Error Probability Estimation for Coherent Optical PDM-QPSK Communications Systems

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ABSTRACT

Two different semi-analytical methods for error probability estimation in PDM-QPSK optical communication systems are investigated. We consider the accuracy of a stochastic semi-analytical method based on Gaussian noise statistics and a deterministic semi-analytical method where the noise probability density function is estimated analytically against Monte-Carlo simulation. Linear coherent PDM-QPSK systems with distortions induced by filtering only, and nonlinear coherent PDM-QPSK systems with or without inline dispersion compensation are studied. Our results suggest that the stochastic semi-analytical method based on Gaussian noise statistics works very well for practical fiber-optic communication systems.
Keywords: Optical fiber communication, error probability, phase shift keying, coherent detection.

1. INTRODUCTION

Error probability estimation through conventional Monte-Carlo simulation requires the transmission of a pseudo-random bit sequence (PRBS), followed by direct error counting. In order to achieve good statistical reliability, it is important to assure that at least 100 errors are observed for each simulation in order to calculate the error probability [1]. This imposes significant constraints on the memory and efficiency for computer simulations, especially for systems with low error probability. Therefore, computationally-efficient analytical or semi-analytical methods are desired for performance characterization of optical communication systems. To design accurate semi-analytical methods, one has to know the probability density function (p.d.f.) of the noise, which can be inherently difficult to evaluate in closed form due to the interaction among the noise, nonlinearities, and digital signal processing algorithms for coherent systems.

In this paper, we compare two different semi-analytical methods for coherent 100 Gb/s non-return-to-zero (NRZ) PDM-QPSK systems performance evaluation, namely, a stochastic and a deterministic semi-analytical methods. We examine the limitations of them, and compare their accuracy against Monte-Carlo

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simulation under different system operating conditions. We find that if additive Gaussian noise is the dominant effect, the agreement of all methods is excellent for all OSNR range. In the presence of strong distortion and negligible noise, the stochastic semi-analytical method is pessimistic for high OSNR. However, for typical terrestrial long-haul systems, the stochastic semi-analytical method based on Gaussian statistics yields accurate results with minimum complexity.

2. MATHEMATICAL MODELS FOR ERROR PROBAILITY

At the input of the decision circuit of a coherent PDM-QPSK transmission system, the k^{th} sample of the complex photocurrent can be written as $x_k = A_k \exp[j(\varphi_k + \theta_k)] + n_{1_k} + jn_{2_k}$, where A_k is the amplitude, φ_k is the phase due to modulation, θ_k is the residual phase noise, and n_{1k}, n_{2k} are two independent Gaussian noise components with standard deviation σ.

According to the deterministic semi-analytical method, the signal is propagated in the absence of noise, and the noise statistics are calculated analytically. The average symbol error probability for each polarization is $P_{e|s} = \sum_{k=1}^{N_s} P_{e|s_k} / N_s$, where N_s denotes the number of simulated symbols and $P_{e|s_k}$ is the conditional error probability for the k^{th} symbol. By projecting all noiseless signal samples x_k to the upper right quadrature of the complex plane, $P_{e|s_k}$ is given by [2]

$$P_{e|s_k} = \int_{-\infty}^{\infty} \left\{ \operatorname{erfc}\left[\sqrt{\rho_{s_k}} \cos(\varphi_k + \theta_k)\right] + \operatorname{erfc}\left[\sqrt{\rho_{s_k}} \sin(\varphi_k + \theta_k)\right] \right\} p_{\theta_k}(\theta_k)\, d\theta_k, \tag{1}$$

where $\rho_{s_k} = A_k^2 / (2\sigma^2)$ is the instantaneous electronic symbol signal-to-noise ratio (SNR) before the decision circuit and $p_{\theta_k}(\theta_k)$ is the p.d.f of the residual laser phase noise, which is considered Gaussian to a first-order approximation with zero mean and variance equal to [3]

$$\sigma_{\theta_k}^2 = Dt_k + \frac{DT}{3} - 2\frac{Dt_k}{T}\left(T - \frac{t_k}{2}\right) + \frac{\sigma^2}{2B_{eq}T}, \tag{2}$$

where $D = 2\pi\Delta\nu$, $\Delta\nu$ is the combined 3-dB spectral linewidth of the lasers, T is the symbol interval, t_k is the sampling time, and B_{eq} is the equivalent noise bandwidth of the coherent optical receiver. Finally, the average bit error probability for Gray coding is approximated by $P_{e|b} = 0.5 P_{e|s}$ [4].

