MIMO Nonlinear Equalizer based on Inverse Volterra Series Transfer Function for Coherent SDM Systems

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Abstract: We propose a novel MIMO Volterra-based nonlinear equalizer. Simulation results revealed a ~ 0.8 dB Q^2 -factor improvement, after transmitting 6×32 Gbaud PM-16QAM signals carried by 6 spatial modes over 1,040 km of an FMF link.

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1. Introduction

Space division multiplexing (SDM), utilizing few mode fibers (FMFs), has been considered as the method of choice for expanding the capacity of future systems [1,2]. Nonetheless, there is always a certain launch power above which the capacity will only degrade. This nonlinear Shannon limit can be mitigated by using digital nonlinear equalizers (NLEs) in the optical transceivers [3]. Contrary to single-mode fibers (SMFs), FMFs are inherently multiple inputmultiple output (MIMO) transmission media that exhibit both linear and nonlinear impairments. Thus, the development of MIMO-based equalizers is required in order to recover the signal quality and increase the maximum transmission reach. Recent publications have studied MIMO NLEs in the context of two-mode fiber system [4], whereas limited work has been done on the development of more scalable schemes.

In this paper, we present a novel MIMO NLE, based on 3rd-order inverse Volterra series transfer function (IVSTF) [5]. The Volterra-based NLE was chosen due to its lower computational complexity and performance compared to the performance and complexity of the digital back-propagation NLE, in multi-channel equalization schemes [6]. In this work, first, we identify the optimum operating conditions of the $2M \times 2M$ 3rd-order IVSTF-NLE. where M denotes the number of spatial weakly-coupled modes, through a two-step optimization process. Second, for these optimum operating conditions, we evaluate its performance after the propagation of $M \times 32$ Gbaud polarization-multiplexed (PM)-16 quadrature amplitude modulation (QAM) signals through 1,040 km of an FMF. The results reveal a ~0.8 Q^2 -factor improvement, compared to maximum Q^2 with linear equalization only, when a 6×32 Gbaud PM-16OAM signal has been transmitted.

2. Operating principle of the MIMO IVSTF-NLE

Similar to the 3rd-order IVSTF-NLE implementation in SMF systems [5], we start from the expanded Manakov equation which describes signal transmission over M spatial modes, each supporting two polarization states [7]:

$$\frac{\partial \mathbf{A}_{p}(z,t)}{\partial z} = -\frac{a_{p}}{2} \mathbf{A}_{p}(z,t) - \beta_{1, p} \frac{\partial \mathbf{A}_{p}(z,t)}{\partial t} + j \frac{\beta_{2, p}}{2} \frac{\partial^{2} \mathbf{A}_{p}(z,t)}{\partial t^{2}} - j \left[\frac{8}{9} \gamma_{pp} \left| \mathbf{A}_{p}(z,t) \right|^{2} + \frac{3}{4} \sum_{p \neq q}^{M} \gamma_{pq} \left| \mathbf{A}_{p}(z,t) \right|^{2} \right] \mathbf{A}_{q}(z,t)$$
(1)

where the variables t and z denote time and distance axes, respectively, while $\mathbf{A}_p(z,t) = [A_{p,x}, A_{p,y}]^T$ and $\mathbf{A}_q(z,t) =$ where the variables t and 2 denote time and distance axes, respectively, while $A_p(2,t) = [A_{p,x}, A_{p,y}]^{-1}$ and $A_q(2,t) = [A_{q,x}, A_{q,y}]^{T}$ are the Jones vectors of the slowly-varying optical signal envelopes for the p^{th} and q^{th} modes, with p,q = 1,...,M. The total power per mode is given by the expressions $|A_p(z,t)|^2 = |A_{p,x}|^2 + |A_{p,y}|^2$ and $|A_q(z,t)|^2 = |A_{q,x}|^2 + |A_{q,y}|^2$. Furthermore, α_p is the attenuation coefficient, $\beta_{1,p}$ and $\beta_{2,p}$ are the group delay per unit length and the chromatic dispersion (CD) parameters, respectively, of the p^{th} mode, while γ_{pp} and γ_{pq} are the intra- and inter-modal nonlinear coefficients. Note that γ_{pp} and γ_{pq} do not exist in the SMF systems, therefore, their use justifies the MIMO extension of the 2×2 3rd-order IVSTF-NLE, as described in [5]. Following the rationale of [5], the solution of (1) can be expanded into IVSTF kernels up to the 3rd-order in order to form the 2M×2M IVSTF-NLE. The block diagram of the latter is depicted in Fig. 1(a). It is comprised of the linear and the nonlinear branches in a parallel structure for each mode. The linear compensation is completed in a single branch per polarization, accounting for the CD introduced by all the N fiber spans, in the frequency domain.



