Performance Outages in CWDM Optical Networks due to the Polarization-Dependent Gain of Semiconductor Optical Amplifiers

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***Abstract*—This letter presents, for the first time, the derivation of analytical formulae for the probability density function and the cumulative density function (cdf) of the optical signal-to-noise ratio variation in optical local and metropolitan area networks employing coarse wavelength-division multiplexing due to the weak polarization-dependent gain (PDG) of cascaded semiconductor optical amplifiers (SOAs). The cdf is used to calculate the outage probability and to derive specifications for the maximum allowable value of PDG per SOA in order to achieve a given network size.**

***Index Terms*—Optical communications, optical metropolitan area networks (MANs), optical signal-to-noise ratio (OSNR), polarization-dependent gain (PDG), semiconductor optical amplifiers (SOAs).**

I. Introduction

SEMICONDUCTOR optical amplifiers (SOAs) are increasingly considered for multichannel amplification in optical local area networks (LANs) and metropolitan area networks (MANs) employing coarse wavelength-division multiplexing (CWDM) due to their low cost, small size, wide bandwidth, and capability of operation in several wavelength bands [1]. Commercially available SOAs for these applications exhibit polarization-dependent gain (PDG) typically of 0.5–1.5 dB [2]. PDG can cause fluctuations of the optical signal-to-noise ratio (OSNR), which, in turn, result in fluctuations of the bit-error rate and outages in the performance of optical communications systems and networks [3]. This effect becomes more pronounced as the number of cascaded SOAs increases and sets an upper limit to the scalability of the optical communications systems and networks.

A recent comprehensive study [4] derived analytical expressions for the outage probability due to polarization-dependent loss (PDL) in long-haul terrestrial optical communications systems, where the number of cascaded PDL elements is large and the PDL can be considered to be continuously distributed along the system.

This letter extends the model of [4] to the case of optical LANs and MANs with a small number of SOAs exhibiting weak PDG. The formalism is developed from first principles considering lumped PDG elements along the system and is simplified by using the approximate PDL vector concatenation rule of [5] for the case of weak PDG. It is shown that the relative OSNR variation after N SOAs, in the case of weak PDG, is approximately equal to the sum of independent, uniformly distributed random variables. Therefore, it is straightforward to derive analytical formulae for the probability density function (pdf) and the cumulative density function (cdf) of the relative OSNR variation. The latter is used to calculate the outage probability and derive specifications for the maximum allowable PDG per SOA as a function of the number of SOAs which can be cascaded. The analytical model is validated using previously published simulation results [6].

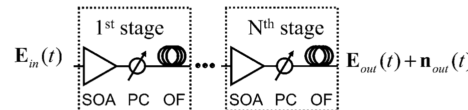

Fig. 1. System topology (Symbols: PC: passive optical components, OF: optical fiber span, $\mathbf{E}_{\rm in}(t)$: input signal electric field, $\mathbf{E}_{\rm out}(t)$: output signal electric field, $\mathbf{n}_{\rm out}(t)$: output ASE noise electric field).

II. Theoretical Model

The system topology under study consists of a chain of N stages (Fig. 1). Each stage is composed of an SOA with average gain $g_{0,i}$, followed by short spans of optical fibers and other passive optical components with total insertion loss $l_i (0 \leq l_i \leq 1)$, such that $g_{0,i} l_i = 1, i = 1, \ldots, N$. In addition, SOAs exhibit weak PDG. This is due to the difference in the confinement factors of the transverse-electric (TE) and transverse-magnetic (TM) modes, since the active region is not rotationally symmetric [7]. The PDG eigenaxes of the ith SOA in Stokes space are denoted by the unit Stokes vectors $\pm\hat{p}_i$. The gains associated with these eigenaxes are $G_{\max,i}, G_{\min,i}$, respectively. The SOA PDG is defined in [3] as $\rho_i \triangleq G_{\max,i}/G_{\min,i}$, the SOA average gain is defined in [4] as $g_{0,i} \triangleq (G_{\max,i} + G_{\min,i})/2$, the PDG coefficient as $\Gamma_i \triangleq (G_{\max,i} - G_{\min,i})/(G_{\max,i} + G_{\min,i})$, and the PDG vector in Stokes space as $\vec{\Gamma}_i \triangleq \Gamma_i \hat{p}_i$.

