Asymmetric Probability Density Function of a Signal with Interferometric Crosstalk in Optically Amplified Systems

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Introduction

Interferometric crosstalk will play an important role in transparent WDM optical communication systems and networks. It can cause severe performance degradation as the systems and networks expand and has been analyzed in previous studies [1-4]. It has been shown that the Probability Density Function (PDF) of the photocurrent follows an arc-sine statistics when one interfering tone is present. These analyses usually assume negligible direct detection terms of the crosstalk and the Amplified Spontaneous Emission (ASE), as well as crosstalk-ASE noise beating. These assumptions together with Gaussian receiver noises lead to a symmetric two pronged distribution for the detected optical power with one crosstalk term. However, in a realistic system with a pre-amplified receiver, the interplay between interferometric crosstalk and ASE noise can not be neglected. In this paper, we present a theoretical and experimental study of the PDF of the received signal power in the presence of interferometric crosstalk for systems with optically pre-amplified receivers. The PDF is found be asymmetric due to the interplay between ASE and the interferometric crosstalk. An accurate model which takes into account the direct detection of the crosstalk and the ASE noise, as well as the ASE noise-crosstalk beating is used to explain the asymmetry. The theoretical PDF is compared with experiment and is found to be in excellent agreement.

Experiments and Results

The experimental set up is shown in Fig. 1. Tunable External Cavity Lasers (ECL) were used to generate the main Continuous Wave (CW) signal (ECL0) and the eight CW crosstalk channels (ECL1-ECL8). A polarizer was used to ensure that the signal and all crosstalk channels were at the same polarization state before the reciever. The ECLs were tuned to the same wavelength using a multi-wavemeter with an accuracy of 0.6GHz. An Erbium Doped Fiber Amplifier (EDFA) was used as the pre-amplifier to the receiver and a digital communication analyzer was used to measure the histogram of the received signal power. The electrical bandwidth of the receiver was nominally 30 GHz.

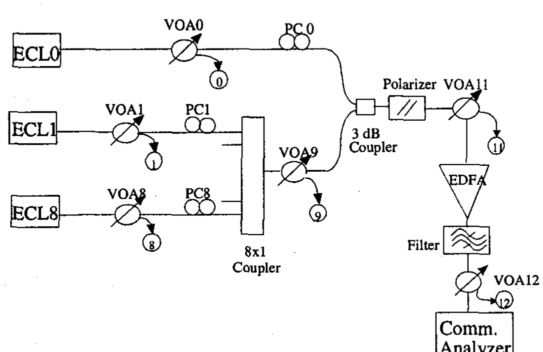
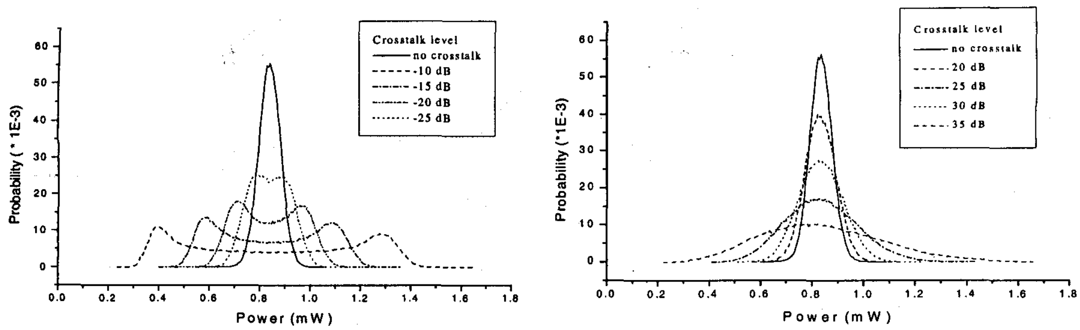


Fig. 1. Experimental setup

The power of the signal channel and crosstalk channels was measured after the polarizer. All crosstalk channels were maintained at the same power level. In this text the term crosstalk level will be used to describe the ratio of the power of each crosstalk channel to the main signal power.

