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# Accurate modeling of incoherent homodyne crosstalk in optically amplified systems


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## Introduction

A number of models with various degrees of accuracy were proposed in the literature for the evaluation of the probability density function (pdf) of the direct detection receiver photocurrent in optically amplified communication systems in the presence of N incoherent homodyne interferers, e.g. most recently /1/. Previous models neglect the crosstalk-crosstalk and amplified spontaneous emission (ASE) noise-crosstalk beatings, as well as the direct detection of the crosstalk and the ASE noise. These effects become important at strong crosstalk values /2/.

This paper presents, for the first time, an accurate derivation of the photocurrent pdf, taking into account the aforementioned neglected terms. It is a generalization of the analysis presented in /2/ for one incoherent homodyne inteferer. The current model is compared with the experiment and is shown to be in excellent agreement. Finally, the approximate model /1/ is compared to the current model and its range of validity is investigated.

## Theoretical model

The electric field impinging upon the photodiode can be written $E(t)=\Re\{\tilde{E}(t)e^{i\omega_0 t}\}$ where $\Re$ denotes the real part, $\omega_0$ is the carrier angular frequency and tilde denotes the complex envelope

$$\tilde{E}(t)=\sum_{k=1}^{N+1} A_k e^{i\phi_k}+n_c(t)+in_s(t) \tag{1}$$

In (1), $A_1$ is the amplitude of the signal, $A_k$, k=2,...,N+1 are the amplitudes of the N interferers, $\phi_1$, $\phi_k$, k=2,...,N+1 are the arguments of the complex envelopes of the signal and interferers, respectively, and $n_c$, $n_s$ are the real and imaginary parts, respectively, of the complex envelope of the ASE noise which are considered independent narrowband bandpass Gaussian stochastic processes with zero mean and variance $\sigma^2$ /3/.

Assuming that the photodiode is an ideal square law detector, the power of the detected signal is given by $P(t)=|\tilde{E}(t)|^2/2$. The photocurrent at the output of the photodiode can be written:

$$\mathrm{I}(t)=\frac{R}{2}\left\{\sum_{k=1}^{N+1}\sum_{m=1}^{N+1} A_k A_m\cos\Delta\phi_{km}+n_c^2(t)+n_s^2(t)+ 2n_c(t)\sum_{k=1}^{N+1}A_k\cos\phi_k+2n_s(t)\sum_{k=1}^{N+1}A_k\sin\phi_k+n_{th}(t)\right\} \tag{2}$$

where R is the responsivity of the photodetector, $\Delta\phi_{km}=\phi_k-\phi_m$ is the difference of the arguments of the k-th and the m-th interfering channel, and $n_{th}(t)$ is the receiver thermal noise which follows a Gaussian distribution with zero mean and variance $\sigma_{th}^2$. In the left hand side of expression (2), we distinguish the following terms: the double sum includes the direct detection of the signal and the crosstalk, as well as the signal-crosstalk and the crosstalk-crosstalk beatings; the second and third terms arise from the direct detection of the ASE noise; the fourth and fifth terms arise from the ASE noise-crosstalk beating.

In the following, as a worst case scenario, it is assumed that the signal and the crosstalk terms have the same frequency (homodyne case), so the argument differences $\Delta\phi_{km}$ are equal to the phase noise differences $\Delta\theta_{km}$. Phase noises $\theta_k$, k=1,...,N+1 can be considered independent Wiener random processes /4/, so they follow Gaussian distributions. Here it is assumed that the variance of the phase noises $\theta_k$, k=1,...,N+1 is large enough so they can be considered independent, identically distributed (i.i.d.) random variables (RVs) following a uniform distribution in the interval $[-\pi,\pi]$ (incoherent crosstalk case).

For the evaluation of the characteristic function of the photocurrent, first, we define the auxiliary variable $z=2I(t)/R$. Using (2) and the definition of the characteristic function /3/, it is straightforward to see that the characteristic function of z is given by

$$\psi_z(\omega)=E\{e^{i\omega z}\}= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\pi}^{\pi}\ldots\int_{-\pi}^{\pi} e^{i\omega z}p[n_c,n_s,\theta_1,\ldots,\theta_{N+1}]dn_c dn_s d\theta_1\ldots d\theta_{N+1}$$
$$=\frac{1}{1-2\sigma^2 i\omega}e^{-\frac{\sigma_{th}^2\omega^2}{2}+\frac{i\omega}{1-2\sigma^2 i\omega}\sum_{k=1}^{N+1}A_k^2}F(\omega) \tag{3}$$

where we defined the auxiliary function

$$F(\omega)=\frac{1}{(2\pi)^{N+1}}\int_{-\pi}^{\pi}\ldots\int_{-\pi}^{\pi}e^{\frac{2i\omega}{1-2\sigma^2 i\omega}\sum_{k=1}^{N}\sum_{m=k+1}^{N+1}A_k A_m\cos\Delta\theta_{km}}d\theta_1\ldots d\theta_{N+1} \tag{4}$$

In the case of one interferer (N=1), (4) can be evaluated analytically /5/ [eq.3.338 (4)], so (3) yields /2/

$$\psi_z(\omega)=\frac{1}{1-2\sigma^2 i\omega}e^{-\frac{\sigma_{th}^2\omega^2}{2}+\frac{i\omega(A_1^2+A_2^2)}{1-2\sigma^2 i\omega}}I_0\left(\frac{2i\omega A_1 A_2}{1-2\sigma^2 i\omega}\right) \tag{5}$$

where $I_0(x)$ is the modified Bessel function of the first kind of order zero.