Fig. 1. (a) Block diagram of the $2M \times 2M$ 3rd-order IVSTF-NLE for 2M modes; (b) The operating principle of the k^{th} nonlinear branch of the MIMO NLE for two adjacent spatial modes, p and q, each supporting two polarization states. (Symbols: Inverse Fast Fourier Transform (IFFT), Linearly Equalized (LE) and Fast Fourier Transform (FFT)).

Each polarization tributary passes through a filter with transfer function $(K_{l,p}(\omega))^N$, where L_s is the span length. On the contrary, the intra- and inter-modal nonlinear compensation is performed in a span-by-span basis through N separate branches, in the time domain.

Fig. 1(b) describes the operating principle of the k^{th} nonlinear branch, where k = 1,...,N. It is divided into four stages instead of three, as it is the case for the 2×2 3rd-order IVSTF-NLE [5,8]. In the first stage, the received polarization tributaries, X_{p} , and Y_{p} are linearly compensated by the filter $(K_{1,p}(\omega))^{k}$, in the frequency domain. The compensation of the intra-modal nonlinearities occurs in the second stage, where the linearly-compensated polarization tributaries, X_{p}^{LE} and Y_{p}^{LE} , are multiplied, in the time domain, with the total power of the p^{th} mode (i.e., $|X_{p}^{\text{LE}}|^{2}+|Y_{p}^{\text{LE}}|^{2}$) and scaled by the adjustable intra-modal parameters c_{pp} . Similar procedure is followed for the q^{th} mode. The third stage is the one that extends the 2×2 3rd-order IVSTF-NLE to its MIMO version and where the inter-modal nonlinear compensation occurs, in the time domain. The polarization components of the p^{th} mode are multiplied with the total power of the q^{th} mode (i.e., $|X_{q}^{\text{LE}}|^{2}+|Y_{q}^{\text{LE}}|^{2}$) and scaled by the inter-modal parameter c_{qp} . The vice versa is realized for the q^{th} mode using the c_{pq} . Generally, for M > 2, each polarization component of each mode is compensated by inter-modal nonlinearities when multiplied with the sum of the total powers of the rest of the modes, and only after each one of these power profiles has been multiplied with the corresponding inter-modal parameter. The c_{pp} and c_{pq} represent the optimized values of the products $c_{intra,pp} = k_{pp} \times \gamma_{pp} \times L_{eff}$ and $c_{inter,pq} = k_{pq} \times \gamma_{pq} \times L_{eff}$, where $L_{eff} = (1-\exp(-\alpha L_s))/\alpha$, achieved by sweeping the dimensionless parameters, k_{pp} and k_{pq} in a range from 0 to 1. Finally, in the fourth stage, the residual CD is compensated, in each nonlinear branch, by passing through the filter $(K_{1,p}(\omega))^{N,k}$, in the frequency domain.

3. Simulation setup

We numerically investigate the performance of the proposed MIMO 3^{rd} -order IVSTF-NLE on the transmission of a single spatial super-channel of *M* weakly-coupled modes, each supporting two polarization states, after 13×80 km of an FMF link. The simulation setup is shown in Fig. 2 and the simulation parameters are given in Table I.



Fig. 2. Simulation setup. Symbols: Few Mode Erbium-Doped Fiber Amplifier (FM-EDFA), Analog-to-Digital converter (ADC).

