Optical Transfer Function (OTF)
Modulation Transfer Function (MTF)

The Optical Transfer Function (OTF) is a complex-valued function describing the response of an imaging system as a function of spatial frequency.

Modulation Transfer Function (MTF) = magnitude of the complex OTF

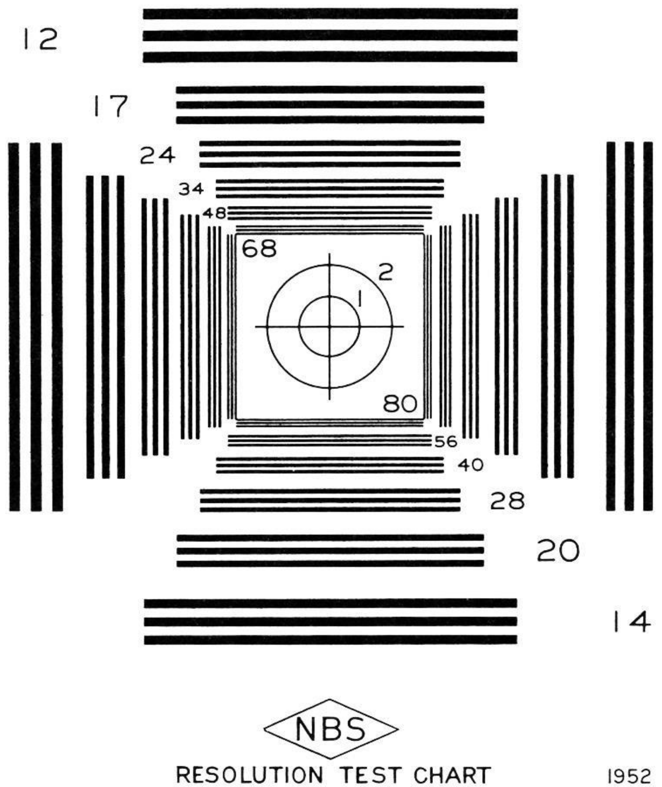
Phase Transfer Function (PTF) = phase of the complex OTF

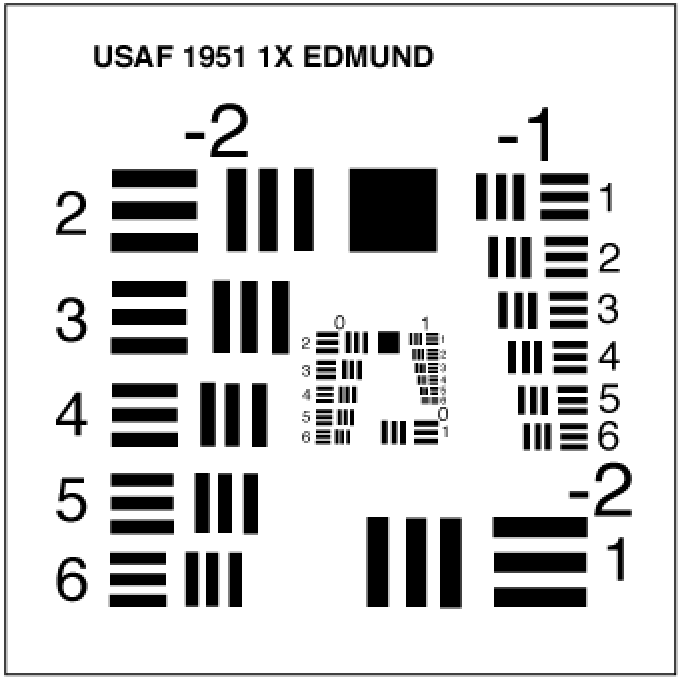
$$OTF(\xi,\eta) = MTF(\xi,\eta)e^{i[PTF(\xi,\eta)]}$$

Incoherent imaging systems are linear in irradiance
Coherent imaging systems are linear in field amplitude

Spatial frequency

Two commonly used resolution bar targets (NBS & USAF) illustrate a variety of low and high spatial frequencies [lines/mm].