According to the stochastic semi-analytical method, both the signal and the noise are propagated through the transmission channel. The conditional p.d.fs of the received currents are assumed to be Gaussian for both quadrants. The stochastic semi-analytical method estimates the conditional mean $\hat{x}$ and variance $\hat{\sigma}^2$ of the received complex samples for each quadrant, given the information of the transmitted bits. The bit error probability assuming Gray coding is

$$p_{e|b} \approx \frac{1}{2}\left[\frac{1}{2}erfc\left(\sqrt{\rho_I/2}\right)+\frac{1}{2}erfc\left(\sqrt{\rho_Q/2}\right)\right], \quad (3)$$

where $erfc(\cdot)$ is the complimentary error function [5], ρ_I, ρ_Q are the electronic symbol SNRs for the in-phase and quadrature components defined as $\rho_{I,Q} = \hat{x}_{I,Q}{}^2 / \hat{\sigma}_{I,Q}{}^2$ [4].

3. SIMULATION RESULTS AND DISCUSSION

We compare the accuracy of the semi-analytical methods for the performance evaluation of practical coherent optical PDM-QPSK communication systems as shown in Fig. 1. For Monte-Carlo simulation, we use a PRBS with length of 2^{16}-1 unless otherwise stated.

First, we consider the performance of the semi-analytical methods in the linear regime. The full-width at half-maximum (FWHM) of the optical filter (MUX and DEMUX in Fig. 1) is B_o, and the half-width at half-maximum (HWHM) of the electrical filter is B_e. We consider two extreme cases, wide filtering (i.e., B_o=2/T_s, B_e=1/T_s) and tight filtering (i.e., B_o=1/T_s, B_e=0.4/T_s). We do not use any DSP and the laser phase noise is neglected so only ASE noise is present. Fig. 2(a) shows the constellation diagrams for the noiseless signals after filtering, and Fig. 2(b) shows the Q value, which is related to the bit error probability by the inverse of the complementary error function [5], versus OSNR. Monte-Carlo simulation can provide accurate results for Q < 10 dB only since we transmit a PRBS sequence of 2^{16} - 1. There is excellent agreement between the semi-analytical methods and the Monte-Carlo simulation in the wide filtering case. However, for tight filtering, there is an increasing discrepancy for higher OSNR values (i.e., above 18 dB), which is due to the fact that the stochastic method treats the ISI distortion as an equivalent Gaussian noise, yielding a spurious error floor in the limit when the OSNR tends to infinity. The deterministic semi-analytical method is accurate in this case in the absence of laser phase noise and DSP.

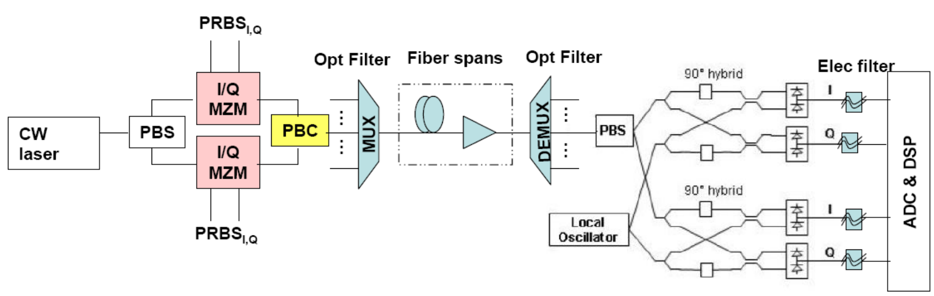


Fig. 1. Block diagram of a coherent optical PDM-QPSK system. PBS and PBC refer to polarization beam splitter and polarization beam combiner, respectively. MUX and DEMUX refer to optical multiplexer and de-multiplexer, respectively.

