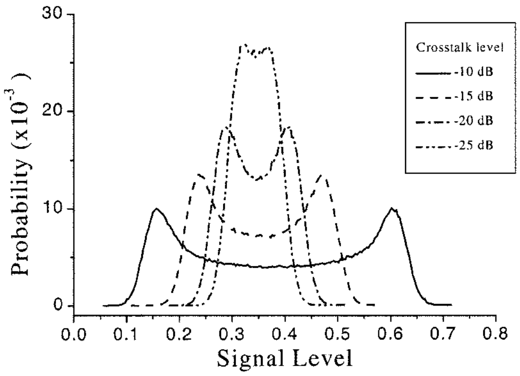


Fig. 2. Histograms of optical power with one crosstalk term without preamplifier. The solid curve is signal without crosstalk; the dashed curves are in the presence of crosstalk.

III. Theoretical Model

The electric field impinging upon the photodiode can be written $E(t) = \Re\{\tilde{E}(t)e^{i\omega_0 t}\}$, where $\Re$ denotes the real part, ω_0 is the carrier angular frequency, and tilde denotes the complex envelope

$$\tilde{E}(t) = \sum_{k=1}^{N+1} A_k e^{i\phi_k} + n_c(t) + i n_s(t). \quad (1)$$

In (1), A_1 is the amplitude of the signal, A_k, $k = 2, \ldots, N+1$ are the amplitudes of the N crosstalk terms or interferers, which are assumed copolarized with the signal, ϕ_1, ϕ_k, $k = 2, \ldots, N+1$ are the arguments of the complex envelopes of the signal and interferers, respectively, and n_c, n_s are the real and imaginary parts, respectively, of the complex envelope of the ASE noise, which are considered independent narrow-band bandpass Gaussian stochastic processes with zero mean and variance σ^2 [7].

Assuming that the photodiode is an ideal square law detector, the power of the detected signal is given by $P(t) = |\tilde{E}(t)|^2/2$. The photocurrent at the output of the photodiode can be written

$$\begin{aligned} I(t) = \frac{R}{2}\Bigg\{ &\sum_{k=1}^{N+1}\sum_{m=1}^{N+1} A_k A_m \cos\Delta\phi_{km} + n_c^2(t) \\ &+ n_s^2(t) + 2n_c(t)\sum_{k=1}^{N+1} A_k \cos\phi_k \\ &+ 2n_s(t)\sum_{k=1}^{N+1} A_k \sin\phi_k \Bigg\} + n_{th}(t) \end{aligned} \quad (2)$$

where R is the responsivity of the photodetector, $\Delta\phi_{km} = \phi_k - \phi_m$ is the difference of the arguments of the kth and the mth interfering channel, and $n_{th}(t)$ is the receiver thermal noise, which follows a Gaussian distribution with zero mean and variance σ_{th}^2. The latter can be increased to approximately account for the contribution of the shot noise. In the right-hand side of expression (2), we distinguish the following terms: the double sum includes the direct detection of the signal and the crosstalk, as well as the signal–crosstalk and the crosstalk–crosstalk beatings; the second and third terms arise from the direct detection of the ASE noise; the fourth and fifth terms arise from the ASE noise-crosstalk beating.

In a worst case scenario, the signal and the crosstalk terms have the same frequency, so the argument differences $\Delta\phi_{km}$ are equal to the phase noise differences $\Delta\theta_{km}$. Phase noises θ_k, $k = 1, \ldots, N+1$ can be considered independent Wiener random processes [10], so they follow Gaussian distributions. In the experiment, the variance of $\Delta\theta_{km}$ is so large that the phase noises θ_k, $k = 1, \ldots, N+1$ can be considered independent, identically distributed random variables following a uniform distribution in the interval $[-\pi, \pi]$.

For the evaluation of the characteristic function of the photocurrent, first, we define the auxiliary RV $z = 2I(t)/R$. Using (2) and the definition of the characteristic function [7], it is straightforward to see that the characteristic function of z is given by (3) at the bottom of the page, where $p[\cdot]$ is the joint PDF of $n_c, n_s, \ldots, \theta_{N+1}$, and we define the auxiliary function

$$\begin{aligned} F(\omega) = &\frac{1}{(2\pi)^{N+1}} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} \\ &\cdot \exp\left(\frac{2i\omega}{1 - 2\sigma^2 i\omega} \sum_{k=1}^{N} \sum_{m=k+1}^{N+1} A_k A_m \cos\Delta\theta_{km} \right) \\ &\cdot d\theta_1 \cdots d\theta_{N+1}. \end{aligned} \quad (4)$$

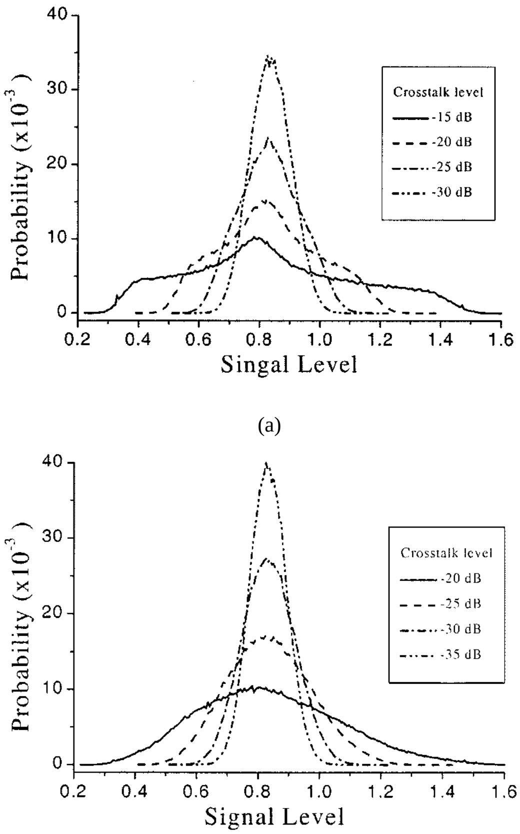
In the case of one interferer ($N = 1$), (4) can be evaluated analytically [8, eq. (3.339)], so (3) yields