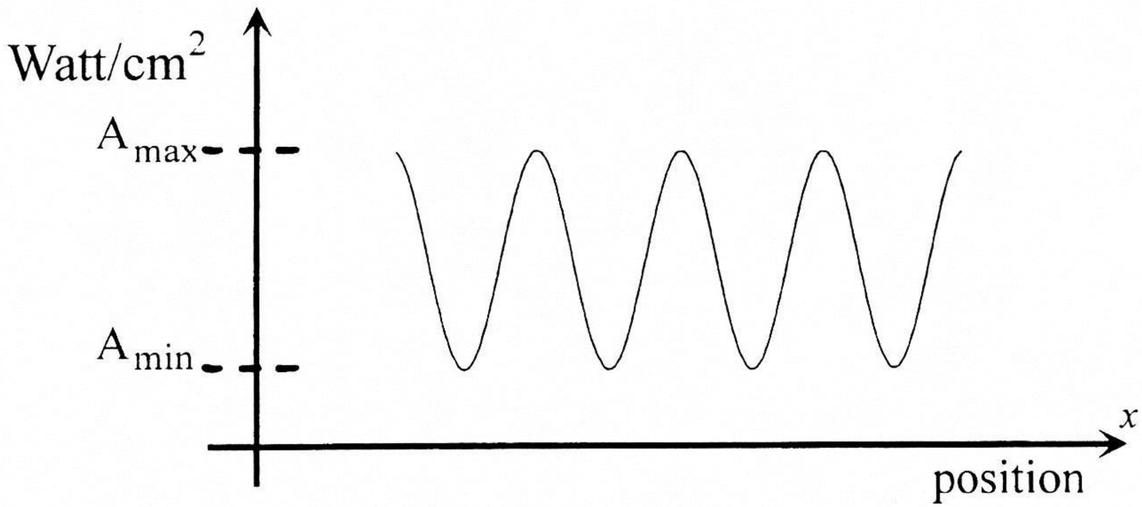




Modulation

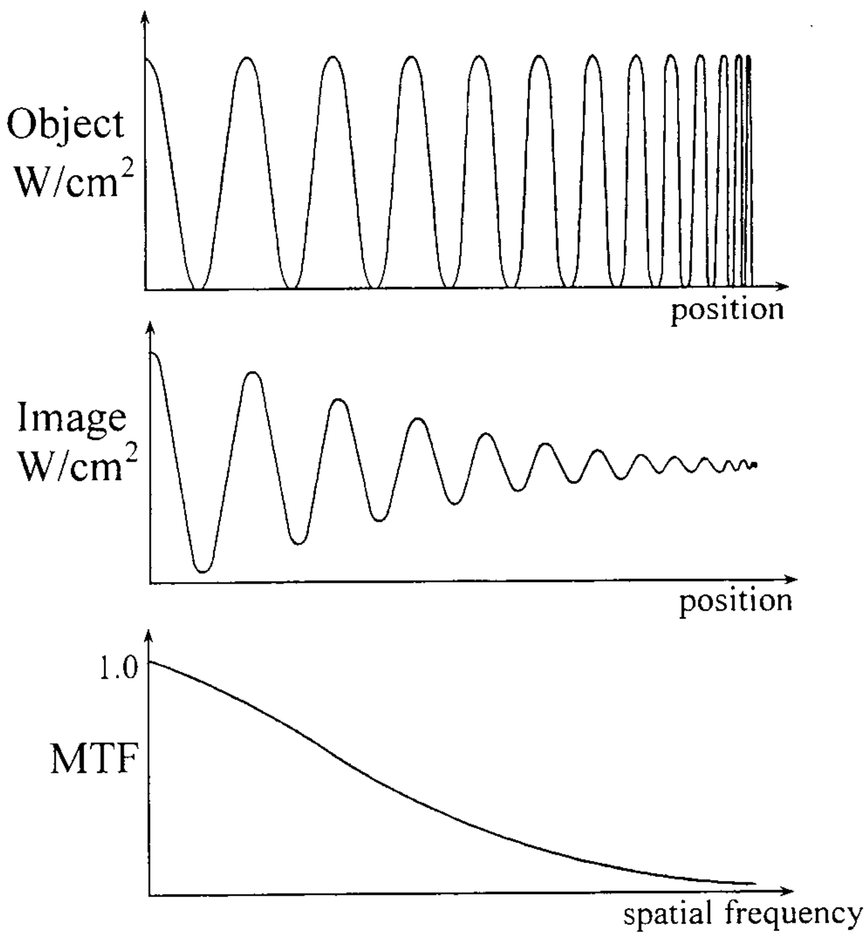
Modulation refers to the contrast between bright and dark regions of an image.

$$\text{contrast} = \frac{\max - \min}{\max + \min}$$



G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

MTF = plot of modulation (contrast) vs spatial frequency



G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

Linear Time-Invariant (LTI) systems

From linear systems theory we are familiar with the concepts of transfer functions and impulse response functions …

$$\begin{array}{c} x(t) \\ X(\omega) \end{array} \longrightarrow \boxed{\begin{array}{c} h(t) \\ H(\omega) \end{array}} \longrightarrow \begin{array}{c} y(t) = x(t) * h(t) \\ Y(\omega) = X(\omega)H(\omega) \end{array}$$

$h(t)$ = "impulse response function"
= output with delta function input

$H(\omega)$ = "transfer function" = $F\{h(t)\}$

$$X(\omega) = F\{x(t)\}$$

$$Y(\omega) = F\{y(t)\}$$

Linear Shift-Invariant (LSI) systems

Shift invariance means that the output of an optical system is the same at all spatial points. However, we know that this is fundamentally not true for aberrated optical systems, which means we need to use LSI systems theory with care.

$$\begin{matrix} \mathrm{o}(x,y) \\ O(\xi,\eta) \end{matrix} \longrightarrow \boxed{\begin{matrix} h(x,y) \\ H(\xi,\eta) \end{matrix}} \longrightarrow \begin{matrix} i(x,y) = o(x,y) * h(x,y) \\ I(\xi,\eta) = O(\xi,\eta)H(\xi,\eta) \end{matrix}$$

$h(x,y)$ = "point spread function" (PSF)
= image of delta function input

$H(\xi,\eta)$ = "optical transfer function" (OTF) = $F\{h(x,y)\}$

$$\mathrm{O}(\xi,\eta) = F\{o(x,y)\}$$

$$I(\xi,\eta) = F\{i(x,y)\}$$

PSF of ideal optical imaging systems

When we consider the point-spread function (PSF) of ideal imaging systems, "ideal" can be defined two ways:

1) No aberrations and no diffraction (not realistic) …

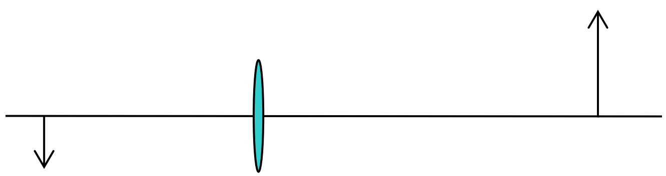


Image = perfect scaled replica of object
(point → point)

2) Diffraction-limited (realistic with enough money, sufficiently narrow field, etc.) …