The measured histograms of the received powers with different crosstalk levels are shown in Fig. 2. Fig. 2a shows histograms with one crosstalk term present. The solid line is the histogram of the signal without any crosstalk while different dashed lines correspond to crosstalk levels from –10 dB to –25 dB. The histogram of the signal with no crosstalk shows a Gaussian-like PDF with a central peak (to be exact, a non-central X^2 PDF [5-6]). When one crosstalk term is added, two peaks appear in the PDF corresponding to constructive and destructive interference between the signal and the crosstalk.

The PDF resembles the well-known arc-sine statistics of interferometric crosstalk, however, the two peaks are obviously asymmetric. The peak at the higher power is clearly broader than the peak at the lower power, which can be explained as follows: due to the interference between the signal and crosstalk, the total optical power entering the receiver changes. This happens on a time scale of the order of μs (~ inverse linewidth), which is slow compared to the response time of the receiver. Therefore the optical PDF will broaden due to the receiver noise. For a pre-amplified receiver, the dominant signal-ASE noise beating, whose variance increases with the signal power, leads to a larger broadening of the part of the PDF associated with the constructive interference, i.e. higher signal levels than the one associated with the destructive interference. A similar asymmetry of the PDF in experiments with multiple crosstalk channels was also observed (Fig. 2b).



(a) 1 crosstalk term

(b) 4 crosstalk terms

Fig. 2. Histograms of the signal consist of interferometric crosstalk. (a) one crosstalk term. (b) 4 crosstalk terms

This asymmetry is different from the one described in [1], where the PDF of the optical power at the output of a Mach-Zehnder interferometer was shown to be strongly asymmetric for mean phase difference of 0 and π when the interferometer was operated in coherent regime. In our experiments the main signal and crosstalk channels are generated by different lasers and the relative phases between the signal and the various crosstalk terms channels are uncorrelated. The crosstalk is incoherent and the phase difference is uniformly distributed between 0 and 2π.

An accurate model which takes into account both interferometric crosstalk and ASE is given below to provide a detailed explanation for the asymmetric behavior observed in the case of one interferer.

Theoretical Model

The power $P(t)$ of an optical signal impinging upon the photodiode of a preamplified receiver can be expressed as a function of the square module of the complex envelope of the electric field, $P(t) = \left|\tilde{E}(t)\right|^2/2$. If we consider one in-band incoherent interference term, the complex envelope is given by

$$\tilde{E}(t) = A_s e^{i\phi_s(t)} + A_x e^{i\phi_x(t)} + \tilde{n}(t) \tag{1}$$

where A_s, A_x are the amplitudes of the signal and interference term respectively, $\phi_s(t)$, $\phi_x(t)$ are the phase noises of the signal and interference term respectively, and $\tilde{n}(t)$ is the complex envelope of the ASE noise.

Substitution of (1) yields the following expression for the power

$$P(t) = \tfrac{1}{2}\left\{[A_s + A_x\cos\Delta\phi(t) + n_c(t)]^2 + [A_x\sin\Delta\phi(t) + n_s(t)]^2\right\} \tag{2}$$

where $\Delta\phi(t) \equiv \phi_x(t) - \phi_s(t)$, and $n_c(t)$, $n_s(t)$ are the quadrature components [5]of the complex envelope of the ASE noise which are assumed independent zero-mean Gaussian processes with variance σ.

