Mode Vector Modulation

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Abstract *We propose a new energy-efficient, short-haul, multidimensional modulation using spatial degrees of freedom in SDM fibers to create well-separated points in the generalized Stokes space. We study the transceiver architecture, geometric constellation shaping, bit-to-symbol mapping, and the performance of the optically-preamplified direct-detection receiver.*

Introduction

Due to the high-volume data traffic in inter-data-center networks (IDCNs) within metropolitan areas, design concepts for high-spectral-efficiency short-haul transmission have attracted considerable interest in recent years[1]. A promising modulation format for this application is $M-$ary Stokes vector modulation (SVM)[2]. Compared to conventional binary intensity modulation, SVM allows for spectral efficiency increases, while still using less expensive direct-detection (DD) receivers[2].

The anticipated deployment of novel optical fibers for space division multiplexing (SDM) in IDCNs offers opportunities for using the spatial degrees of freedom of these fibers jointly rather than independently to create new modulation formats, e.g.,[3]. Following this philosophy, we propose an energy-efficient, short-haul, multidimensional modulation that is a direct extension of the SVM-DD for SDM fibers. This format uses jointly the states of polarization per mode/core, as well as groups of modes/cores in SDM fibers to create well-separated points on the unit Poincaré hypersphere in the generalized Stokes space[4]–[6]. We refer to this modulation technique here as *mode vector modulation* (MVM) to distinguish it from the conventional SVM over single-mode fibers (SMFs). It is worth noting that Ji et al.[7] recently studied the feasibility of a self-homodyne, optically-preamplified, MVM receiver and demonstrated its ability to detect information transmitted over the LP$_{11}$ mode group of a six-mode few-mode fiber (FMF).

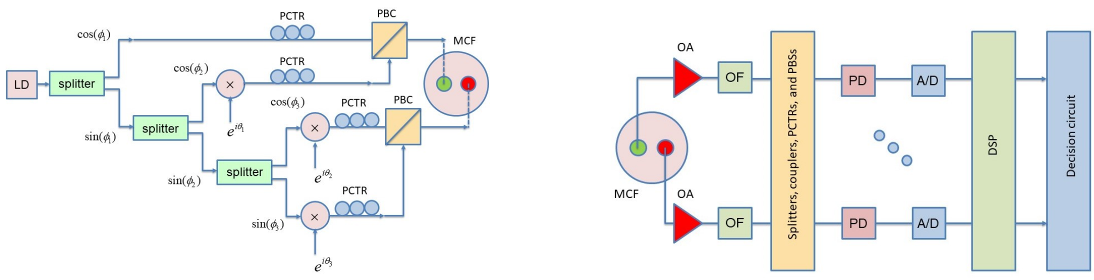
In this paper, first, we derive the MVM transceiver architecture[8]. Then, we propose two MVM constellation families with well-separated points on the unit Poincaré hypersphere: (a) Simplex constellations based on symmetric, informationally complete, positive operator valued measure (SIC-POVM) vectors[6],[9]; (b) Geometrically-shaped constellations optimized with mathematical and physical analogs[10],[11] using gradient descent[12]. We also optimize the bit-to-symbol encoding of these constellations via simulated annealing[13]. Finally, we perform a thorough theoretical calculation of the performance limits of MVM optically-preamplified DD receivers using both Monte Carlo simulation and a new analytical formula that we derived for the union bound[8] in the generalized Stokes space.

MVM transceiver design

The launched states in an SDM fiber with N spatial and polarization modes can be geometrically represented by vectors in a $N-$dimensional Jones space or in a generalized $(N^2-1)-$dimensional Stokes space[4]–[6]. The dimensionality of the generalized Stokes space grows quadratically with the number of spatial and polarization modes in MMFs/MCFs. Instead of using SVM-DD in conjunction with the conventional 3D Stokes space, we can generate more energy-efficient constellations by spreading the constellation points in the generalized Stokes space.

We parameterize a unit Jones vector $|s\rangle$ using $2N-2$ hyperspherical coordinates[6], i.e., $|s\rangle := \left[\cos(\phi_1), \sin(\phi_1)\cos(\phi_2)e^{i\theta_1}, \ldots, \sin(\phi_1)\cdots\sin(\phi_{N-2})\sin(\phi_{N-1})e^{i\theta_{N-1}}\right]^T$.