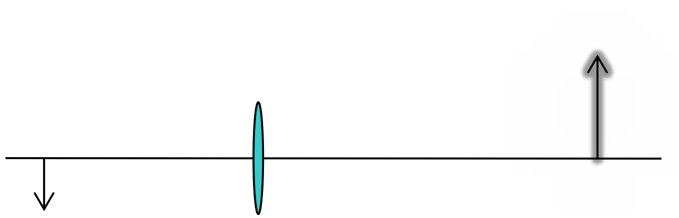
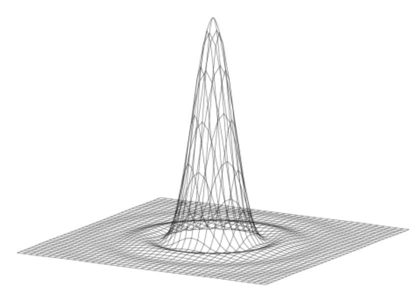


Image = slightly blurred and scaled replica of object
(point → PSF)

Central lobe half-width: $1.22\lambda(F\#)$



PSF of diffraction-limited imaging systems

For an imaging system with a circular pupil, the diffraction-limited PSF is a set of concentric rings called the "Airy pattern" …

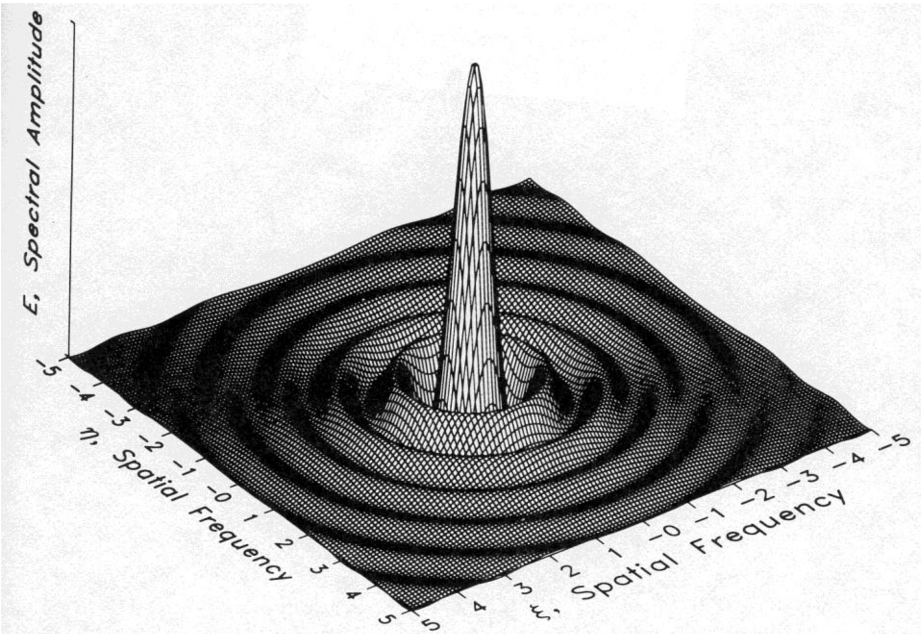


Figure 7.8 The form of $J_1(2\pi as)/(2\pi as)$, the Fourier transform of the circular aperture, $s^2 = \xi^2 + \eta^2$.

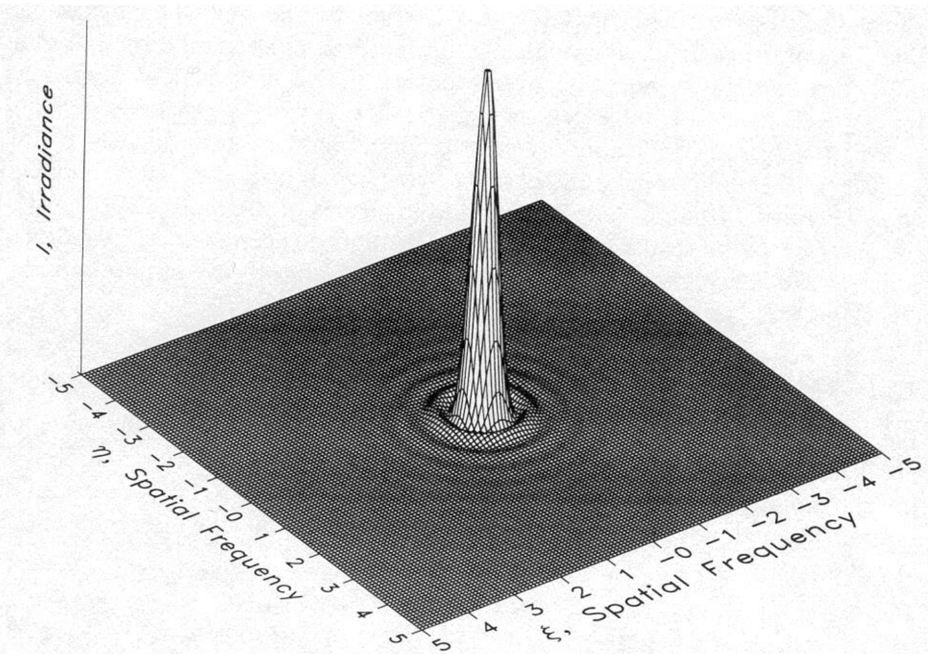
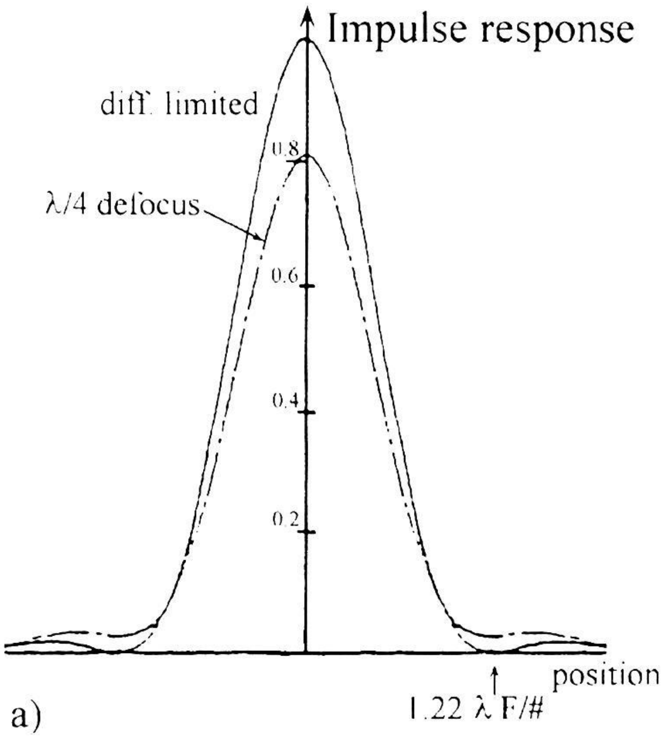