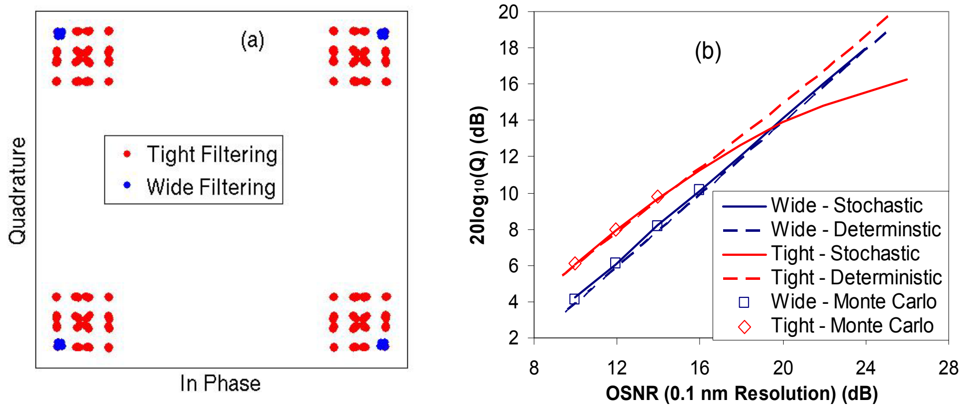


Fig. 2. (a) Noiseless constellation diagram for wide filtering (blue) and tight filtering (red); (b) Q-factor as a function of OSNR in the absence of phase noise using different techniques.

Next, we compare the accuracy of the semi-analytical methods for an 8-channel 100 Gb/s PDM-QPSK coherent optical system, with 50-GHz channel spacing. The transmission system consists of 30 spans of 100 km of standard single-mode fiber (SSMF) with in-line optical dispersion compensation. The residual dispersion per span is 17 ps/nm. The total residual dispersion is pre-compensated. The average differential group delay of the link is chosen to be 15 ps. The combined laser linewidth is 200 kHz. In the coherent receiver, we use a fixed transversal filter to compensate the residual dispersion of the edge channels and a two-stage CMA-based PMD adaptive equalizer with 13 taps to compensate for the polarization mode dispersion [6]. We use the Viterbi-Viterbi algorithm for phase noise estimation [7] with a fixed block size of 40 symbol intervals.

Fig. 3 shows the estimated Q value as a function of the launch power. We observe excellent agreement between Monte-Carlo simulation and the stochastic method up to the optimum launch power. For high launch power (e.g., 2 dBm per channel), the Q value difference between the stochastic method and the Monte-Carlo simulation can be as large as 0.5 dB. We also observe that the deterministic semi-analytical method has a larger disagreement with the Monte-Carlo method for large launch power, which is due to the fact that adaptive equalizers work differently with and without noise, and due to the omission of the nonlinear phase noise effect in the deterministic method. Figs. 4 (a) and (b) show the constellation diagrams for a launch power of -1 dBm and 2 dBm, with and without ASE noise, respectively. Compared with that for -1 dBm, the constellation diagram for 2 dBm is significantly elliptical. In addition, the constellation diagram in the presence of ASE noise is rotated compared to the noiseless constellation diagram due to the nonlinear phase noise. These two factors account for the discrepancy of 0.5 dB in Q value in the high-power region between stochastic semi-analytical method and Monte-Carlo simulation.

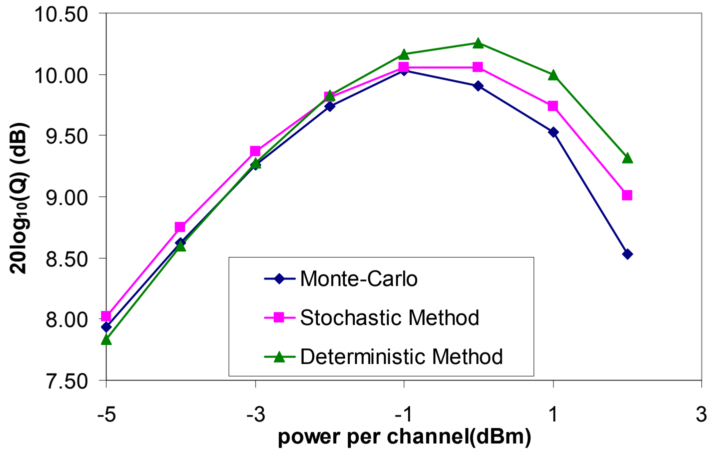


Fig. 3. Q-factor of the central channel as a function of channel launch power for a 3000 km PDM-QPSK system with inline DCF