According to the above equation, we can construct the MVM transmitter using concatenated electro-optic Y-junctions and phase shifters. Fig. 1 (a) shows an implementation example of the MVM transmitter for a two-core homogeneous multicore fiber with identical single-mode cores $(N=4)$. The first Y-junction splits the incoming signal into its two output ports with an arbitrary power splitting ratio $\cos^2(\phi_1) : \sin^2(\phi_1)$, the second Y-junction splits the incoming signal into its



(a) (b)

Fig. 1: Schematic of the proposed (a) MVM transmitter and (b) Optically-preamplified MVM DD receiver. Symbols: LD=laser diode, OA=optical amplifier, OF=optical filter, PCTR=polarization controller, PBC/S=polarization beam combiner/splitter, PD=photodiode, A/D=Analog-to-digital converter. Condition: $N = 4$.

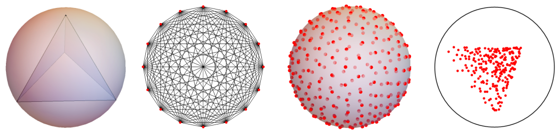
two output ports with an arbitrary power splitting ratio $\cos^2(\phi_2) : \sin^2(\phi_2)$, and so forth. The phase modulators at different branches generate the phase differences $\theta_1, \theta_2, \ldots$ among Jones vector components. Signals from two subsequent signal paths are set into orthogonal states of polarization (SOPs) using polarization controllers and polarization beam combiners and are launched into different fiber cores.

The direct-detection receiver front-end for $N = 4$ is shown in Fig. 1 (b). The reverse procedure is followed compared to the transmitter: First, the spatial and polarization components of the Jones vectors representing the received MVM signals are separated. Then, they are mixed pairwise to create $2(N^2 - 1)$ combinations, using a network of power splitters/couplers, polarization controllers, and polarization beam splitters. Finally, an array of $2(N^2 - 1)$ identical photodiodes are used pairwise for direct-detection of the $N^2 - 1$ Stokes components of the received signals. It can be shown that the front-end complexity can be reduced significantly taking into account that Stokes vector components are interdependent since they are functions of the $2N - 2$ hyperspherical coordinates in $|s\rangle$. The number of required photodiodes can therefore be reduced to only $3N - 2$ without loss of information.

Constellation design

Simplex constellations can be easily implemented using SIC-POVM vectors, which have been found either analytically or numerically for various N, and were initially intended for quantum mechanical measurements[9]. Examples of SIC-POVM constellations for $N = 2$ and $N = 4$ are shown in Fig. 2(a),(b), respectively. In general, for arbitrary N, SIC-POVM states correspond to $M = N^2$ equipower and equiangular generalized Stokes vectors.

For $M \neq N^2$, we resort to geometric constellation shaping. To facilitate calculations, we adopt a cost function borrowed from electrostatics[11], where one assumes that the constellation points are identical charges on the surface of the Poincaré hypersphere. Starting from given initial positions, the charges are free to relax to their final positions under the action of Coulomb forces. That is to say, we adapt the original 3D Thomson problem[11] to higher-dimensional Stokes space. This adaptation requires that the M constellation points are constrained to move inside a $(2N-2)$–dimensional manifold due to the relationships connecting the higher-dimensional Jones and Stokes spaces[4]–[6]. Examples of constellations of $M = 256$ approximately equidistributed points obtained by numerically solving the Thomson problem for $N = 2$ and $N = 4$ are shown in Fig. 2(c),(d), respectively. An alternative approach that gives slightly sharper bounds than the Thomson method is reported in[10] but the cataloged vectors are limited to small M.



(a) (b) (c) (d)

Fig. 2: Vectors in Stokes space. (a) Simplex MVM constellation with $M = 4$ points for $N = 2$. (b) Equiangular 2D projection of a simplex MVM constellation with $M = 16$ points for $N = 4$. (c) MVM constellation with $M = 256$ approximately equally separated points for $N = 2$. (d) Two-dimensional projection of an MVM constellation with $M = 256$ approximately equally separated points for $N = 4$. Condition: (a),(b) SIC-POVMs; (c),(d) Thomson method.

Upon geometrically-shaping a constellation, we turn to the task of optimizing the bit-to-symbol mapping. To do so, we create a potential function based off an asymptotic expansion of the Union Bound; minimizing this objective function over all possible bit encodings serves to minimize the bit error probability. As the number of possible encodings is $M!$, we use simulated annealing[13] to find a numerical approximation of the minimum.

Performance limits

In Fig. 3, we present the results of Monte Carlo simulation of the bit error rate (BER) for our optimized constellations and encodings for optically-preamplified DD receivers. For a given BER, we observe a significant gain by increasing the number of modes and moderate penalty for quadrupling the number of Jones vectors in a constellation. This general trend persists across all (N, M) pairs and attests to the superiority of MVM over SVM.