Figure 7.9 The square of the function of Figure 7.8.

R. G. Wilson, *Fourier Series and Optical Transform Techniques in Contemporary Optics*, Wiley, 1995.

PSF of aberrated optical imaging systems

Aberrations broaden the central lobe of the PSF and shift energy away from the center to the outer regions. The ratio of the aberrated PSF central peak height to the diffraction-limited central peak height is called the Strehl ratio.



Strehl ratio = 0.8 (as shown here)

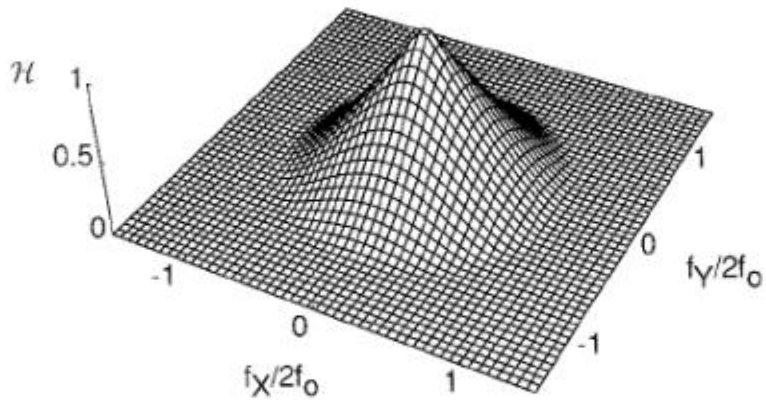
G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

MTF of diffraction-limited optical imaging systems

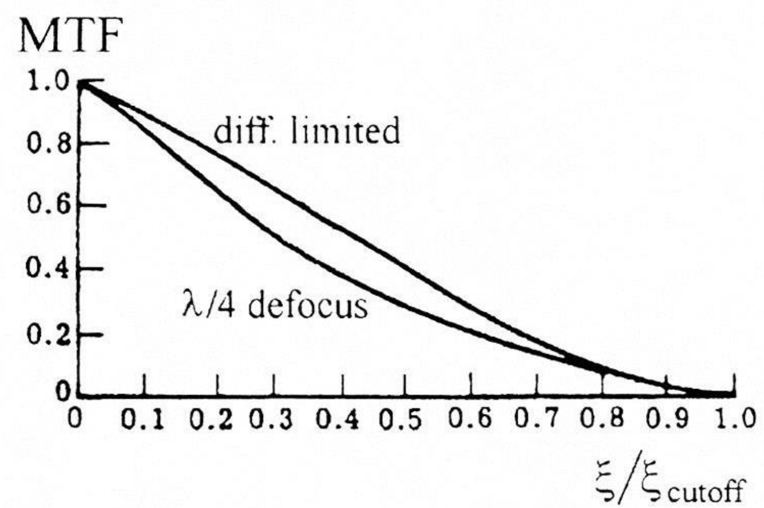
The MTF can be calculated as the magnitude of the Fourier transform of the PSF or as an autocorrelation of the pupil function.

Note that a diffraction-limited imaging system behaves as a low-pass filter that reproduces low spatial frequencies with higher contrast than high frequencies.

$$MTF(\rho) = \begin{cases} \dfrac{2}{\rho}\left[\cos^{-1}\left(\dfrac{\rho}{2\rho_0}\right) - \dfrac{\rho}{2\rho_0}\sqrt{1-\left(\dfrac{\rho}{2\rho_0}\right)^2}\right] & \rho \le 2\rho_0 \\ 0, & \text{otherwise} \end{cases}$$

