Outage Probability Due to PMD in Coherent PDM QPSK Systems With Electronic Equalization

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***Abstract*—Polarization-division-multiplexed (PDM) quadrature phase-shift-keying (QPSK) coherent optical systems employ blind adaptive electronic equalizers for polarization-mode dispersion compensation. In this letter, we compare the performance of fractionally spaced, linear electronic equalizers, composed of four parallel finite impulse response (FIR) filters of various lengths, using the outage probability as a performance criterion. The constant modulus algorithm is applied for the adaptation of FIR filter coefficients. A parallel programming implementation of the multicanonical Monte Carlo method is adopted for the estimation of the tails of the outage probability distribution. It is shown that less than 20 complex, half-symbol-period-spaced taps per FIR filter suffice, in order to reduce the outage probability of PDM QPSK coherent optical systems to less than 10^{-5}, for a mean differential group delay up to twice the symbol period.**

***Index Terms*—Multicanonical Monte Carlo (MMC), optical communications, outage probability, parallel programming, polarization-mode dispersion (PMD).**

I. Introduction

In contrast to conventional intensity-modulation/direct-detection systems, polarization-division-multiplexed (PDM) quadrature phase-shift-keying (QPSK) coherent optical communications systems are extremely vulnerable to polarization impairments. Therefore, adaptive electronic equalizers are indispensible in order to counteract rapid polarization rotations, to perform polarization demultiplexing, and to combat polarization-mode dispersion (PMD) and polarization-dependent loss (PDL) [1]–[4]. Nevertheless, due to the limited length of their finite impulse response (FIR) filters [1], as well as to singularities in their coefficient adaptation algorithms [4], [5], these equalizers cannot fully invert the transfer function of the optical channel. Therefore, there is always a non-negligible, albeit small, power outage probability, even after equalization.

To the best of the authors' knowledge, there is no in-depth theoretical study of the performance of fractionally spaced, linear electronic equalizers of various lengths and coefficient adaptation algorithms for coherent PDM QPSK systems, using the outage probability as a performance criterion. The reason lies in the inherent difficulty of this endeavor. Power outages after electronic equalization have very low probability of occurrence and hence, even experimental measurements may be unable to assess the system performance [2]–[4]. Using conventional Monte Carlo (MC) simulations to estimate very low outage probabilities would require a prohibitive number of iterations. Importance sampling schemes, such as multicanonical Monte Carlo (MMC) [6], can be used to reduce the number of iterations required to estimate the probability of such rare events. The advantage of the MMC technique over other importance sampling methods is that little (if any) *a priori* knowledge of the target probability density function (pdf) is required. The execution time can be further reduced using parallel programming and high-performance computing systems.

Recently, we have shown that a coherent PDM QPSK system with a 5-tap half-symbol-period-spaced adaptive electronic equalizer, based on the constant modulus algorithm (CMA) [7], exhibited an outage probability lower than 10^{-5} at 1-dB system margin for a mean differential group delay (DGD) equal to half of the symbol period [8]. In this letter, we elaborate on this initial study. More specifically, we use a parallel implementation of the MMC method for the efficient evaluation of the outage probability of a coherent PDM QPSK system with adaptive electronic PMD equalizers of various lengths. The letter's contribution is twofold: 1) it is the first study of its kind for coherent PDM QPSK systems, and 2) parallel MMC simulation is applied, for the first time, to evaluate the performance of adaptive PMD equalizers. Our simulations show that using system margins of 1, 2, and 3 dB, the adaptive PMD equalizers of 5, 10, 15, and 20 taps, respectively, produce outage probabilities lower than 10^{-5}, even for mean DGD twice the symbol period.