The signal propagation is described in (1). Each of the *M* spatial modes carries a PM-16QAM signal shaped by a square root raised cosine filter (raised cosine roll off equal to 0.1). The transmitted signal is $M \times 32$ Gbaud PM-16QAM. For simplification, we use the same CD parameters, as well as intra- and inter-modal nonlinear coefficients for every mode. At the receiver, the super-channel is selected by an optical bandpass filter of rectangular shape and bandwidth equal to 1.2 times the symbol rate. After mode de-multiplexing and matched filtering, the received signals are down-sampled at 2 samples per symbol (SpS) and pass to the MIMO-NLE. Prior to demodulation, further down-sampling to 1 SpS. As a figure of merit, we use the Q^2 -factor as calculated by the relationship $Q^2 = 10\log_{10}(1/EVM)^2$, where EVM is the error vector magnitude. Also, the Q^2 -factor improvement is used and defined as $\Delta Q^2 = Q^2_{\text{max.wo.NLE}} - Q^2_{\text{max.wo.NLE}}$, where $Q^2_{\text{max.wo.NLE}}$ are the maximum values of Q^2 with and without the use of the MIMO IVSTF-NLE, respectively, at the corresponding optimum powers. In the case where the MIMO NLE is not applied, linear equalization does occur.

Table I: Simulation parameters.

# of weakly-coupled modes M	1,3,6	Symbol rate (Gbaud)	32	α (dB/km)	0.2
Simulated symbols	32768	Noise figure (dB/km)	4.5	γ_{pp} (W ⁻¹ km ⁻¹)	1.4
Oversampling	16	Dispersion (ps/nm/km)	17	γ_{pq} (W ⁻¹ km ⁻¹)	0.7

4. Results and discussion

Initially, a two-step optimization procedure is applied to achieve the optimum performance of the MIMO IVSTF-NLE. Firstly, we identify the optimum input power. We sweep simultaneously the adjustable parameters k_{pp} and k_{pq} (i.e., $k_{pp} = k_{pq} = c$) and we perform a joint optimization of their single value with the average input power for each of the *M* spatial modes. The results of this parallel processing are shown in Fig. 3(a). Specifically, for M = 6, the $2M \times 2M$ IVSTF-NLE provides $\Delta Q^2 \approx 0.737$ dB at optimum input power equal to -5 dBm and $k_{pp} = k_{pq} \approx 0.5$. Secondly, at the optimum input power (i.e., -5 dBm), we independently sweep k_{pp} and k_{pq} until we obtain the highest ΔQ^2 . In Fig. 3(b), we observe that when the values of k_{pp} and k_{pq} are equal to ~0.6 and ~0.5, respectively, then the maximum ΔQ^2 is ~0.793. The same two optimization steps are repeated for M = 1 and M = 3. In Fig. 3(c), we show the absolute values of Q^2 with and without MIMO NLE and as a function of the input power, after transmitting $M \times 32$ Gbaud PM-16QAM signals, where M = 1,3,6, over 13×80 km of an FMF. Every time the capacity is tripled, the optimum input power (i.e., the nonlinear threshold) is decreased by 1 dB while the maximum ΔQ^2 also decreases from $\Delta Q^2 \approx 1.3$ dB for M = 1, to $\Delta Q^2 \approx 1$ dB for M = 3 and to $\Delta Q^2 \approx 0.8$ dB for M = 6.



Fig. 3. (a) Contour plot of ΔQ^2 as a function of the input power and the $k_{pp} = k_{pq} = c$ adjustable parameter for M = 6; (b) Contour plot of ΔQ^2 as a function of the k_{pp} and k_{pq} at the optimum input power; (c) Q^2 -factor versus input power, with and without MIMO-NLE, after transmitting $M \times 32$ Gbaud PM-16QAM signals, where M = 1,3,6 over 13×80 km.

5. Summary

The performance of the proposed MIMO IVSTF-NLE is numerically studied after transmitting $M \times 32$ Gbaud PM-16QAM signals, where M = 1,3,6, over 13×80 km of an FMF. The results reveal a ~0.8 dB Q^2 -factor improvement, compared to linear equalization case, after a 6 times spatial increase of the transmitted capacity. In this work, the $2M \times 2M$ IVSTF-NLE is studied in a benchmarking case, whereas future work may focus on examining the statistical impact of differential group delay on the equalization performance.

6. References

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