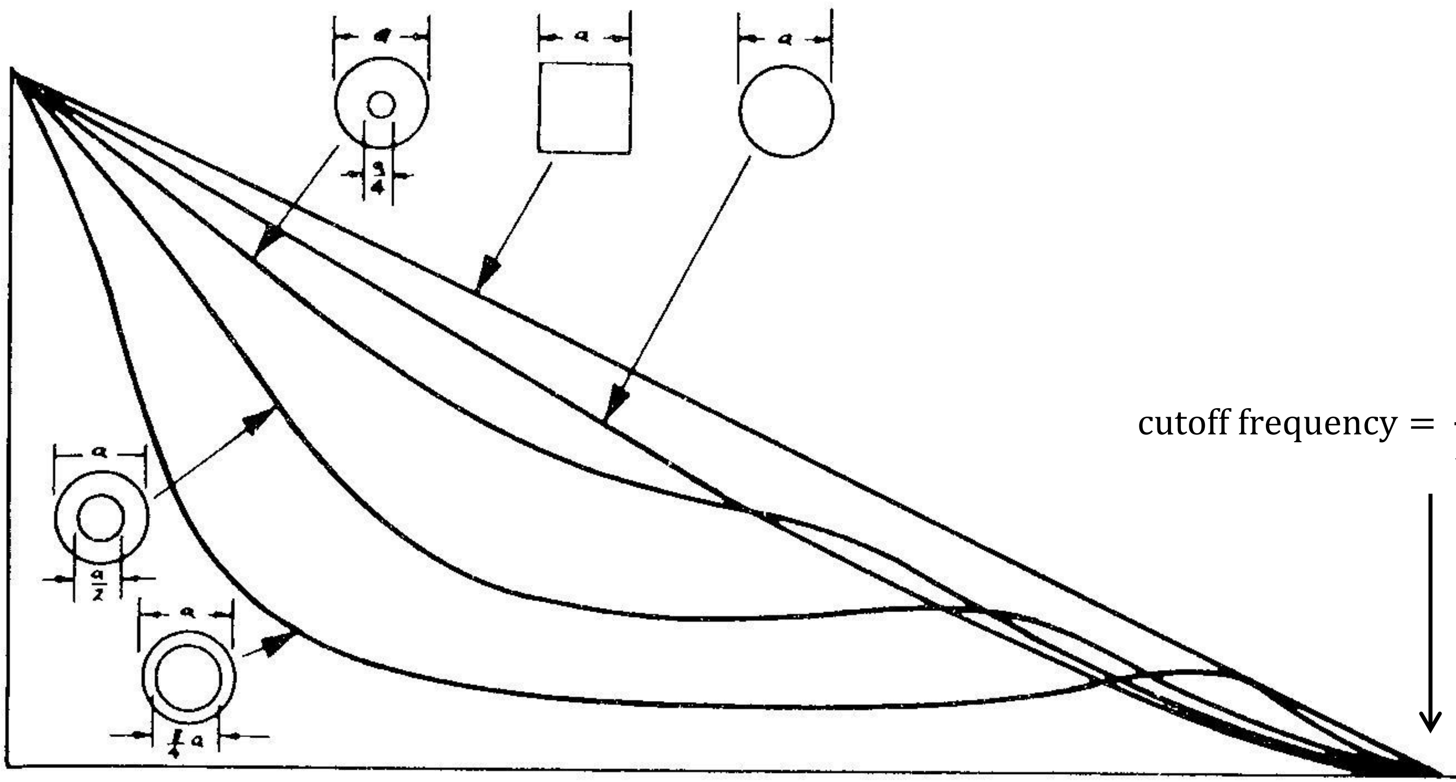


J. Goodman, *Intro. to Fourier Optics,* McGraw-Hill 1996



G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

Diffraction-limited MTF for different pupils

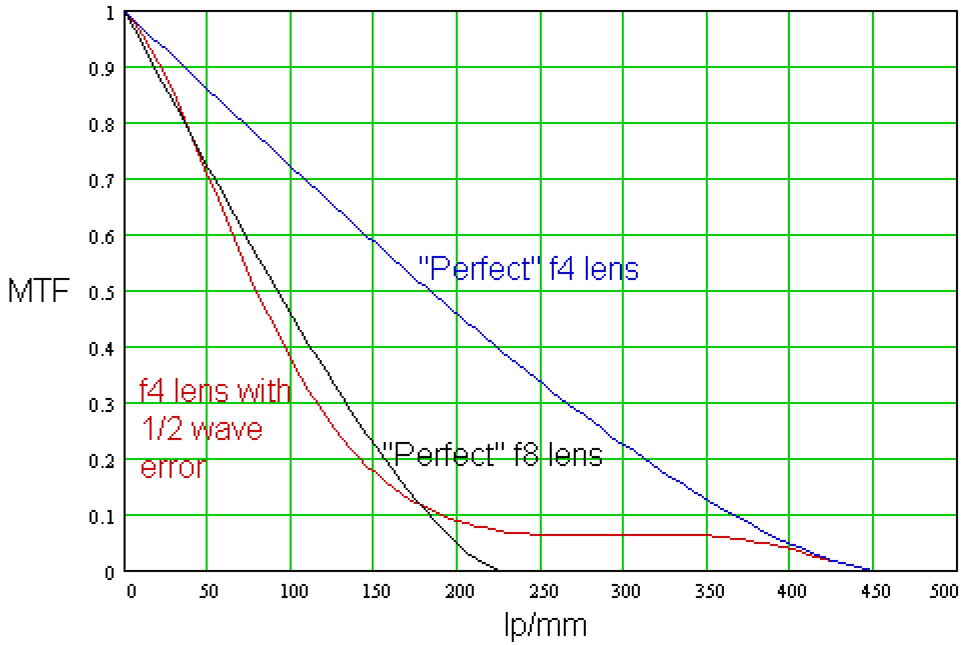


$$\text{cutoff frequency} = \frac{1}{\lambda(f\#)}$$

G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

