Computer modeling of the nonuniform FM transfer function of semiconductor lasers for the study of coherent optical systems

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ABSTRACT

This paper proposes an accurate computer model of the nonuniform FM response of semiconductor lasers, to be used in the computer-aided design of coherent optical communication systems. The model is communications engineer oriented and does not involve the physical insight of the device. The main idea of this approach is that the FM response of the laser can be approximated by a recursive digital filter based directly on measurements of the FM response. The procedure is divided into two steps : First, measurements of the FM response are fitted by a rational interpolant using the theory of multi-point Padé approximants. Then, the impulse invariant transformation is used to calculate digital filter coefficients. The procedure is applied in the case of a conventional single-electrode distributed-feedback (DFB) laser. The calculated digital filter is used to study the influence of the nonuniform FM response on the performance of a coherent heterodyne CPFSK system with differential receiver operating at 1 Gb/s. The sensitivity penalty is given as a function of signal-to-noise ratio, phase noise and sequence length by a semi-analytical technique. Theoretical and experimental results are in excellent agreement .

1 INTRODUCTION

Coherent optical communication systems are promising candidates for long-haul high density multichannel transmission. A suitable modulation format for these systems with respect to the actual technological limitations is the Continuous Phase Frequency Shift Keying (CPFSK).[1] Its principle advantages are the possibility of direct laser modulation, the tolerance to phase noise and the compactness of spectrum allowing dense optical frequency division multiplexing (OFDM).

Direct CPFSK modulation can be obtained by biasing the semiconductor laser with a DC current far above threshold and adding a small modulation current. The modulation current changes periodically the refractive index of the laser cavity. We can distinguish two different mechanisms which contribute to the refractive index changes, the temperature modulation effect and the carrier density modulation effect.[2] The temperature modulation effect dominates at low frequencies and the carrier density modulation effect at high frequencies. The two effects have a phase difference and their vectorial addition results, in general, in a nonuniform FM response.

The exact form of the FM response depends mainly on the structure of the device. As light source in the transmitter of coherent optical CPFSK systems, conventional single-electrode and three-electrode distributed feedback (DFB) lasers are most often used. The FM response of a conventional single-electrode DFB laser typically presents a magnitude dip in the 10 kHz to 10 MHz region and a phase transition from 180 deg to less than 0 deg as the frequency increases.[3]

The nonuniform FM response of the lasers causes intersymbol interference (ISI) which deteriorates the performance of coherent optical systems. This effect has been studied via digital simulation by several authors.[4-8] In these works, physical models[4,5] or analytical approximations[6-8] of the FM response were used for the calculation of the instantaneous frequency of the laser.

The diversity of these previous approaches shows that there exists no general formal procedure for computer modeling of the nonuniform FM response. Obviously, modeling depends on the application. Here we are interested in a model suitable for integration in a software package for computer aided design of coherent communication systems. For this purpose, an abstract, communications engineer oriented model, which does not involve the detailed physical theory of the laser is desirable. The model must be accurate, to allow for the calculation of low error probabilities and fast, to allow for the simulation of long sequences.

This paper proposes an algorithm for accurate computer modeling of the nonuniform FM response. The main idea of this approach is that the FM response of the laser can be approximated by a recursive digital filter based directly on measurements of the FM response. The procedure is divided into two steps : First, measurements of the FM response are fitted by a rational interpolant using the theory of multi-point Padé approximants. Then, the impulse invariant transformation is used to calculate digital filter coefficients.

The procedure is applied in the case of a conventional single-electrode DFB laser. The calculated digital filter is used to study the influence of the nonuniform FM response on the performance of a coherent heterodyne CPFSK system with differential receiver operating at 1 Gb/s. The simulation includes both laser phase noise and shot noise and makes use of a semi-analytical technique in order to evaluate low error probabilities ($\sim 10^{-9}$). The experiment verifies the theoretical results.

The paper is organized as follows: In section 2, the computer modeling procedure of the nonuniform FM response is described. In section 3, a design example is given. In section 4, the simulation model of a coherent heterodyne CPFSK system with differential receiver is presented. In section 5, the experimental set-up is described. Finally, in section 6, the influence of the nonuniform FM response on the system's performance is studied both theoretically and experimentally.

2 DIGITAL FILTER DESIGN

We are concerned in this section with the design of a recursive digital filter representing the nonuniform FM response. In the general case the design procedure can be divided into two parts, the approximation problem and the digitization problem.

In the following subsections, each problem is treated separately.

2.1 Approximation problem: modified Thacher-Tukey algorithm

The approximation problem can be stated as follows: Given a set of N distinct values of the amplitude and phase of the laser FM response, find a rational interpolant which passes from these points (the restriction to a rational interpolant is implied by the digitization problem, as it will be explained in the next subsection).

A rational interpolant which fits given function values at N points is called a N-point Padé approximant.[9]

Obviously, the N-point Padé approximant will tend to the real FM response as $N \rightarrow \infty$. However, it will be shown in the following that it is convenient for the simulation to keep the number of points N sufficiently low. For this reason, we must take care that the N-point Padé approximant possesses at least some of the properties of the true solution.

