Error probability of Mode Vector Modulation optically-preamplified direct-detection receivers

J. Kwapisz[1,*], I. Roudas[2], and E. Fink[1]

[1] *Department of Mathematical Sciences, Montana State University, Bozeman, MT 59717, USA*

[2] *Electrical and Computer Engineering, Montana State University, Bozeman, MT 59717, USA*

[*] *jkwapisz@montana.edu*

Abstract: Mode Vector Modulation (MVM) is a spatial, multidimensional modulation format for short-haul, optical interconnects. We evaluate the back-to-back performance of optically-preamplified MVM direct-detection receivers and prove that MVM is superior to conventional binary modulations.

1. Introduction

There is keen interest in short- and medium-haul optical interconnects utilizing direct-detection (DD) receivers and novel modulation formats [1, 2]. Among others, Stokes vector modulation (SVM) [3–5] allows for more power-efficient signaling than M-ary pulse amplitude modulation (M-PAM) by spreading the constellation points in the three-dimensional Stokes space. To further increase the energy efficiency of SVM, a transition to a higher-dimensional Stokes space is necessary, which can be achieved by using few-mode [6] or multicore fibers, or free space orthogonal mode sets. We call this new modulation format mode vector modulation (MVM) [7].

We give a closed-form upper bound for the symbol and bit error probabilities of optically-preamplified MVM direct-detection receivers by applying the Union Bound formalism [8] to higher-dimensional Stokes spaces. We validate the bound via Monte Carlo simulation and use it to compare the performance of MVM to conventional modulation formats.

2. Mathematical Model and the Union Bound

MVM consists of simultaneously sending optical pulses of the same shape (but ofvdifferent amplitudes and initial phases) over N identical spatial and polarization degrees of freedom (SDOFs). We consider M-ary transmission, where the transmitter uses an alphabet of M symbols, each representing a word of $k := \log_2(M)$ bits. For the $m-$th symbol, the electric fields of the optical waves at the transmitter output are combined into a single column vector $\mathbf{s}_m(t) = A_m e^{\iota\phi_m} g(t)|s_m\rangle$, where A_m is the common amplitude of the electric fields, ϕ_m is the common phase, $g(t)$ is the pulse shape, and $|s_m\rangle$ is a generalized unit Jones vector with elements describing the complex excitations of the cores, i.e., the relative amplitudes and phases of electric fields of the optical waves.

We analytically calculate the back-to-back performance of M-ary MVM over N SDOFs in the amplified spontaneous emission (ASE) noise-limited regime. For mathematical tractability, we neglect transceiver imperfections and implementation penalties. These assumptions are justified in the sense that we seek to quantify the ultimate potential of MVM for use in optical interconnects. We model the communication link as an additive white Gaussian noise (AWGN) channel. The received signal vector equals $\mathbf{r}(t) = e^{\iota\theta}\mathbf{s}_m(t) + \mathbf{n}(t)$, where θ is a random carrier phase shift uniformly distributed in the interval $[0, 2\pi)$ and $\mathbf{n}(t)$ is a noise vector with N independent and identically distributed (i.i.d.) entries following the complex Gaussian distribution with zero mean and variance σ^2 per quadrature. For equiprobable and equipower signals, the optimal detection scheme based on the *maximum likelihood criterion*, after averaging over all possible random carrier phase shifts [9], is $\hat{m} := \operatorname{argmax}_{1\le m\le M} |\langle r|s_m\rangle|$.

The general form of the Union Bound is described in [8] and reads

$$P_{e|s} \le \sum_{m=1}^{M} P_m \sum_{m'\neq m} P_{\mathrm{e,bin}}^{m'|m}, \tag{1}$$

where P_m is the occurrence probability of the m^{th} symbol and $P_{\mathrm{e,bin}}^{m'|m}$ is the pairwise error probability of deciding on $|s_{m'}\rangle$ when $|s_m\rangle$ was sent in a binary decision where no other symbols are considered. We relate the pairwise error probability $P_{\mathrm{e,bin}}^{m'|m}$ to the difference between two independent Rician random variables [10] and find that

$$P_{\mathrm{e,bin}}^{m'|m} = Q_1\left(\frac{\sqrt{1-\sqrt{1-\gamma^2}}}{2\sqrt{\sigma^2}}, \frac{\sqrt{1+\sqrt{1-\gamma^2}}}{2\sqrt{\sigma^2}}\right) - \frac{1}{2}\exp\left(-\frac{1}{4\sigma^2}\right) I_0\left(\frac{\gamma}{4\sigma^2}\right), \tag{2}$$

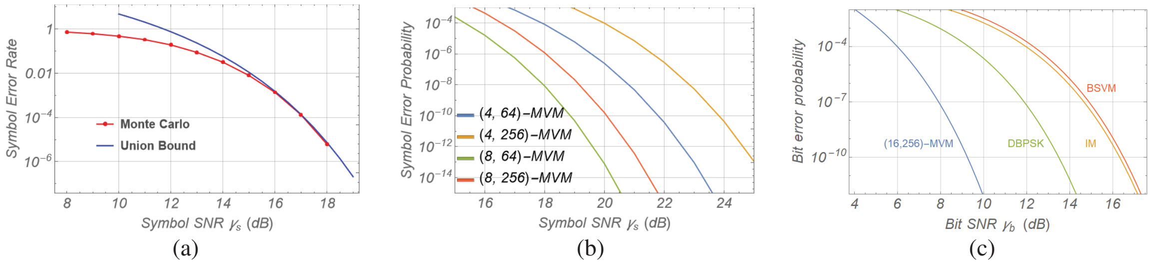

Fig. 1: (a) Symbol error rate (SER) for a $(8,512)$-MVM constellation via Monte Carlo simulation (red) and the Union Bound (blue). (b) Symbol error probabilities for various geometrically optimized (N,M)-MVM constellations. (c) Bit error probability for $(16,256)$-MVM in comparison to conventional modulation formats for an 8-core MCF.