II. System Model

A. MMC Method and the PMD Emulator

The MMC method is a recursive simulation technique for the efficient and accurate estimation of very low occurrence probabilities. In essence, MMC simulations are adaptively biased with *a priori* unknown weights. We assume that the output random variable whose pdf is unknown (referred to as control parameter) $Y = f(X_1, \ldots, X_M)$ is a function of input random variables $X_1, \ldots, X_M$ whose joint pdf is known. Note that the function $f(\cdot)$ need not be known in analytic form. MMC consists of a predetermined number of iterations n, during which a set of the input random variables $\{\{X_1^{(1)}, \ldots, X_M^{(1)}\}, \ldots, \{X_1^{(n)}, \ldots, X_M^{(n)}\}\}$ is generated, based on the accept/reject rule of the Metropolis algorithm

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[9]. The corresponding candidate values of the output control parameter $\{Y_1^{(1)}, \ldots, Y_M^{(n)}\}$ are obtained and a histogram $\hat{H}_k^{(n)}$, over a prespecified number of bins, is constructed. At the end of the nth iteration, the probability for the kth bin $\hat{P}_k^{(n)}$ is calculated and the transition from the old to the new state is accepted/rejected using a properly specified criterion [9]. In subsequent iterations, the information of the estimated probability $\hat{P}_k^{(n)}$ is used to bias the input joint pdf in order to force the output samples to fall on tails of the desired output pdf. As the number of iterations increases, one obtains a more accurate estimation of the output pdf.

For the problem under study, the MMC method is used for the efficient emulation of PMD. Thus, the optical fiber is modeled as a concatenation of 30 birefringent sections. The MMC method consists of 10 iterations, during which 1000 realizations of the PMD fiber are generated. In this case, the input random variables $X_1, \ldots, X_M$ of the MMC are the parameters of each birefringent section, i.e., the triplets $\{\alpha_m, \varepsilon_m, \Delta\tau_m\}$, where α_m, ε_m are the azimuth and ellipticity of the slow principal axis and $\Delta\tau_m$ the differential delay of the mth birefringent section, respectively. The statistics of α_m, ε_m are such that the corresponding PMD vectors are uniformly distributed over the Poincaré sphere [8]. The differential delays $\Delta\tau_m$ are independent, identically distributed Gaussian random variables. These assumptions lead to a Maxwellian pdf for the instantaneous DGD [8]. In direct detection, it is customary to neglect the random differential carrier phase shift between the principal axes of each waveplate due to local birefringence. However, given the phase sensitivity of coherent detection, these phase shifts should be taken into account because they slightly worsen the performance of coherent PDM QPSK systems. Their impact is omitted here but will be addressed in future work.

The OSNR penalty is the control parameter Y and is defined as the difference in OSNR (measured in decibels using a resolution bandwidth equal to the symbol rate) between the back-to-back and the PMD distorted system, required to achieve a target error probability of 10^{-9}. The error probability is estimated using a semi-analytical method [10]. The outage probability is defined as the probability that the OSNR penalty exceeds a specified threshold (system margin). We arbitrarily choose an outage probability of 10^{-5} as a design criterion, which translates to a fractional outage time of 5.4 min per year [11]. The outage probability is evaluated at the last MMC iteration for thresholds equal to 1, 2, and 3 dB.

Furthermore, the structure of the MMC algorithm lends itself to parallel computing. Parallelization is performed at two levels: 1) different runs corresponding to different mean DGD values are launched on different computers (embarrassingly parallel application), 2) within a single run, concurrent execution of commands within "for loops" is achieved by using different cores on a multicore computer. For instance, we managed to accelerate the execution time for each separate MMC simulation up to three times by using four cores of a Quad Intel processor, so that the generation of each point in Fig. 3 requires about 4 h.

B. Coherent PDM QPSK System Simulation Model

The block diagram of the system under study is shown in Fig. 1. The two orthogonal polarization components, x, y, of a laser source are independently QPSK modulated in the PDM QPSK transmitter, and subsequently combined and fed into the transmission fiber. By appropriately selecting the modulation bit sequences, the transmitted QPSK symbols form De Bruijn sequences of length 4^5, in order to accurately model five-symbol long ISI caused by PMD.