From a physical point of view, the interpolant can be viewed as the transfer function of an analog filter whose entry is the laser injection current and whose output is the instantaneous optical frequency. Consequently, it should satisfy the following requirements in order to correctly simulate the laser behavior:

i) The filter should exhibit a conjugate symmetry around the origin (i. e. $H(s^*) = H^*(s)$). The physical interpretation of this requirement is that the impulse response of the filter should be real.

ii) The filter must be lowpass (i. e. the order of the denominator polynomial must be greater than the order of the numerator polynomial so that the transfer function of the filter vanishes as $s \rightarrow \infty$).

iii) The filter must be stable (i. e. the poles of the denominator must be located at the left half of the s-plane).

For the calculation of a multi-point Padé approximant specific to the above requirements, we can use a highly efficient, always convergent computational scheme which is known as the modified Thacher-Tukey algorithm.[9] According to this scheme, consider the continuous fraction interpolant $r_N(s)$

$$r_N(s) = a_0 + \cfrac{a_1(s-s_0)}{1+\cfrac{a_2(s-s_1)}{1+\cfrac{a_3(s-s_2)}{1+\cfrac{\cdots}{1+a_{N-1}(s-s_{N-2})}}}} \tag{1}$$

In the above expression, $S_0 = \{s_0, \ldots, s_{N-1}\}$ is the set of the complex frequency points where the nonuniform FM response is known and a_i, $i = 0, \ldots, N-1$ are the coefficients we have to calculate. (The point s_{N-1} does not appear explicitely in (1) but it is used in the following for the calculation of a_{N-1}).

In order to preserve the property (i), the $s_0, \ldots, s_{N-1}$ should be chosen symmetrically around the origin (i. e. $N/2$ measurements of the nonuniform FM response and their complex conjugates at the negative frequencies should be used). Thus the number of points N in our case is always even. Obviously, for an even number of points, the order of the denominator polynomial of the $r_N(s)$ is always smaller than the order of the numerator polynomial.

The evaluation of the coefficients a_i, $i = 0, \ldots, N-1$ is done in order to satisfy the constraints

$$r_N(s_i) = H^{-1}(s_i) \quad i = 0, \ldots, N-1 \tag{2}$$

The reason for which we approximate the inverse of the FM response $H^{-1}(s)$ and not the FM response $H(s)$ itself, is that the $r_N^{-1}(s)$ has the desired property (ii).

The solution of the system of equations (2) is straightforward and can be done in analytical form by successive substitutions. The expressions for the a_i become intricate as i increases, so we give here only the three first coefficients in closed form:

$$a_0 = r_N(s_0) = H^{-1}(s_0)$$

$$a_1 = \frac{r_N(s_1) - a_0}{s_1 - s_0} = \frac{H^{-1}(s_1) - a_0}{s_1 - s_0}$$

$$a_2 = \left[\frac{a_1(s_2 - s_0)}{r_N(s_2) - a_0} - 1\right](s_2 - s_1)^{-1} = \left[\frac{a_1(s_2 - s_0)}{H^{-1}(s_2) - a_0} - 1\right](s_2 - s_1)^{-1}$$

After the evaluation of the coefficients a_i, $i = 0, \ldots, N-1$, $r_N(s)$ can be transformed into a simple fraction by a simple recursive procedure (theorem of Euler, 1757).[9]

Notice that the procedure converges only when the coefficients a_i, $i = 0, \ldots, N-1$ are finite and non-zero. This condition is always satisfied in practice. However, a somewhat subtler constraint is the stability requirement (property (iii)). The user must choose (by a trial and error procedure) the optimum number and location of the complex frequency points in order to obtain an approximant whose denominator poles are located at the left half of the complex frequency plane. As a rule of thumb, more points must be selected from the regions of the FM response which vary rapidly.

The same procedure with slight modifications could be used, at least in principle, to obtain a simple analytical expression for the transfer function of an FM equalizer.

The modified Thacher-Tukey algorithm is one of the numerous fitting procedures that we can apply to the approximation problem. Its advantages are that it is fast and accurate (it fits exactly the given points). Its drawback is that it does not give the minimum number of poles. This is equal to $N/2$, so it depends on the number of points chosen. Obviously, we could give some degree of freedom to the rational interpolant to diverge slightly from the measurements (using a minimum error criterion) in order to gain in number of poles.

2.2 Digitization problem: Impulse invariant transformation

The digitization problem can be stated as follows: Given the rational transfer function of an analog filter, design a digital filter whose transfer function approximates the analog one as closest as possible, at least at the region of interest.