In the subsequent analysis, it is assumed that the SOAs are operating at the linear (i.e., unsaturated) regime, that the interconnecting fibers between consecutive SOAs randomly change the signal state of polarization (SOP) so it becomes uniformly distributed on the surface of the Poincaré sphere, and that there is no dynamic gain equalization or PDG equalization in order to minimize cost. Conforming to the above assumptions, SOAs

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can be considered as equivalent partial polarizers with gain instead of insertion loss. In order to isolate the impact of PDG, all other transmission effects are neglected.

In the following, we outline the steps of the derivation of the OSNR statistics. As a starting point, we calculate the first-order Taylor expansion of the transmittance of N concatenated stages in terms of Γ_i (cf. [5, expressions (6), (10), and (11)])

$$T_{\rm tot}(N) = 1 + \vec{\Gamma}_{\rm tot}(N)\hat{s} + O\left(\Gamma_i^2\right) \tag{1}$$

where $\hat{s}$ is the launched signal SOP and $\vec{\Gamma}_{\rm tot}(N)$ is the total PDG vector of the SOA chain given by $\vec{\Gamma}_{\rm tot}(N) \triangleq \sum_{i=1}^{N}\vec{\Gamma}_i$. It is worth noting that the total average SOA gain $g_{0,\rm tot}(N) \triangleq \prod_{i=1}^{N} g_{0,i}$ does not appear in (1) since it is assumed equal to the total insertion loss.

The power of the optical signal at the output of the SOA chain is written based on (1) as

$$P_{s,\rm out}(N) = T_{\rm tot}(N)P_s \cong P_s[1 + \vec{\Gamma}_{\rm tot}(N)\hat{s}] \tag{2}$$

where P_s is the average launched optical signal power.

For the calculation of the amplified spontaneous emission (ASE) noise power at the output of the SOA chain, it is observed that CWDM optical LANs and MANs employ optical multiplexers-demultiplexers with 13-nm bandwidth [8]. In this case, the contribution of the ASE noise orthogonal to the received signal SOP in the error probability might be significant. Therefore, both ASE noise polarization components must be taken into account.

The power of the total ASE noise at the output of the SOA chain is given by (cf. [4, expression (12)] with slight changes in notation) $P_{n,\rm out}(N) \cong NP_n$, where P_n denotes the total equivalent ASE noise power at the input of the individual SOAs in both polarizations at optical equivalent noise bandwidth Δf. The OSNR after N stages is

$$R(N) \triangleq \frac{P_{s,\rm out}(N)}{P_{n,\rm out}(N)} \cong R_N[1 + \vec{\Gamma}_{\rm tot}(N)\hat{s}] \tag{3}$$

where $R_N \triangleq P_s/(NP_n)$ is the OSNR after N stages in the absence of PDG.

It is convenient to introduce the relative OSNR variation $y \triangleq [R(N) - R_N]/R_N$. It can be shown that the scalar product $\hat{p}_i\hat{s}$ of two independent unit vectors $\hat{p}_i, \hat{s}$ with random direction in Stokes space is uniformly distributed over the interval $[-1, 1]$ [9]. Then, the relative OSNR variation y can be expressed as a weighted sum of N independent, uniformly distributed random variables $\vec{\Gamma}_i\hat{s}$, each taking values in the interval $[-\Gamma_i, \Gamma_i]$.

In the case of identical SOAs with PDG coefficients $\Gamma_i = \Gamma$, the pdf of the relative OSNR variation at the output of the Nth stage can be expressed in a variety of closed forms, e.g., [10]

$$p_y(y) = \frac{\sum_{k=0}^{N}(-1)^k \binom{N}{k} [y + (N-2k)\Gamma]_+^{N-1}}{(2\Gamma)^N (N-1)!} \tag{4}$$

where the "plus" function x_+^n vanishes for $x \leq 0$ and equals x^n for $x \geq 0$ [10].