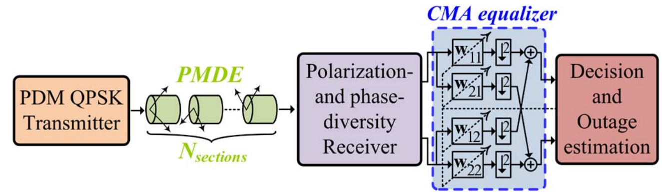

Fig. 1. System block diagram. (Abbreviations: PMDE: PMD Emulator; $\mathbf{w}_{ij}$: FIR filters' taps.)

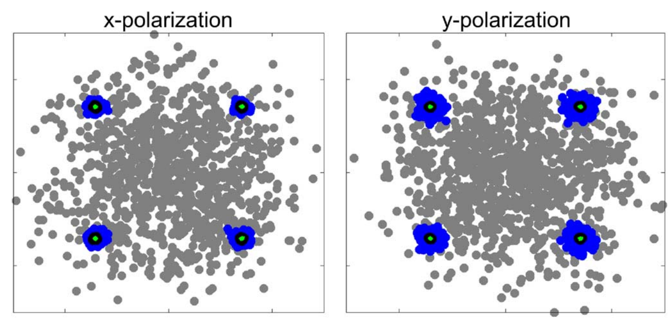

Fig. 2. Representative constellation diagrams of the two polarization tributaries, in the presence of PMD, before and after compensation. Symbols: uncompensated (gray dots), 5-tap (blue dots), 10-tap (black dots), and 15-tap (green dots) CMA equalizer for instantaneous DGD equal to 2.8 T_s.

A coherent, homodyne, polarization-, and phase-diversity receiver is used, consisting of ideal 2×4 90° optical hybrids and balanced detectors. The photocurrents are filtered and sampled at twice the symbol rate.

PMD compensation is performed electronically, using a linear adaptive electronic CMA-based equalizer [4], [7] with a butterfly structure. The latter consists of four transversal FIR filters (whose impulse responses are denoted in Fig. 1 by $\boldsymbol{w}_{ij}$, $i, j = 1, 2$). This results in an equalizer transfer matrix similar to the one of the optical channel. The filters' taps are $T_s/2$-spaced, where T_s is the symbol period. Synchronization is achieved by scanning all possible sampling instances for a small time window, and choosing the one with the largest average constellation opening. Feed-forward frequency and phase-error estimators are used [2] to remove any small residual constellation rotations.

III. Results and Discussion

A qualitative comparison of the performance of adaptive electronic CMA equalizers with respect to the uncompensated case, with various numbers of taps, is shown in Fig. 2. Here, we assume a back-to-back system with a one-tap electronic demultiplexer [5] as the uncompensated case. Fig. 2 depicts representative constellation diagrams obtained from the last MMC iteration for the cases of an uncompensated PDM QPSK system (gray dots), and after a CMA equalizer with 5 taps (blue dots), 10 taps (black dots), and 15 taps (green dots), for an instantaneous DGD value equal to 2.8 T_s. As the number of taps increases, the constellation diagram of both polarization tributaries opens, indicating the efficient convergence of the CMA equalizer for this indicative simulation instant. Furthermore, the

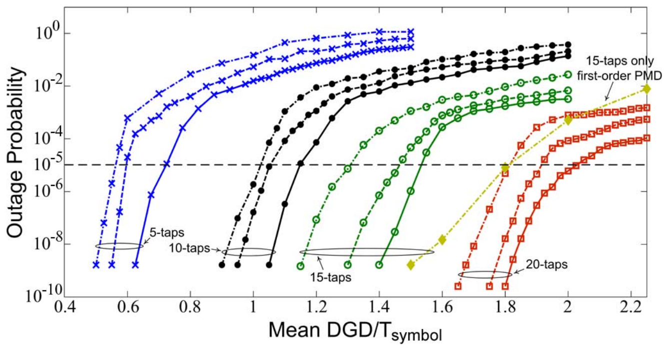

Fig. 3. Outage probability as a function of the normalized mean DGD for a CMA-based equalizer with 5 taps (crosses), 10 taps (dots), 15 taps (circles), and 20 taps (squares) per FIR filter for 1-dB (dashed–dotted line), 2-dB (dashed line), and 3-dB (solid line) threshold, respectively. The yellow line represents only the first-order PMD equalization with a 15-tap CMA equalizer.