Filtering is usually performed in the frequency domain by means of the fast Fourier transform (FFT). Unfortunately, in our case this approach is not efficient. The region of interest of the nonuniform FM response extends between $10^4 - 10^{10}$ Hz. If we choose 10^4 Hz as the fundamental frequency of the FFT, we need as many as 10^6 samples for simulating the whole FM response. In addition to that, double precision arithmetic is required to preserve the accuracy of the numerical calculations (a rule of thumb for choosing the precision of the arithmetic for the FFT routine, is to examine whether the first term in the Fourier series, i. e. $\cos(2\pi/N)$, is correctly evaluated. Since $\cos(2\pi/N) \simeq 1 - (2\pi/N)^2$ and $N = 10^6$, we need at least 12 significant digits for the computer representation). Clearly, these requirements in computer memory storage and CPU time, make FFT quite impractical for a fast simulation of long sequences.

An alternative way to perform filtering is to use a recursive digital filter working, of course, in the time domain. This filter will be a computer implementation of the analog filter whose transfer function was found in the previous section by use of the modified Thacher-Tukey algorithm. This procedure is known as digitization. Digitization of an analog filter corresponds to a transformation of the continuous time operation to discrete time operation. This transformation inevitably introduces distortion.

Several methods for digitizing an analog filter exist in the literature.[11,12] We have used a technique called *impulse invariant transformation.* This technique is treated in detail in the above references. Here we describe only its basic features.

Impulse invariant transformation is based on the principle that the impulse response of the digital filter must be a sampled version of the impulse response of the analog filter. Sampling of the time domain results in spectral overlap and aliasing in the frequency domain. In consequence, the digital filter has the same impulse response as the analog one but not the same FM response. However, the digital FM response can coincide with the analog one at least at the region of interest $10^4 - 10^{10}$ Hz by proper choice of the sampling rate. The most attractive property of the impulse invariant transformation is that it preserves both magnitude and phase characteristics in the region where no aliasing occurs.

The object of the impulse invariant transformation is to calculate the coefficients of a recursive filter simulating the nonuniform FM response. The evaluation procedure is given by the following algorithm[12] :

i) Expansion of the nonuniform FM response in partial fractions

$$r_N(s) = \sum_{i=1}^{N/2} \frac{r_i}{s - p_i} \tag{3}$$

where N is the number of complex frequency points which were used in the modified Thacher-Tukey algorithm.

ii) Calculation of the impulse response by inversion of the Laplace transform

$$h(t) = \sum_{i=1}^{N/2} r_i e^{p_i t} \tag{4}$$

iii) Calculation of the impulse response of the digital filter by sampling the analog one at sample intervals T_s

$$h_d(n) \triangleq h(nT_s) = \sum_{i=1}^{N/2} r_i e^{p_i n T_s} \tag{5}$$

iv) Calculation of the z-Transform of the digital filter

$$H_d(z) = \sum_{i=0}^{\infty} h(n) z^{-n} = \sum_{i=1}^{N/2} \frac{r_i}{1 - e^{p_i T_s} z^{-1}} \tag{6}$$

v) Evaluation of the instantaneous optical frequency $f(nT_s)$ by the following recursive relations

$$f(nT_s) = T_s \sum_{i=1}^{N/2} f_i(nT_s) \tag{7}$$

$$f_i(nT_s) = r_i i(nT_s) + e^{p_i T_s} f_i[(n-1)T_s] \tag{8}$$

where $i(nT_s)$ is the instantaneous injection current and $f_i(nT_s)$ are auxiliary parameters. The scaling factor T_s is added in (7) to compensate the gain induced by the sampling of the analog impulse response in Eq. (5).[12] With this adjustment $H_d(z)|_{z=e^{i\omega T_s}} \approx r_N(s)|_{s=i\omega}$.

In fact, the steps (ii)-(iv) are implicit to the procedure and can be omitted. They are used to establish a mapping relation between the step (i) and the step (v).

The relations (7), (8) can be viewed as a digital filter consisting of $N/2$ first order filters in parallel form (fig. 1).

3 EXAMPLE OF DIGITAL FILTER DESIGN

The design method outlined in the previous section, has been applied to the synthesis of a recursive filter representing the nonuniform FM response of a double-channel planar buried heterostructure (DCPBH)[13] DFB laser of our laboratory. The measurements are summarized in Table 1.

The values of the Table 1 and their complex conjugates at the negative frequencies were used in the modified Thacher-Tukey algorithm to calculate a 26-point Padé approximant in the continuous fraction form (1). The algorithm is fast (typically, a program written in double precision arithmetic needs $\sim$ 10 ms of CPU time on a DEC/μVAX computer).

The amplitude and the phase of the interpolant $r_{26}(s)|_{s=i\omega}$ (curves) are plotted in fig. 2 together with the measurements (points). The approximation of the nonuniform FM response is exact at the given points (precision of fifteen decimal digits) and very good in the frequency intervals between them (precision of three decimal digits). This accuracy is sufficient for the study of the influence of the nonuniform FM response on the performance of coherent optical systems.

A symbolic calculation package (Mathematica[14]) was used to convert the approximant to a simple fraction form and then to rewrite the simple fraction function in a partial fraction expansion like (3). The poles p_i and the residues r_i of $r_{26}(s)$ are summarized in Table 2.