$$\begin{aligned} \psi_z(\omega) = &\frac{1}{1 - 2\sigma^2 i\omega} e^{-(\sigma_{th}^2\omega^2/2) + [i\omega(A_1^2 + A_2^2)/(1 - 2\sigma^2 i\omega)]} \\ &\cdot I_0\left(\frac{2i\omega A_1 A_2}{1 - 2\sigma^2 i\omega} \right) \end{aligned} \quad (5)$$

where $I_0(x)$ is the modified Bessel function of the first kind of order zero.

In the case of multiple interferers ($N > 1$), the evaluation of (4) requires $N+1$ numerical integrations. For this purpose, Gauss–Legendre quadrature can be used [9]. The selection of the number of integration nodes is a compromise between accuracy and computational complexity. Since the number of operations grows proportionally to the $(N+1)$th power of the number of integration nodes, the latter must be kept as small as possible. It is possible to reduce by one the number of integrals in (4) using a transformation of variables from $\theta_1, \ldots, \theta_{N+1}$ to

$$\begin{aligned} \psi_z(\omega) = E\{e^{i\omega z}\} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} e^{i\omega z} p[n_c, n_s, n_{th}, \theta_1, \cdots, \theta_{N+1}] dn_c\, dn_s\, dn_{th}\, d\theta_1 \cdots d\theta_{N+1} \\ &= \frac{1}{1 - 2\sigma^2 i\omega} \exp\left(-\frac{\sigma_{th}^2 \omega^2}{2} + \frac{i\omega}{1 - 2\sigma^2 i\omega} \sum_{k=1}^{N+1} A_k^2 \right) F(\omega) \end{aligned} \quad (3)$$



(b)

Fig. 3. Histograms of optical power with multiple crosstalk terms with (a) two crosstalk terms, and (b) four crosstalk terms.

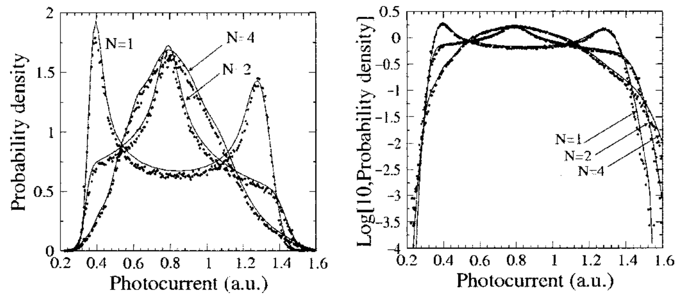
$\theta_1 - \theta_2,\ \theta_2 - \theta_3,\ \ldots,\ \theta_{N+1}$ and integrate over θ_{N+1}. In addition, it is possible to reduce the number of operations in (4) using symmetries for the integration variables.

From the characteristic function (3), it is possible to calculate the probability density function using a Fourier transform [7]

$$p_z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_z(\omega) e^{-i\omega z}\, d\omega. \tag{6}$$

Despite the fact that (6) is relatively simple, no analytical expression for the integral in (6) can be found and the Fourier transform must be evaluated numerically.

Fig. 4 shows histograms of the photocurrent $I(t)$ for three different numbers of interfering channels ($N = 1, 2,$ and 4). The points are experimental data taken from Figs. 1 and 3. The lines are calculated from the theoretical model. For convenience, $R/2 = 1$ is assumed. The other parameters used for the fitting are: 1) For $N = 1$, $A_1 = 0.88$, $A_2 = 0.27$, $\sigma = 0.022$ (corresponding to -10 dB crosstalk level); 2) For $N = 2$, $A_1 = 0.89$, $A_2 - A_3 = 0.155$, $\sigma = 0.022$ (corresponding to -15 dB crosstalk level/interferer). Gauss–Legendre quadrature with 40 nodes is used for the evaluation of (4); 3) For $N = 4$, $A_1 = 0.9$, $A_2 - A_5 = 0.09$, $\sigma = 0.022$ (corresponding to -20 dB crosstalk level/interferer). Gauss–Legendre quadrature with ten nodes is used for the evaluation of (4). In all cases, a fast Fourier transform with 2048 points is used for the numerical evaluation of (6). The receiver thermal noise variance σ_{th}^2 is set to zero. The theoretical model accurately describes both the center and the tails of the measured PDF. The shape of the PDF is clearly asymmetric, with steeper left edge and smoother right edge. The asymmetry is attributed to the ASE noise-crosstalk beating, as well as the direct detection of the ASE noise.



(a) (b)

Fig. 4. Fit of measurements of Fig. 3 with the expression (6). (a) Linear; (b) Logarithmic (Symbols: lines = model; points = experiments).

IV. Summary

An asymmetric photocurrent PDF at the output of an optically amplified, direct-detection receiver in the presence of N homodyne incoherent interferers is observed experimentally and explained theoretically for the first time. An accurate expression of the PDF of the photocurrent is derived. The model is compared with the experiment and is shown to be in excellent agreement.

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