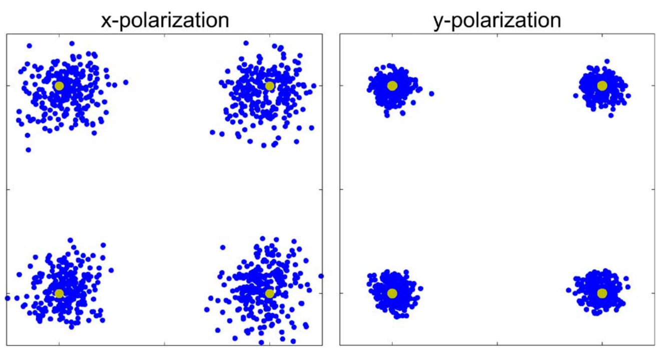

Fig. 4. Constellation diagrams of the two polarization tributaries after equalization for an all-order (blue dots) and first-order only (yellow dots) PMD case study. Symbols: 15-tap (blue dots) CMA equalizer for PMD emulator with 30 sections (all-order PMD case) and 15-tap (yellow dots) CMA equalization for first-order only PMD (first-order PMD case).

value of the OSNR penalty differs from the ideal case by only 0.052 dB with the 15-tap CMA equalizer.

Quantitatively, the performance of adaptive CMA equalizers, as a function of normalized mean DGD, using different thresholds to calculate the outage probability, is presented in Fig. 3. We observe that the CMA equalizer significantly reduces the impact of PMD. For instance, it considerably increases the tolerable mean DGD to 0.58 T_s for 5 taps per FIR filter and to 1.8 T_s for 20 taps per FIR filter for a 1-dB threshold at an outage probability of 10^{-5}. The 20-taps per FIR filter CMA equalizer can increase the tolerable *instantaneous* DGD values to 2.05 T_s for a 3-dB threshold. It is worth noting that the filter length requirements for equalizers provided by first-order PMD studies like [1] are optimistic, compared to the present work. The reason for this discrepancy is that in [1] only the first-order PMD is emulated, in contrast to this work where higher order PMD has been included. In Fig. 3, a 15-taps per FIR filter CMA equalizer increases the acceptable mean DGD to 1.8 T_s, if only first-order PMD is simulated. In contrast, the same equalizer succeeds in increasing the tolerable mean DGD by only 1.3 T_s when higher order PMD is also taken into account. In Fig. 4, indicative constellation diagrams reflect PMD equalization in first- and higher order PMD conditions with a 15-tap CMA-based equalizer, for instantaneous DGD equal to 1.5 T_s.

IV. Conclusion

In this letter, we compared the performance of fractionally spaced, CMA-based PMD equalizers for coherent optical PDM QPSK systems in the exclusive presence of PMD. Very rare PMD events were generated using a parallel programming implementation of the MMC method. The outage probability was used as a performance metric. Using system margins of 1, 2, and 3 dB, it was shown that $T_s/2$-spaced, CMA-based PMD equalizers of 5, 10, 15, and 20 taps per FIR filter, respectively, reduced the outage probability below 10^{-5}, even for a mean DGD twice the symbol period. For instance, the mean allowable DGD ranges from 0.58 T_s for 5-taps per FIR filter to 1.8 T_s for 20 taps per FIR filter for an outage probability of 10^{-5} and a system margin of 1 dB.

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