From the values of the Table 2, it is straightforward to program the digital filter by using the relations (7), (8). The sampling rate was chosen equal to 1 THz in order to avoid aliasing. For this sampling rate $H_d(z)|_{z=e^{i\omega T_s}}$ was equal to $r_{26}(s)|_{s=i\omega}$ up to 10 GHz.

To test the validity of the model, the simulated impulse and step response of the digital filter were compared with the theoretical ones and were shown to be in excellent agreement.

In the following, the DFB model of this example is used to study the performance degradation of a coherent heterodyne CPFSK system with differential receiver operating at 1 Gb/s.

4 SIMULATION MODEL FOR A COHERENT HETERODYNE CPFSK SYSTEM WITH DIFFERENTIAL RECEIVER

A block diagram of the computer model of a coherent optical CPFSK system with differential receiver is shown in Fig. 3. The input data consists of a pseudorandom binary sequence of variable length (periods from $2^7 - 1$ to $2^{15} - 1$). The sequence represents a non-return to zero (NRZ) current of amplitude I_0 which is injected into the digital filter of the previous section. The output of this filter represents the instantaneous optical frequency $f(t)$.

It is well known[15] that the instantaneous optical frequency of a semiconductor laser presents fluctuations $\dot{\phi}(t)$. The frequency fluctuations $\dot{\phi}(t)$ are simulated as a white gaussian noise with two side power spectral density equal to $2\pi\Delta\nu$, where $\Delta\nu$ is the 3-dB spectral linewidth. This modeling does not take into account the $1/f$ noise behavior at low frequencies and the laser resonance peak.

The sum $2\pi f(t) + \dot{\phi}(t)$ is integrated by use of the trapezoidal rule to obtain the instantaneous phase $\phi(t)$ of the transmitted optical signal. The amplitude of the optical signal is considered as constant. This is only approximately true. In direct modulation, even for small modulation currents, there is a residual amplitude modulation (AM) of the optical signal coexisting with the FM. Its contribution at 1 Gb/s is not expected to be important, as it will be shown by the agreement between theoretical and experimental results. However, it is possible to incorporate this spurious effect in the calculations, simulating the AM response by an additional digital filter with the procedure developed in section 2.

Fiber and polarization dispersion and fiber nonlinearities are not included in the model. The local oscillator is assumed to have negligible phase noise.

The microwave current after the photodiode resulting by the mixing of the received and the local oscillator optical signals, can be written in equivalent baseband signal notation as

$$\tilde{i}_{ph}(t) = A_{IF} \exp\left\{ i \int_0^t \left[2\pi f(t') + \dot{\phi}(t') \right] dt' \right\} + \tilde{n}(t) \tag{9}$$

where $A_{IF} = 2R\sqrt{P_s P_{lo}}$, R is the responsivity of the photodiode, P_s is the received optical power from the transmitter, P_{lo} is the received optical power from the local oscillator and $\tilde{n}(t)$ is the sum of shot and thermal noises, which can be approximated as an additive white gaussian noise. Tilde denotes the complex envelope of the signals.

In Fig. 3, the transmitter, the local oscillator and the photodiode do not appear like separate units. The program generates directly the signal given by the relation (9).

The bandpass filter (BPF) and lowpass filter (LPF) bandwidths are chosen large in comparison to the signal bandwidth (20 and 10 GHz respectively). Thus the only contribution of the bandpass filter is to reduce noise and of the lowpass filter to eliminate the high frequency component produced by the multiplication. Their influence on the signal is negligible.

The differential delay was chosen equal to $\tau = T_b/2$. This is the optimum value of the delay for a modulation index of $m = 1$ when the laser's FM response is flat.[16] However this is not the case when the FM response is nonuniform. In order to optimize the system's performance, the amplitude I_0 of the injection NRZ current was modified until the achievement of a maximum eye opening at the output of the receiver for a sequence length $2^7 - 1$.

The degradation of the system's sensitivity in prior works is estimated by calculation of the amount of eye closure at the receiver output.[4,5,8] This approach takes into account only the signal distortion and does not consider the phase noise and the non gaussian shot noise statistics at the receiver output. Moreover, for long sequences and high phase noise levels, bit error rate floors can appear and the estimations of sensitivity penalty given by this method do not hold (cf. fig. 7).

In a more accurate approach, the error probability must be used as performance criterion. For the evaluation of the error probability we dealt with a semi-analytical technique.[17,18] According to this method, the signal is simulated in the absence of noise in order to compute the distortion induced by the nonuniform FM response. The nonuniform FM response changes the phase difference $\Delta\theta_n$ between the two entries of the microwave mixer. Then we can use the relation (10) of Jacobsen et al.[6] to evaluate the conditional error probability $P_{e|n}$ of the n-th bit of the sequence

$$\begin{aligned} P_{e|n} &= \frac{1}{2} - \frac{1}{2}\rho_n \, e^{-\rho_n} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left[I_k\left(\frac{\rho_n}{2}\right) + I_{k+1}\left(\frac{\rho_n}{2}\right) \right]^2 \\ &\quad e^{-(2k+1)^2 \pi \Delta\nu \tau} \cos[(2k+1)\Delta\theta_n] \end{aligned} \tag{10}$$

where ρ_n is the instantaneous signal-to-noise ratio at the entries of the microwave mixer, $I_k(x)$ are the modified Bessel functions of the first kind and $\Delta\nu$ is the 3-dB spectral linewidth of the transmitter.