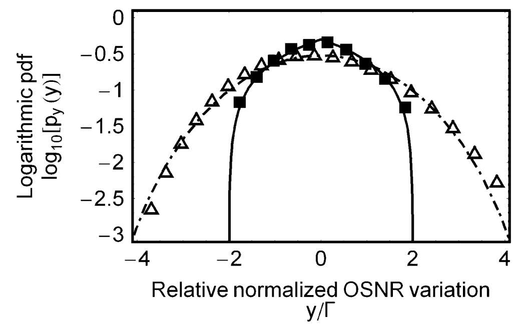

Fig. 2. Analytic pdfs of the relative OSNR variation for $N = 2$ (solid curve) and $N = 5$ (dashed–dotted curve) SOA stages plotted in logarithmic scale and verification by Monte Carlo simulation (squares and triangles, respectively). (Condition: Number of simulation runs: 122^N).

The cdf of the relative OSNR variation in the case of identical SOAs with PDG coefficients $\Gamma_i = \Gamma$ is [10]

$$F_y(y) = \frac{\sum_{k=0}^{N}(-1)^k \binom{N}{k} [y + (N-2k)\Gamma]_+^{N}}{(2\Gamma)^N N!}. \tag{5}$$

The cdf of the relative OSNR variation in the case of nonidentical SOAs with PDG coefficients Γ_i can be calculated analytically using [11, expression (26.57b)].

Similar to [4], the outage probability can be approximately defined as the probability that the normalized OSNR $R(N)/R_N$ falls below a threshold χ (referred to as OSNR margin) and is given by

$$P_{\rm outage} = F_y(\chi - 1). \tag{6}$$

It must be stressed that the above definition of the outage probability is unconventional [3]. Formally, one needs to first calculate the error probability, e.g., generalizing the formalism of [12] and [13]. However, this calculation is outside of the scope of the current letter and will be part of future work.

Finally, it is worth noting that, in the case of a large number of concatenated SOAs, the pdf of the relative OSNR variation $p_y(y)$ might be asymptotically approximated by a Gaussian with zero mean and variance $\sigma_y^2 = \sum_{i=1}^{N}\Gamma_i^2/3$, using the central limit theorem [14].

III. Results and Discussion

Fig. 2 shows semilogarithmic plots of the relative OSNR variation pdf, as given by (4), at the output of the SOA chain, for $N = 2$ (solid curve) and $N = 5$ (dashed–dotted curve) cascaded stages. The validity of the theoretical expression (4) is checked by comparison with Monte Carlo simulation for SOA PDG $\rho_i = 0.5$ dB (see [6] for details). Numerical results are depicted by squares for $N = 2$ and triangles for $N = 5$, respectively. For small values of $N\Gamma$ (e.g., for the case $N = 2$), there is excellent agreement between the analytical and numerical results. For larger values of $N\Gamma$ (e.g., for the case $N = 5$), the left tail of the numerical pdf decreases more rapidly than the right tail and the numerical results deviate from the theoretical prediction. This indicates that (6) yields slightly pessimistic results for the outage probability.

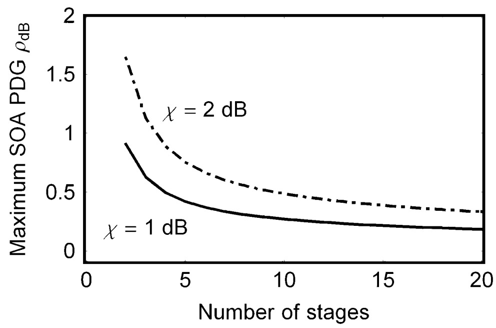

Fig. 3. Maximum allowable PDG per SOA as a function of the number of SOA stages for OSNR margin $\chi = 1$ dB (solid curve) and $\chi = 2$ dB (dashed–dotted curve).

Fig. 3 shows the maximum allowable PDG per SOA, calculated using (6), as a function of the number of stages traversed in order to achieve an outage probability of 1/17, 520. If this ensemble average is translated into a time average, assuming a single experiment where the signal SOP between consecutive SOAs is rapidly changing to uniformly cover the Poincaré sphere, this criterion corresponds to an outage time of 30 min per year [3]. The OSNR margin χ allocated for PDG is assumed 1 dB (solid curve) and 2 dB (dashed–dotted curve). For example, if the OSNR margin allocated for PDG is 1 dB and the maximum allowable PDG per SOA is 0.5 dB, up to four SOAs can be cascaded in series. If the OSNR margin is increased to 2 dB, then the maximum allowable PDG per SOA, for a network containing four SOAs in series, is 0.89 dB.