In the case of multiple interferers (N>1), the evaluation of (4) requires N+1 numerical integrations. For this purpose, Gauss-Legendre quadrature can be used /6/. The selection of the number of integration nodes is a compromise between accuracy and computational complexity. Since the number of operations grows proportionally to the (N+1)-th power of the number of integration nodes, the latter must be kept as small as possible. Notice that in the special case when the crosstalk terms have all the same power level, it is possible to reduce the number of operations in (4) using symmetries for the integration variables $\theta_k$, k=1,...,N+1.

From the characteristic function (3), it is possible to calculate the probability density function (pdf) using a Fourier transform /3/

$$p_z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi_z(\omega)e^{-i\omega z}d\omega \qquad (7)$$

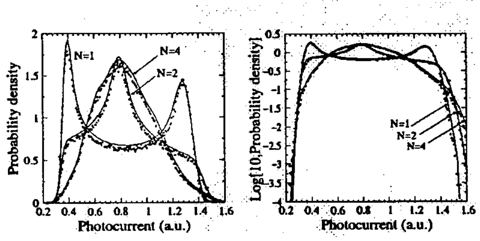
No analytical expression for the integral (7) can be found and the Fourier transform must be evaluated numerically.

Finally the pdf of the photocurrent can be evaluated using the relationship /3/ $p_I(I) = (2/R)p_z(2z/R)$ .

In the following, the theoretical results are compared with the experiment. The experimental setup was described in /2/.

## Results and discussion

Fig.1 shows histograms of the photocurrent I(t) for three different numbers of interfering channels (N=1, 2 and 4). The points are experimental data taken from /2/. The lines are calculated from the theoretical model. For convenience, R/2=1 is assumed. The other parameters used for the fitting are: I) For N=1, $A_1$=0.88, $A_2$=0.27, $\sigma$=0.022 (corresponding to –10 dB crosstalk level); II) For N=2, $A_1$=0.89, $A_2$-$A_3$=0.155, $\sigma$=0.022 (corresponding to –15 dB crosstalk level/interferer). Gauss-Legendre quadrature with 40 nodes is used for the evaluation of (4); III) For N=4, $A_1$=0.9, $A_2$-$A_5$=0.09, $\sigma$=0.022 (corresponding to –20 dB crosstalk level/interferer). Gauss-Legendre quadrature with 10 nodes is used for the evaluation of (4). In all cases, a fast Fourier transform (FFT) with 2048 points is used for the numerical evaluation of (7). The receiver thermal noise variance $\sigma_{th}^2$ is set to zero.




(a) (b)

**Fig. 1** Photocurrent histograms (a) Linear; (b) Logarithmic (Symbols: lines= model; points=experiment /2/).

The theoretical model describes accurately both the center and the tails of the measured pdf. The shape of the pdf is clearly asymmetric, with steeper left edge and smoother right edge. The asymmetry is due to the ASE noise-crosstalk beating, as well as the direct detection of the ASE noise. It is worth noting that the asymmetry of the pdf was observed experimentally before, e.g. /7/, but it was not understood.

Fig. 2 shows a comparison of the photocurrent pdfs as calculated using the accurate current model (solid lines) and the previous approximate model /1/ (dotted lines), for the same conditions of Fig.1. The approximate model /1/ predicts a symmetric shape for the photocurrent pdf and therefore can never match perfectly the whole exact pdf, but only parts of it. Moreover, the approximate model /1/ yields a small shift in the mean of the pdf due to the omission of the terms related to the direct detection of the crosstalk and the ASE noise. These discrepancies between the two models diminish when the crosstalk level decreases. A detailed study for the case of one interferer (N=1) and the same conditions as in Fig. 1 reveals that the tops of the approximate and the exact pdf match for a crosstalk level below –30 dB but the difference at the pdf tails persists even for a crosstalk level below –40 dB.

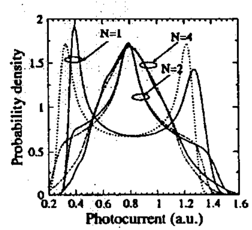



**Fig. 2 Model comparison (Symbols: solid lines= current model; dotted lines= previous model /1/).**

In conclusion, an accurate expression of the pdf of the photocurrent for optically amplified, direct detection communication systems in the presence of N homodyne incoherent interferers is derived. The model is compared with the experiment and is shown to be in excellent agreement.

## References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