The total error probability can then be estimated by averaging over all the output samples M

$$P_e = \frac{1}{M}\sum_{n=1}^{M} P_{e|n} \tag{11}$$

where M denotes the sequence length.

The semi-analytical approach is advantageous in comparison with the analytical method proposed by Jacobsen et al.[6] because it takes into account the whole sequence and not only the parts where several repeated marks or spaces occur. It was already used with success for the study of the influence of the bandpass filter (BPF) on the system's performance.[18,19]

For the simulations, we used TOPSIM,[20,21] a software package for simulation of analog and digital communication systems.

5 EXPERIMENT

For the verification of the theoretical evaluations, the experimental arrangement shown in Fig. 4 was used. The DCPBH DFB laser diode of the Table 1 was used as CPFSK transmitter. Its wavelength was 1520 nm and its 3-dB linewidth for the bias current chosen was about 29 MHz. The laser was directly modulated at a bit rate $R_b = 1$ Gb/s and the optical signal was launched into a single-mode fiber. An optical isolator providing more than 60 dB isolation was used at the laser output to avoid undesired feedback.

A tunable 10 KHz-linewidth external cavity laser was used as local oscillator. The local oscillator power received at the photodetector was $P_{lo} = -5.3$ dBm. This power was not sufficient to produce shot noise limited operation and the contribution of the thermal noise was significant. A polarization controller was used to match the state of polarization of the two lasers. The two optical fields were combined with an $1:1$ fiber coupler and detected by a PIN photodiode. The photodiode bandwidth was 13 GHz and its responsivity $R = 0.8$ A/W. The IF frequency was fixed at 3 GHz. The IF signal was amplified by a three stage wide-band preamplifier. After the first stage of amplification, the signal was filtered by a bandpass filter with bandwidth $B_{IF} = 2$ GHz. The delay-line discriminator had a 3 GHz zero crossing and delay $\tau = T_b/2$. The lowpass filter (LPF) had a 3-dB cut-off frequency equal to $0.8R_b$.

As for the simulation, the amplitude of the modulation current was adjusted to achieve a maximum eye opening at the output of the receiver for a sequence length $2^7 - 1$. The same amplitude was conserved for all the other sequences during the measurements of the bit error rate (BER).

6 RESULTS AND DISCUSSION

Fig. 5(a) and (b) compare respectively theoretical and experimental spectra of the CPFSK in the presence of phase noise. An ideal CPFSK spectrum for a modulation index $m = 1$ consists of a central lobe $3R_b$ large. Two discrete spectral lines appear at $\pm R_b/2$ MHz around the central frequency.[22] The effect of the nonuniform FM response is that the discrete lines have been smeared and the separation between the central and the secondary lobes has disappeared.

The agreement between the two spectra verifies that the modulation index was almost the same for the simulation and the experiment. Note that the experimental spectrum is asymmetric. This is due to the residual AM modulation of the optical signal which enhances the spectrum at lower frequencies.

The effect of the nonuniform FM response on the waveform at the output of the receiver is illustrated in the Fig. 6. The transmitted sequence was 1010101-32×0-10101010-011. Each bit of the sequence was transmitted consecutively 10 times (It is equivalent to say that the bit rate for this measurement was equal to $R_b = 100$ Mb/s and the modulation index m was ten times higher than at 1 Gb/s to yield the same frequency deviation). The solid line shows the waveform predicted in the theory and the dashed one shows the waveform measured in the laboratory. The theoretical waveform was plotted without phase noise. The experimental one was averaged.

For an ideal CPFSK system, the amplitude of the theoretical output signal in the Fig. 6 should take the values ± 0.2 V. The nonuniform FM response causes a reduction of the amplitude of the output signal whenever a long sequence of consecutive 1 or 0 is transmitted.

Finally, fig. 7 shows the estimated probability of error as a function of the signal-to-noise ratio for three different sequences (1010, $2^7 - 1$, and $2^{15} - 1$) together with the experimental points. For comparison, the error probability for a coherent DPSK or CPFSK system with a uniform FM response is plotted (curve 1). This curve is obtained by the formula (12) of Nicholson.[23] The sensitivity penalties at 10^{-9} are 2.8 dB for the sequence 1010, and 4 dB for the sequence $2^7 - 1$. A probability of error equal to 10^{-9} can not be achieved with a sequence of $2^{15} - 1$ because of the appearance of an error floor.