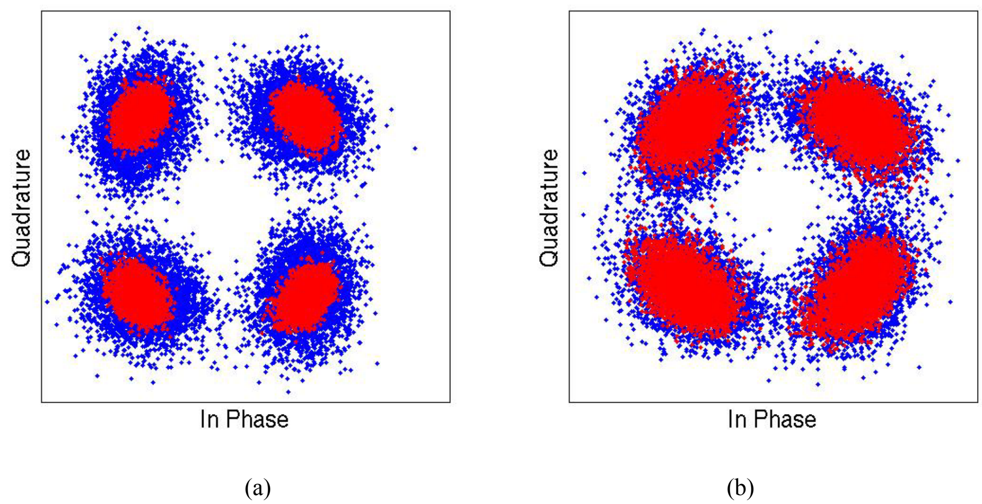
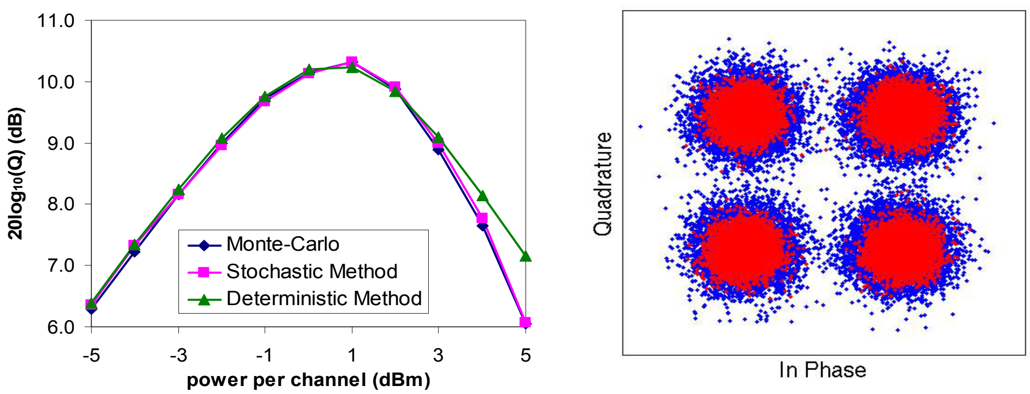


Fig. 4. Constellation diagram for (a) -1 dBm, and (b) 2 dBm launch power with and without noise. Blue and red denote the noisy and noiseless constellation diagrams, respectively.