IV. Summary

This letter presents, for the first time, the derivation of approximate analytical formulae for the pdf and cdf of the relative OSNR variation in small networks comprising N cascaded SOAs with PDG coefficients $\Gamma_i \ll 1$. The cdf is used to calculate the outage probability and derive specifications for the maximum allowable PDG per SOA. For large values of $N\Gamma_i$, the aforementioned expression for the outage probability becomes pessimistic, so the specifications can be applied *a fortiori* in all cases.

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References

[1] D. R. Zimmerman and L. H. Spiekman, "Amplifiers for the masses: EDFA, EDWA, and SOA amplets for metro and access applications," *J. Lightw. Technol.*, vol. 22, no. 1, pp. 63–70, Jan. 2004.

[2] S. Park, R. Leavitt, R. Enck, V. Luciani, Y. Hu, P. J. S. Heim, D. Bowler, and M. Dagenais, "Semiconductor optical amplifier for CWDM operating over 1540–1620 nm," *IEEE Photon. Technol. Lett.*, vol. 17, no. 5, pp. 980–982, May 2005.

[3] C. D. Poole and J. Nagel, "Polarization effects in lightwave systems," in *Optical Fiber Telecommunications IIIA*, I. P. Kaminow and T. L. Koch, Eds. San Diego, CA: Academic, 1997, ch. 6.

[4] A. Mecozzi and M. Shtaif, "Signal-to-noise-ratio degradation caused by polarization-dependent loss and the effect of dynamic gain equalization," *J. Lightw. Technol.*, vol. 22, no. 8, pp. 1856–1871, Aug. 2004.

[5] M. Karlsson and M. Petersson, "Quaternion approach to PMD and PDL phenomena in optical fiber systems," *J. Lightw. Technol.*, vol. 22, no. 4, pp. 1137–1146, Apr. 2004.

[6] N. Antoniades, K. C. Reichmann, P. P. Iannone, N. J. Frigo, A. M. Levine, and I. Roudas, "The impact of polarization-dependent gain on the design of cascaded semiconductor optical amplifier CWDM systems," *IEEE Photon. Technol. Lett.*, vol. 18, no. 20, pp. 2099–2101, Oct. 15, 2006.

[7] M. J. O'Mahony, "Semiconductor laser optical amplifiers for use in future fiber systems," *J. Lightw. Technol.*, vol. 6, no. 4, pp. 531–544, Apr. 1988.

[8] K. C. Reichmann, P. P. Iannone, X. Zhou, N. J. Frigo, and B. R. Hemenway, "240-km CWDM transmission using cascaded SOA Raman hybrid amplifiers with 70-nm bandwidth," *IEEE Photon. Technol. Lett.*, vol. 18, no. 2, pp. 328–330, Jan. 15, 2006.

[9] C. D. Poole and D. L. Favin, "Polarization-mode dispersion measurements based on transmission spectra through a polarizer," *J. Lightw. Technol.*, vol. 12, no. 6, pp. 917–929, Jun. 1994.

[10] W. Feller, *An Introduction to Probability Theory and Its Applications*, 2nd ed. New York: Wiley, 1971, vol. 2.

[11] N. Johnson, S. Kotz, and N. Balakrishnan, *Discrete Univariate Distributions*, 2nd ed. New York: Wiley, 1995, vol. 2.

[12] I. T. Lima Jr., A. O. Lima, Y. Sun, H. Jiao, J. Zweck, C. R. Menyuk, and G. M. Carter, "A receiver model for optical fiber communication systems with arbitrarily polarized noise," *J. Lightw. Technol.*, vol. 23, no. 3, pp. 1478–1490, Mar. 2005.

[13] J. L. Rebola and A. V. T. Cartaxo, "Performance evaluation of optically preamplified receivers with partially polarized noise and arbitrary optical filtering: A rigorous approach," *Inst. Elect. Eng. Proc.-Optoelectron.*, vol. 152, no. 5, pp. 251–262, Oct. 2005.

[14] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. New York: McGraw-Hill, 2002.