To account for the phase noise filtering, the phase noise variance in the simulations of fig. 7 was chosen so that the slope of the theoretical curves fits the slope of the experimental data. A 3 dB linewidth after filtering equal to $0.3\Delta\nu_{measured}$ (8.7 MHz instead of 29 MHz) seems to provide the closer approximation. This approximation is necessary since, to the authors knowledge, no theory exists which considers the phase noise filtering done by the two filters. Indicatively, we report that the formula (13) of Jacobsen and Garrett[25] predicts a phase noise variance reduction at $0.68\Delta\nu_{initial}$ due to the bandpass filter only, for $m = 1$ and $B_{IF} = 2R_b$. In addition to that, expressions (51), (52) of Kazovsky[15] predict a sensitivity improvement of 1.76 dB due only to the lowpass filter.

The difference between the experimental and theoretical results can be attributed to the residual AM modulation, the ISI induced by the narrow experimental filters and the non-ideal characteristics of the receiver.

The above results show that the proposed technique for the modeling of the nonuniform FM response, together with an accurate semi-analytical method can be a very efficient tool in the computer-aided design of the coherent optical systems. The semi-analytical method is very general and permits to incorporate other effects as well, like chromatic dispersion and signal distortions due to filtering in the study. However, the phase noise filtering is not accurately calculated and is a subject of further study.

7 SUMMARY

This paper presented an accurate computer model of the nonuniform FM response of the semiconductor lasers, to be used in the computer-aided design of coherent optical communication systems. The theory of multi-point Padé approximants and the impulse invariant transformation were used to conceive a digital filter simulating the laser's operation. The influence of the nonuniform FM response on the spectrum, the waveform at the output of the receiver, and the probability of error of a coherent optical CPFSK system were studied both theoretically and experimentaly. The agreement of theoretical and experimental results confirms the validity of the approach.

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f_m (MHz)	$\|H(s)\|$ (GHz/mA)	$arg[H(s)]$ (rad)
0.010	0.386	2.341
0.032	0.285	1.765
0.102	0.304	1.027
0.312	0.405	0.591
1.013	0.523	0.354
3.153	0.622	0.224
10.223	0.705	0.130
31.826	0.769	0.026
103.180	0.801	-0.218
200.000	0.738	-0.487
400.000	0.558	-0.843
700.000	0.377	-1.104
1000.000	0.278	-1.232

Table 1: Measurements of the nonuniform FM response $H(s)$ of a double-channel planar buried heterostructure (DCPBH) DFB laser.

p_i (MHz)	r_i (GHz MHz/mA)
-34121.348	12.252
-7408.376	11.517
-2137.558	1847.214
-1528.305	66.472
-512.503	-56.793
-184.971	-11.309
-63.383	-4.572
-20.928	-2.010
-6.745	-0.874
-2.118	-0.362
-0.640	-0.139
-0.176	-0.047
-0.034	-0.013

Table 2: Poles p_i and residues r_i of the 26-point Padé approximant $r_{26}(s)$ which fits the measurements of the Table 1.

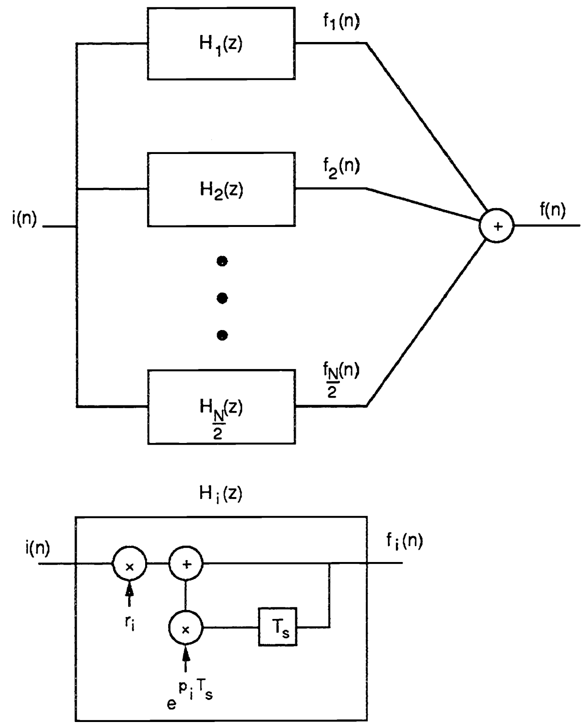

Figure 1: Digital filter structure.

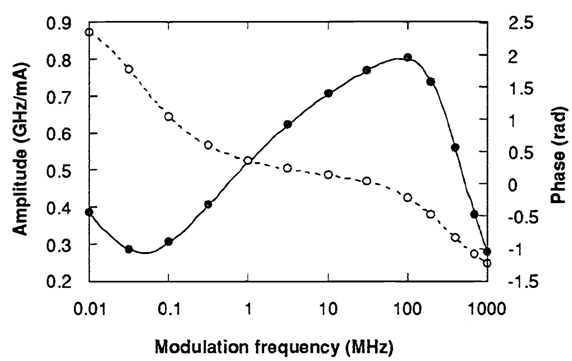

Figure 2: Comparison of the measurements of Table 1 and their fitting by the 26-point Padé approximant $r_{26}(s)|_{s=i\omega}$. (Symbols used : Solid line : $|r_{26}(s)|$, Dashed line : $arg[r_{26}(s)]$, $\bullet$: $|H(s)|$, $\circ$: $arg[H(s)]$).