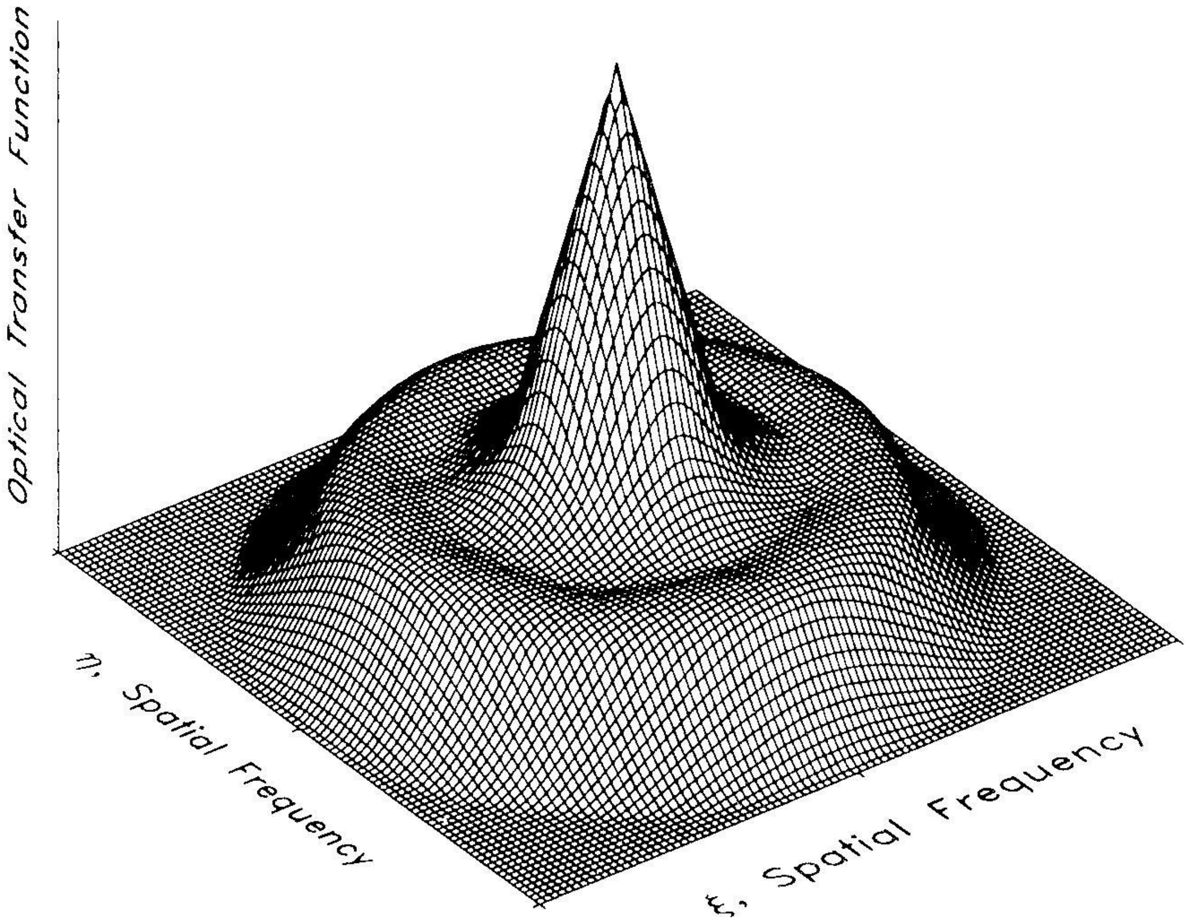
Diffraction-limited MTF for different f/# values

Because the diffraction-limited cutoff frequency is $\frac{1}{\lambda(f\#)}$…



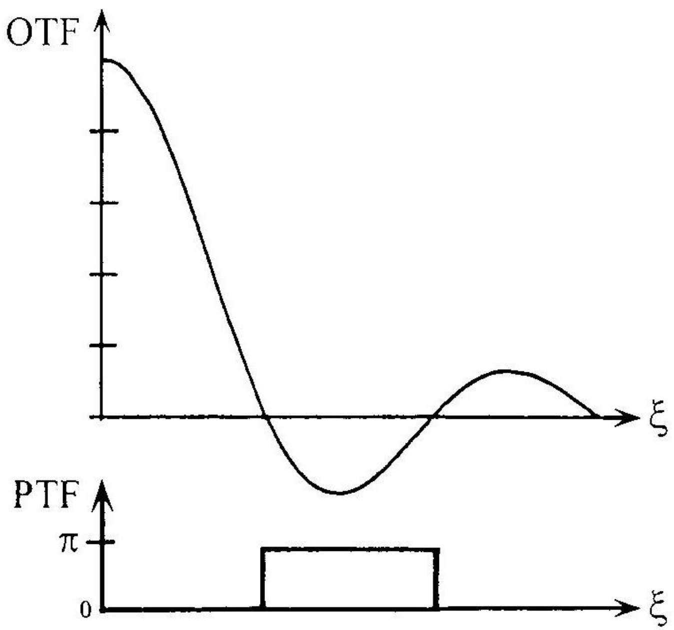
 http://photo.net/learn/optics/mtf/

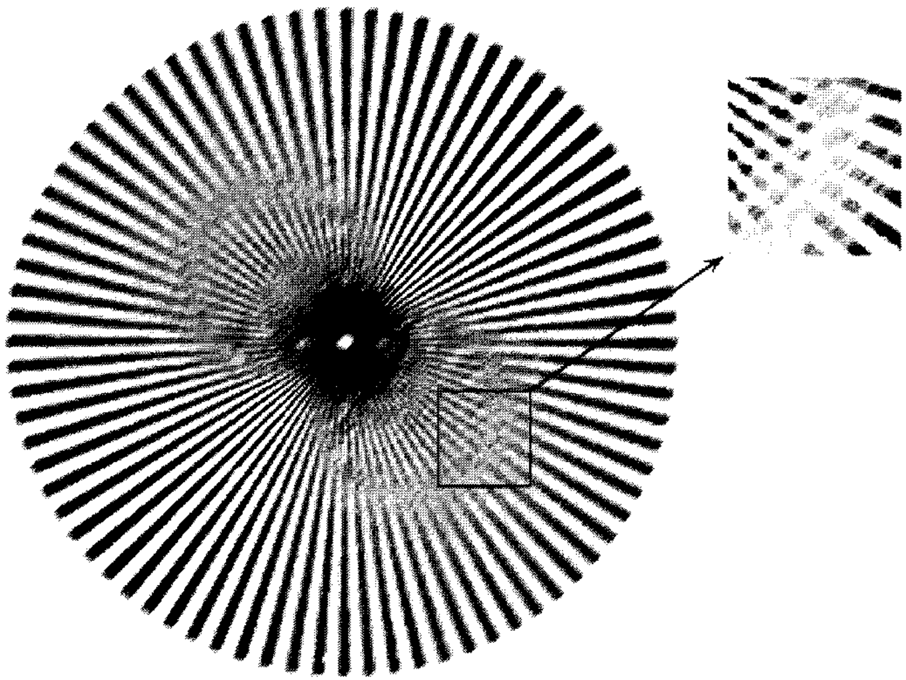
Pupil obstruction creates high-pass filter effect