It is possible to find the characteristic function of the power [6, Expr. 8-20]

$$\psi_P(P) \equiv E\{e^{i\upsilon P}\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\upsilon P}\, p(\Delta\phi, n_c, n_s)\, d(\Delta\phi)\, dn_c\, dn_s \tag{3}$$

where $p(\Delta\phi, n_c, n_s)$ is the joint PDF of the Random Variables (r.v.) $\Delta\phi, n_c, n_s$. Since the r.v.s are independent, the joint PDF can be expressed as a product $p(\Delta\phi, n_c, n_s) = p(\Delta\phi)p(n_c)p(n_s)$. It is assumed that $\Delta\phi$ is uniformly distributed in the interval [-π,π]. Substitution of $p(\Delta\phi), p(n_c), p(n_s)$ in (3) yields after some algebra

$$\psi_P(\upsilon) = \frac{1}{1-\sigma^2 i\upsilon} e^{i\upsilon\frac{(A_s^2+A_x^2)}{2(1-\sigma^2 i\upsilon)}} I_0\left(\frac{i\upsilon A_s A_x}{1-\sigma^2 i\upsilon}\right) \tag{4}$$

where $I_0(.)$ is the modified Bessel function of the first kind of order zero.

From (4), it is possible to evaluate the PDF of the detected power [6] using a Fourier transform

$$p_P(P) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_P(\upsilon) e^{-i\upsilon P} d\upsilon \tag{5}$$

Despite the fact that (4) is relatively simple, no analytical expression for the integral in (5) can be found and the Fourier transform must be evaluated numerically.

To illustrate the above formalism, Fig. 3 shows a plot of the accurate expression (5) (black curve) and the approximate expression [2] (gray curve) for $A_s = 1, A_x = 0.2, \sigma = 0.05$. The results were verified by using Monte Carlo simulation with 10^5 points but they are not shown in the graph for the sake of clarity. It is observed that the two curves are clearly offset but this offset decreases when the crosstalk level decreases. The shift is due mainly to the direct detection of crosstalk and to a lesser extent to the direct detection of the ASE noise (also referred to as ASE-ASE beating). The asymmetry is mainly due to ASE-crosstalk beating. Direct detection of the ASE noise, also causes a small asymmetry, even in the absence of crosstalk, but this effect is not so pronounced for the (linear) scale of the graph.

Fig. 4 shows a fit of measurements (points) with the expression (5) (curve) for $A_s = 0.88$, $A_x = 0.27, \sigma = 0.022$. It is observed that the theoretical curve approximates well the experimental points. There is a small discrepancy in the sense that the theoretical curve is always above the experimental points and slightly overestimates the left peak. This is attributed to the inaccuracies of the digital oscilloscope used in the experiment.

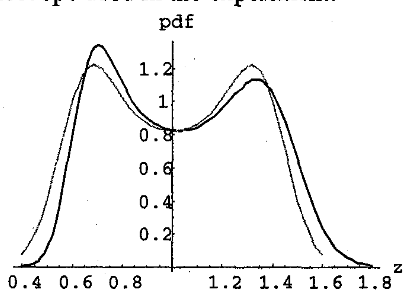


Fig. 3. Comparison of PDF's given by the expression (5) (black curve) and the approximate expression [2] (gray curve). Parameters: A_s=1, A_x=0.2, σ=0.05.

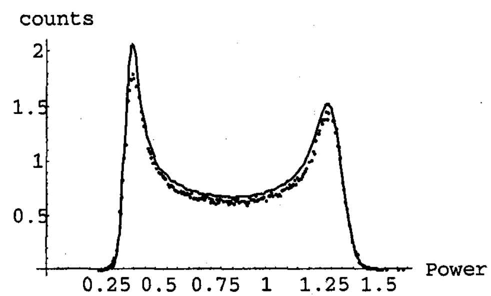


Fig. 4. Fit of measurements (points) with the expression (5) (curve). Parameters: As=0.88, A_x=0.27, σ=0.022.

Summary

An asymmetric PDF due to the beating of interferometric crosstalk and ASE noise in a system with optical preamplifiers was observed experimentally and explained theoretically for the first time. The theoretical PDF of a directly detected signal with one in-band incoherent interference term and ASE noise was compared with the experiment and is found to be in excellent agreement.

References

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