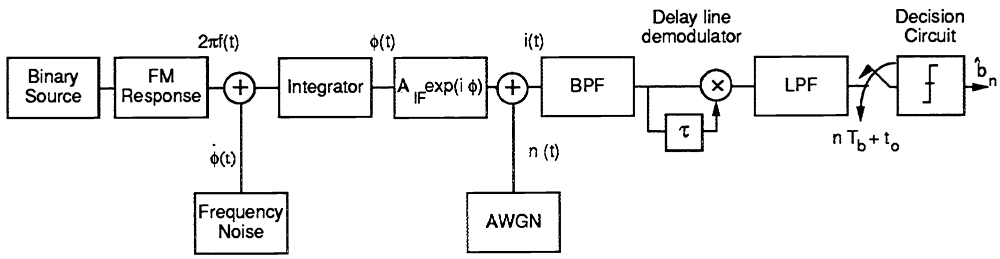

Figure 3: Block diagram of the simulation model (Abbreviations used: AWGN=additive white gaussian noise, BPF=bandpass filter, LPF=lowpass filter, τ=delay).

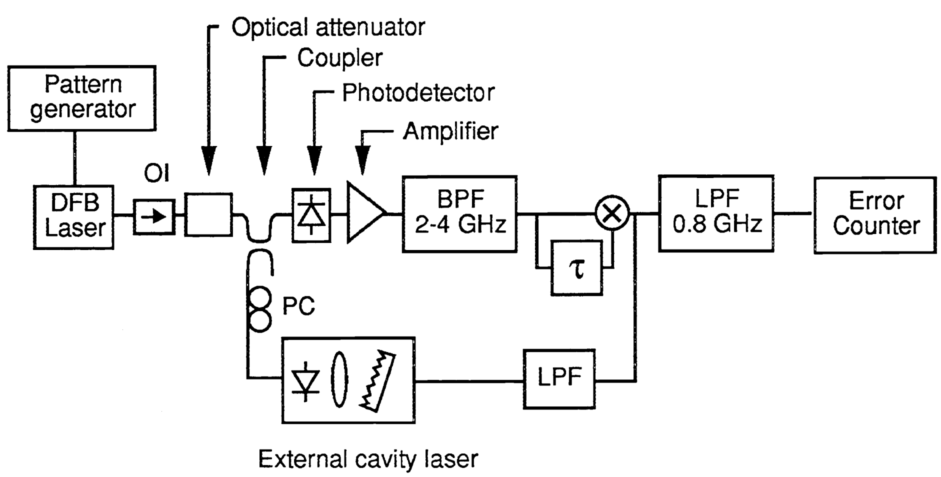

Figure 4: Experimental set-up (Abbreviations used: OI=Optical Isolator, PC=Polarization controller, BPF=bandpass filter, LPF=lowpass filter, τ=delay).