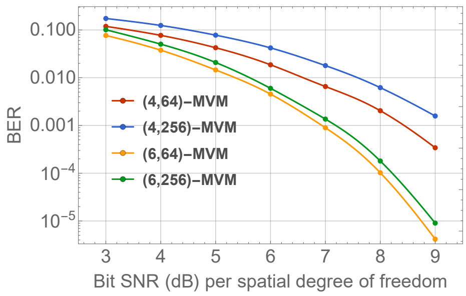


Fig. 3: Monte Carlo simulation of the bit error rates for our optimized (N, M)-MVM constellations.

In Fig. 4(a), we plot the upper limit of the bit error probability, given by the union bound, for an optically-preamplified SIC-POVM MVM DD receiver with matched optical filters, as a function of the electronic bit signal-to-noise ratio (SNR) per spatial degree of freedom. The Jones space dimension varies in the interval $N = 2 - 16$ in unit increments for different lines from top to bottom. The accuracy of the curves has also been checked by Monte Carlo simulation and the numerical data agree asymptotically with the analytical curves but the Monte Carlo simulation results have been omitted from Fig. 4(a) to avoid clutter. We observe that the bit SNR required to achieve a given bit error probability decreases as N increases since the Euclidean distance between two Stokes vectors increases as $d = \sqrt{2N^2/(N^2-1)}$.

At the same time, the spectral efficiency increases with N as $2\log_2 N$. Consequently, the normalized spectral efficiency per spatial degree of freedom $2\log_2 N/N$ decreases with increasing dimensionality N of Jones space. Fig. 4(b) shows the change in spectral efficiency per spatial degree of freedom (SDOF) for SIC-POVM MVM for different N as a function of the bit SNR per SDOF required to achieve a bit error probability of 10^{-4} (in blue). For comparison, we plot in the same figure the spectral efficiency given by Shannon's formula for channels with additive white Gaussian noise (AWGN) (in red)[8]. We observe that

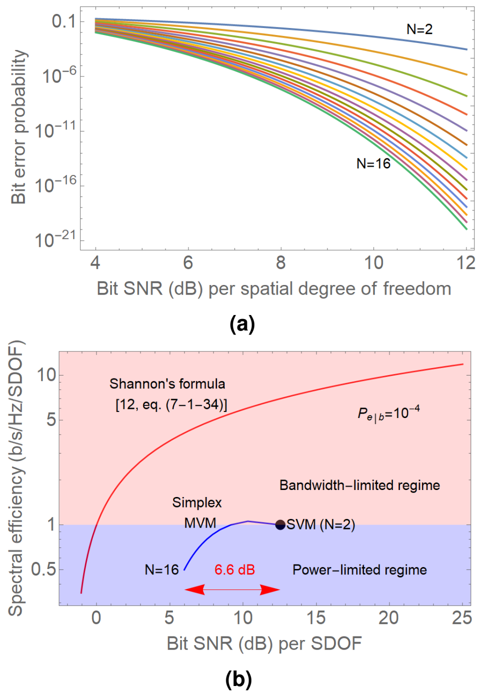


Fig. 4: (a) Bit error probability of $M-$ary MVM based on SIC-POVM vectors vs the bit SNR per spatial degree of freedom. (b) SIC-POVM MVM spectral efficiencies per spatial degree of freedom (SDOF) vs the bit SNR per SDOF required to achieve a bit error probability of 10^{-4} (in blue).

the maximum spectral efficiency for SIC-POVM MVM appears for $N = 3$ and is 1.06 b/s/Hz/SDOF. As a comparison, the spectral efficiency per spatial degree of freedom is equal to 1 b/s/Hz/SDOF for $N = 2, 4$. Finally, it is worth noting that for $N = 16$, SIC-POVM MVM over SDM fibers offers a 6.6 dB sensitivity improvement compared to conventional simplex SVM over SMFs ($N = 2$), at the expense of spectral efficiency per SDOF. We conclude that one can potentially benefit from utilizing MVM DD over SDM fibers, where spatial degrees of freedom in SDM fibers are used in unison as a *single channel* rather than as independent parallel channels, which is the standard engineering practice. The reason is that MVM offers greater flexibility for better trade-offs between energy consumption and spectral efficiency compared to SVM DD over SMFs.

Summary

MVM is an extension of SVM-DD to SDM fibers. This modulation scheme uses multiple spatial and polarization degrees of freedom simultaneously to transmit information over short SDM links. We studied the architecture of the MVM transceiver, two MVM constellation families with well-separated points on the unit Poincaré hypersphere, and the performance of the optically-preamplified MVM DD receiver.

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