For SMF systems without inline DCF, the Q-factor as a function of launch power for a 5000 km system (100 km per span) is shown in Fig. 5(a). We observe excellent agreement between the stochastic semi-analytical method and Monte-Carlo simulation. This can be explained by the fact that for systems without inline DCF, the pulses are spread over hundreds of symbol intervals due to chromatic dispersion, which makes the ISI-induced distortion more Gaussian noise-like. Therefore, the inherent assumption of the stochastic semi-analytical method holds. Fig. 5(b) shows the constellation diagram at 1 dBm power per channel. We see that the constellation diagram for the system without inline DCF is circular. For the deterministic method, we see a good agreement for launch power up to 3 dBm, and a discrepancy for very high powers such as 5 dBm, which is due to the exclusion of the nonlinear phase noise.

Finally, we compare and validate the two semi-analytical methods for uncompensated systems with 8 channels and various transmission distances. We fix the launch power at 1 dBm per channel. The deterministic method is believed to be accurate for uncompensated systems with optimum launch power of 1 dBm, since it accurately takes into account the deterministic distortions. The stochastic method based on Gaussian statistics works well for long distances when the OSNR is relatively low. However, the stochastic method might encounter problems for high OSNR region (i.e., short distances), as observed in Fig. 2. Thus it is necessary to validate the stochastic method for short distances. In Fig. 6, we show the Q value as a function of OSNR (or equivalently, as a function of transmission distance) for the stochastic method, deterministic method, and the Monte-Carlo method. We observe excellent agreement among the three methods.



(a) (b)

Fig. 5. (a) Q-factor of the central channel for systems without inline dispersion compensation after 5000 km transmission; (b) Constellation diagram at 1 dBm per channel launch power. Blue and red denote the noisy and noiseless constellation diagrams, respectively

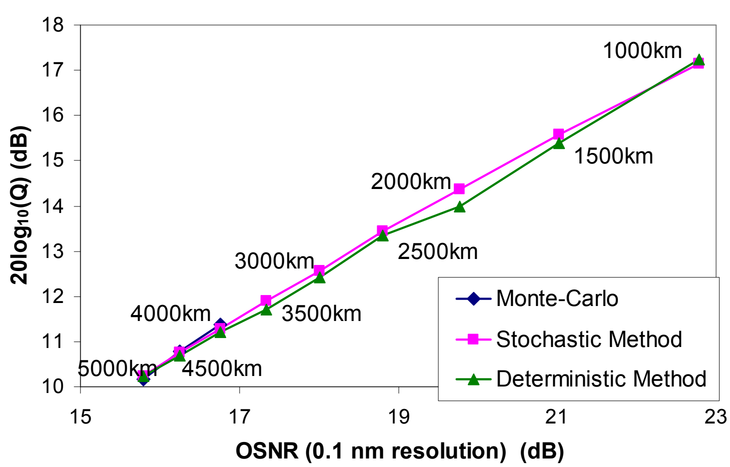


Fig. 6. Q-factor as a function of OSNR and transmission distance with 1dBm per channel launch power.

4. CONCLUSION

We have compared stochastic and deterministic semi-analytical methods for the error probability evaluation of coherent optical PDM-QPSK systems. The stochastic method based on Gaussian statistics suffers from inaccuracies when significant fiber nonlinearities impact the performance for dispersion-managed systems. However, for all of the terrestrial systems we studied, the stochastic semi-analytical method produced accurate results, and thus is preferred as it yields accurate results with minimum complexity for the performance evaluation of practical communication systems.

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REFERENCES

[1] R. Rubinstein, and P. Kroese, *Simulation and the Monte Carlo Method,* 2nd Ed., John Wiley & Sons: New York, 2007.

[2] R. Noé, *J. Lightw. Technol*., vol. 23, pp. 802–808, Feb. 2005.

[3] G. Goldfarb and G. Li, *Opt. Exp.*, vol. 14, pp. 8043–8053, Sept. 2006.

[4] J. G. Proakis, *Digital Communications*, 4th Ed., McGraw-Hill: New York, 2001.

[5] G. P. Agrawal, *Fiber-optic Communication Systems*, 3rd Ed., Wiley: New York, 2003.

[6] B. Spinnler, *J. Sel. Topics in Quantum Electron.*, vol. 16, no.9, pp. 1180-1192, 2010.

[7] A. Vertibi and A. Viterbi*, IEEE Trans on Inform. Theory*, vol. 29, no. 4, pp. 543-551, 1983.