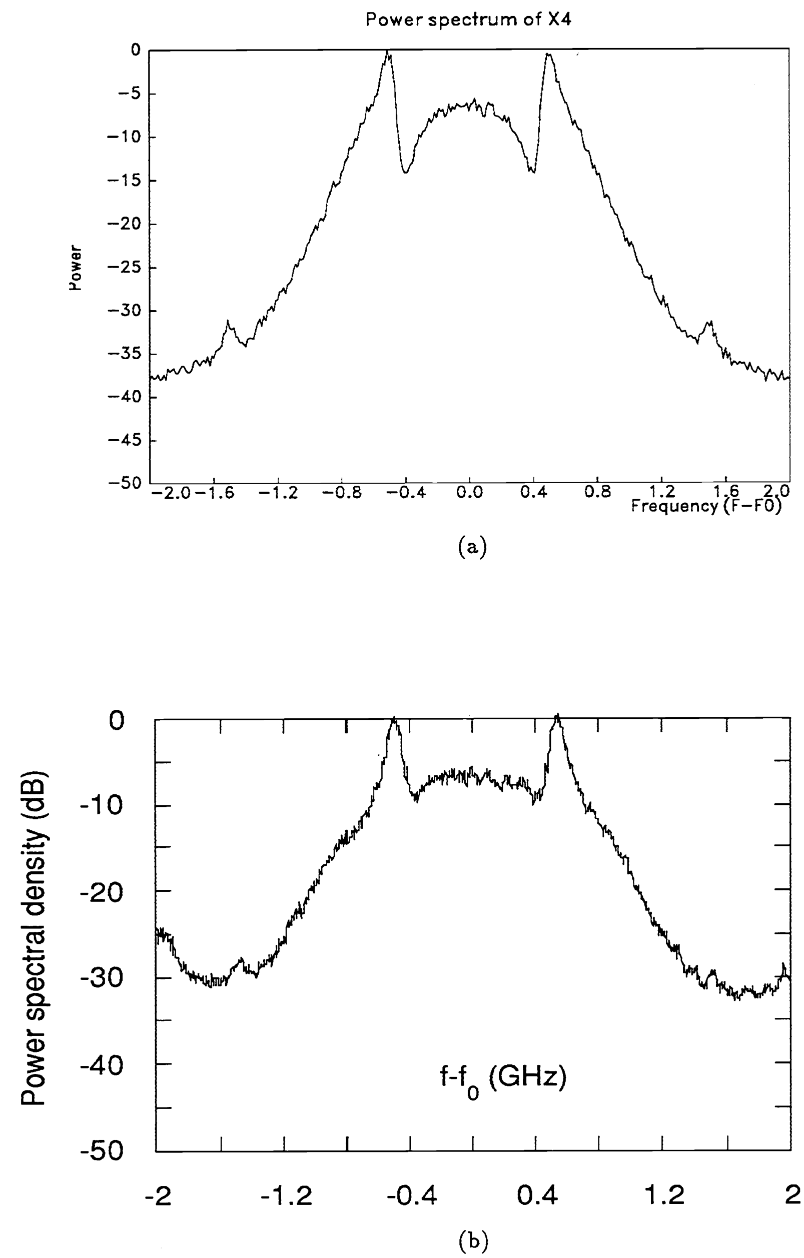

Figure 5: (a) Theoretical and (b) experimental spectrum of the CPFSK in the presence of phase noise. (Conditions : A mixed-radix FFT with 800 points was used for the calculation of the theoretical spectrum. The resolution bandwidth was 125 MHz and the transmitted sequence length was $2^{13} - 1$. The resolution bandwidth of the experiment was 3 MHz and the sequence length was $2^{23} - 1$. Both curves were obtained by averaging over 100 spectra).

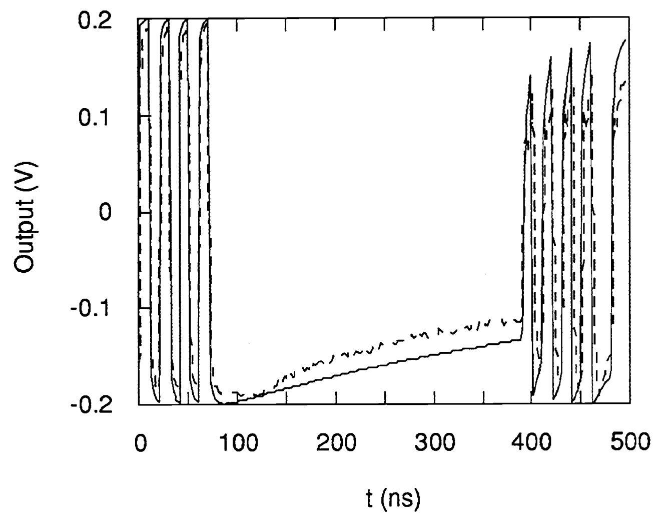

Figure 6: Effect of the nonuniform FM response on the output signal of the differential receiver. Solid line : theory, Dashed line : experiment.

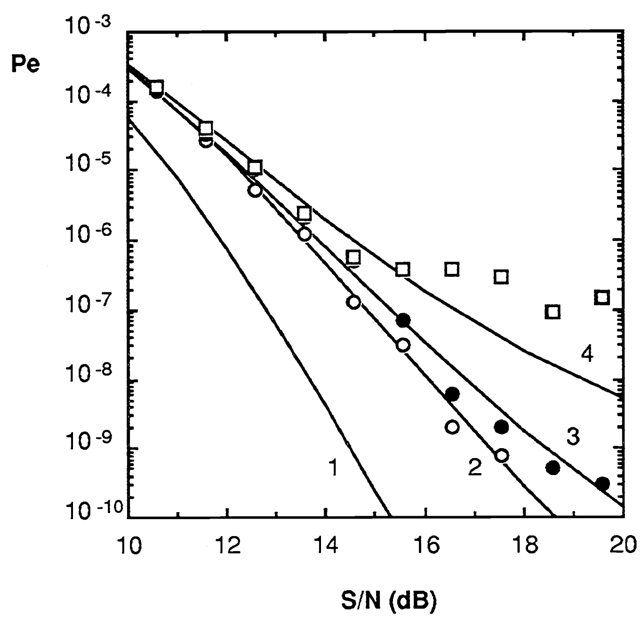

Figure 7: Influence of the sequence length on the probability of error (Symbols used : Curve 1 : coherent DPSK or CPFSK system with a uniform FM response, Curve 2 : Seq. 1010, Curve 3 : 2^7-1, Curve 4 : $2^{15}-1$, $\circ$: Measurements with the Seq. 1010, $\bullet$: Measurements with the Seq. 2^7-1, $\square$: Measurements with the Seq. $2^{15}-1$).