 R. G. Wilson, *Fourier Series and Optical Transform Techniques in Contemporary Optics*, Wiley, 1995.

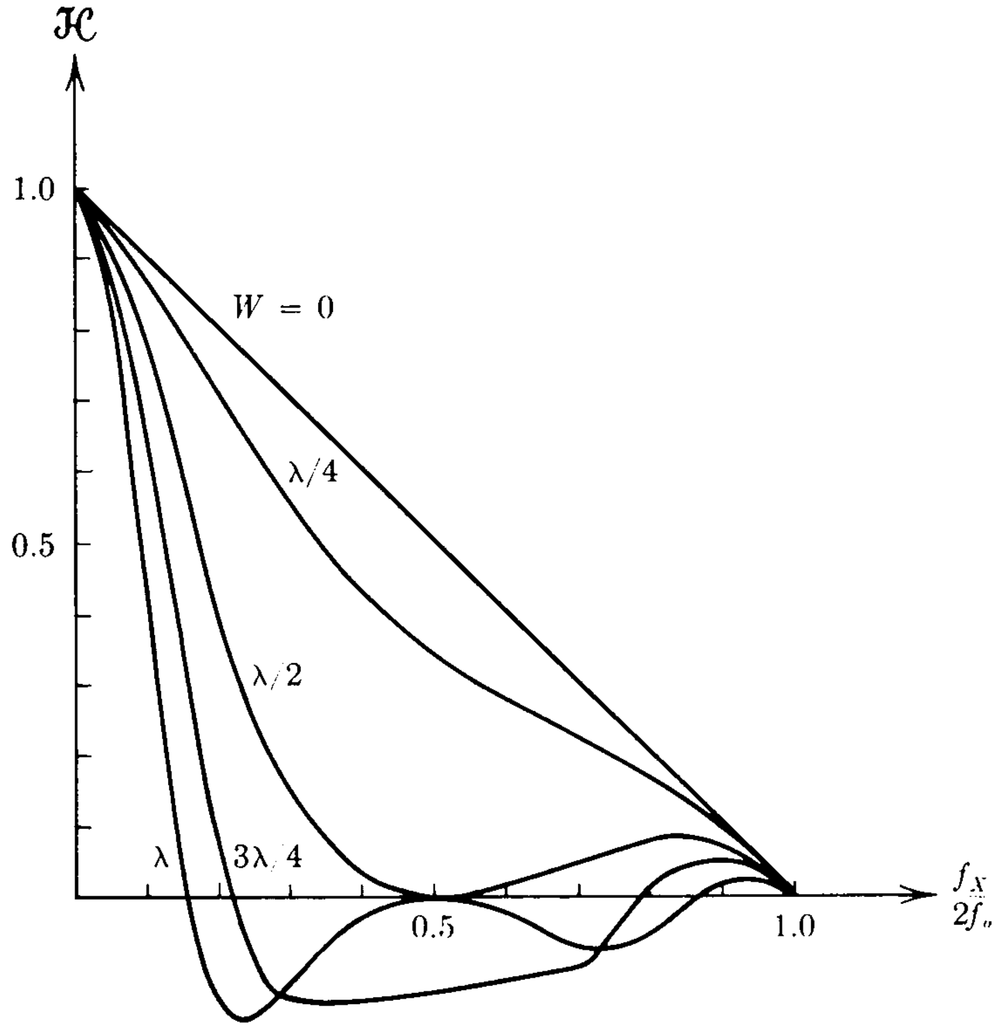
Aberrated OTF with contrast reversal





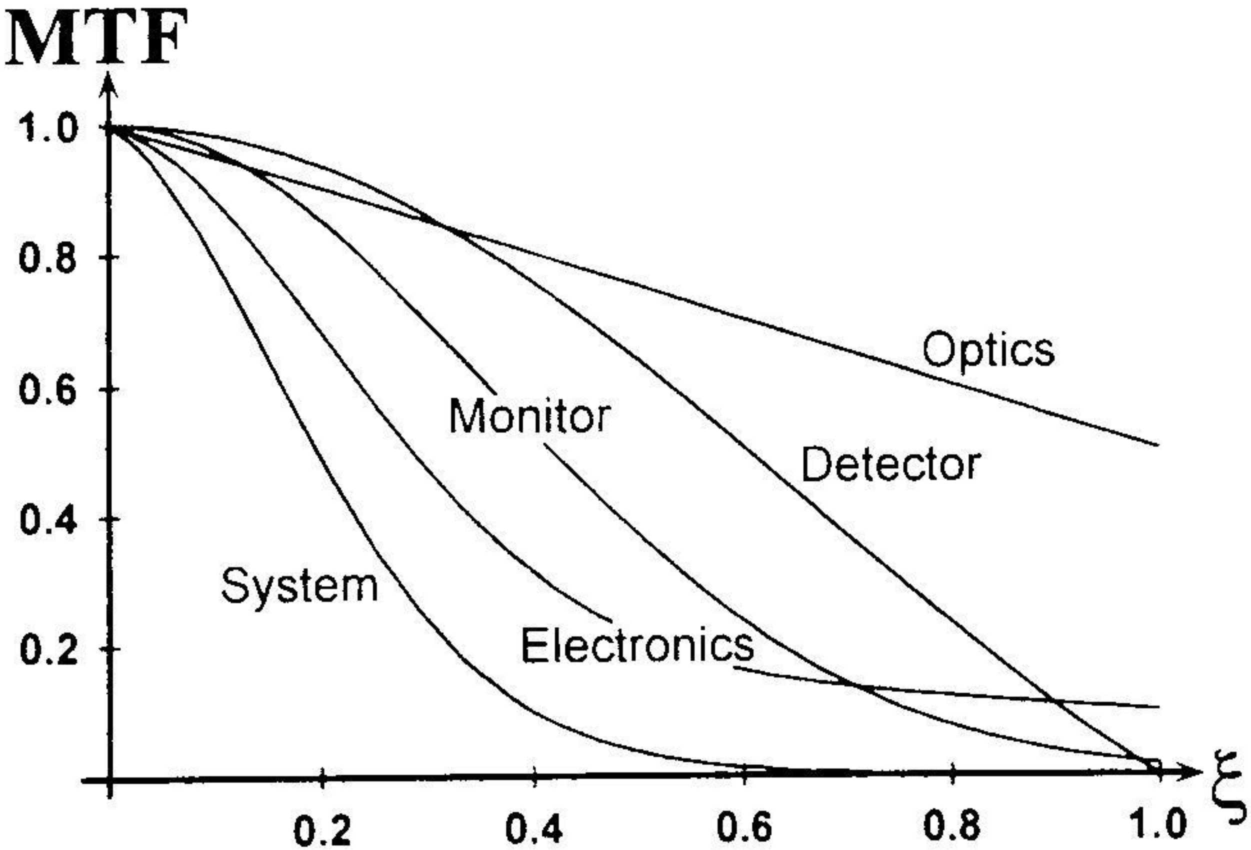
G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

MTF with defocus



J. Goodman, *Intro. to Fourier Optics,* McGraw-Hill 1996

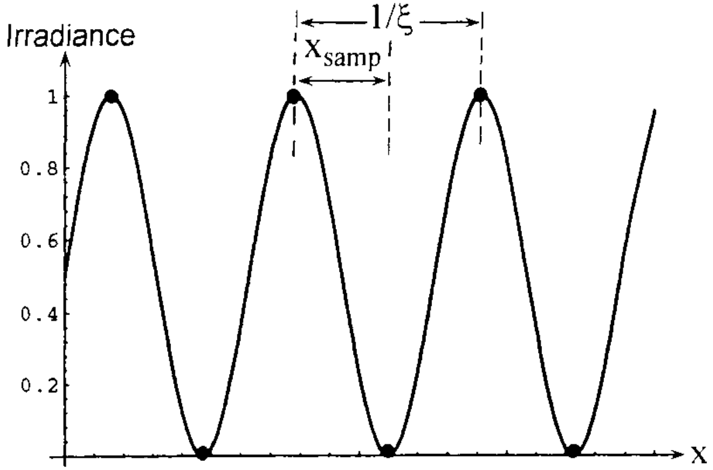
The system transfer function is a product of components



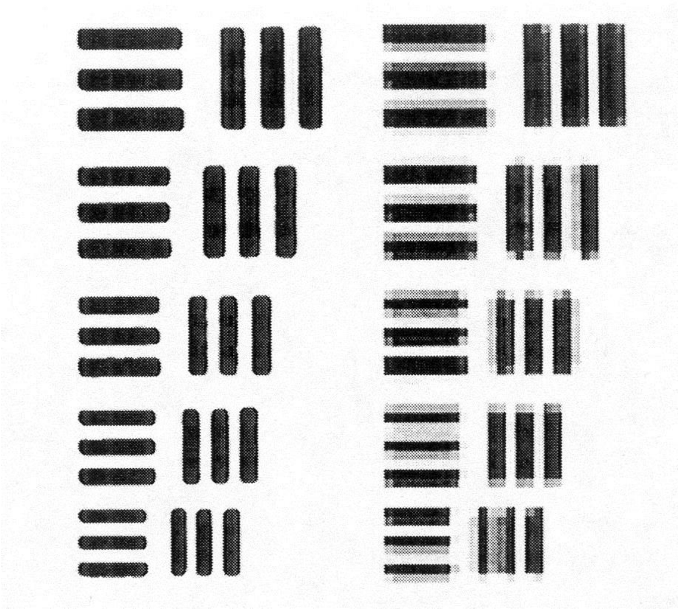
G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

Detector sampling

Nyquist sampling



Wider pixel spacing produces aliasing



MTF cutoff frequency for detector of dimension *d*: $f_{cd} = \frac{1}{d}$

To avoid aliasing, we need 2 pixels per line pair, which defines the detector Nyquist frequency: $f_N = \frac{f_{cd}}{2} = \frac{1}{2d}$

G. Boreman, *Modulation Transfer Function in Optical and Electro-Optical Systems,* SPIE, 2001.

MTF in Zemax

Geometric MTF … Geometric optics approximation calculated from ray data
(useful for heavily aberrated systems with OPD $\geq 10\lambda$)

FFT MTF … Diffraction-based MTF calculated from FFT of pupil

Huygens MTF … Complex sum of Huygens wavelet PSFs …
(useful for tilted image planes, but much slower than FFT)