where $\gamma := |\langle s_m|s_{m'}\rangle|$, $Q_1(a,b)$ is the first-order *Marcum Q function* [8], and $I_0(x)$ is the zeroth order *modified Bessel function* of the fist kind. Upon multiplying the pairwise symbol error probability $P_{\mathrm{e,bin}}^{m'|m}$ by the Hamming distance $h_{m'm}$ between the bit sequences assigned to symbols m',m, the average bit error probability satisfies

$$\bar{P}_{e|b} \leq \frac{1}{kM} \sum_{m=1}^{M} \sum_{m' \neq m} h_{m'm} P_{\mathrm{e,bin}}^{m'|m}. \tag{3}$$

3. Results and Discussion

In Figure 1a, we show the symbol error rate (SER) as a function of the symbol SNR per SODF, $\gamma_s := 1/(2\sigma^2)$, for a $(8,512)$-MVM constellation via Monte Carlo simulation (red) and via the Union Bound (blue). The Union Bound (1)–(2) gives meaningless values at low SNR but becomes sharp as we move to a higher SNR regime. In particular, the Union Bound becomes asymptotically tight for symbol error probabilities below roughly 0.1%. In Figure 1b, we show the symbol error probability as a function of the symbol SNR, as computed by the Union Bound (1)–(2), for various geometrically optimized (N,M)-MVM constellations (designed per the methodology in [11]).

To illustrate how to compare the performance of MVM with conventional modulation formats for short-haul transmission and optically-preamplified direct-detection, we consider the special case of a homogeneous 8-core MCF with single-mode cores. We can transmit 8 parallel channels using either binary intensity modulation (IM) [12], binary DPSK (BDPSK) [12], or binary SVM (BSVM) [13] per single-mode core. We can also transmit $2^8 = 256$-ary MVM using all 8 cores with 16 SDOFs. The optimal $(16,256)$-MVM constellation corresponds to a 256-simplex in the 255-dimensional generalized Stokes space [7]. All aforementioned modulation formats can achieve a theoretical spectral efficiency of $0.5\,\mathrm{b/s/Hz/SDOF}$ if ideal Nyquist pulses with zero roll-off factor are used, ensuring a fair comparison. Figure 1c shows plots of the bit error probability as a function of the bit SNR per SDOF, $\gamma_b := \gamma_s/k$, at the decision device in the optically-preamplified direct-detection receiver. At a bit error probability of 10^{-9}, $(16,256)$-MVM has bit SNR gains of approximately 4.2 dB, 7.0 dB, and 7.2 dB over DBPSK, binary IM, and BSVM, respectively. Therefore, the use of MVM can dramatically improve the system performance compared to conventional modulation formats at the expense of transceiver complexity [7].

4. Conclusion

We derived a tight upper-bound for the error probability of optically pre-amplified DD receivers and used it to show that MVM has higher sensitivity than other DD modulations, e.g., gaining 4–7 dB of SNR in an 8-core MCF.

References

1. M. H. Eiselt, A. Dochhan, and J.-P. Elbers, in *23rd Opto-Electronics and Communications Conference,* (2018).
2. J. K. Perin, A. Shastri, and J. M. Kahn, "Coherent data center links," J. Light. Technol. **39**, 730–741 (2021).
3. K. Kikuchi and S. Kawakami, Opt. Express **22**, 7374 (2014).
4. D. Che, A. Li, X. Chen, Q. Hu, Y. Wang, and W. Shieh, J. Light. Technol. **33**, 678–684 (2015).
5. M. Morsy-Osman, M. Chagnon, and D. V. Plant, J. Light. Technol. **34**, 1585–1592 (2016).
6. H. Ji, D. Che, C. Sun, J. Fang, G. Milione, R. R. Unnithan, and W. Shieh, Opt. Lett. **44**, 2065 (2019).
7. I. Roudas, J. Kwapisz, and E. Fink, in *European Conference on Optical Communication,* (2021). Tu2D.5.
8. J. G. Proakis, *Digital Communications 5th Edition* (McGraw Hill, 2007).
9. D. Divsalar and M. Simon, IEEE Transactions on Commun. **38**, 300–308 (1990).
10. S. Benedetto, E. Biglieri, and V. Castellani, *Digital transmission theory* (Prentice-Hall, Englewood Cliffs, N.J, 1987).
11. E. Fink, J. Kwapisz, and I. Roudas, in *2021 IEEE Photonics Conference (IPC),* (2021).
12. P. A. Humblet and M. Azizoglu, J. Light. Technol. **9**, 1576–1582 (1991).
13. K. Kikuchi, Opt. Express **28**, 26007–26017 (2020).