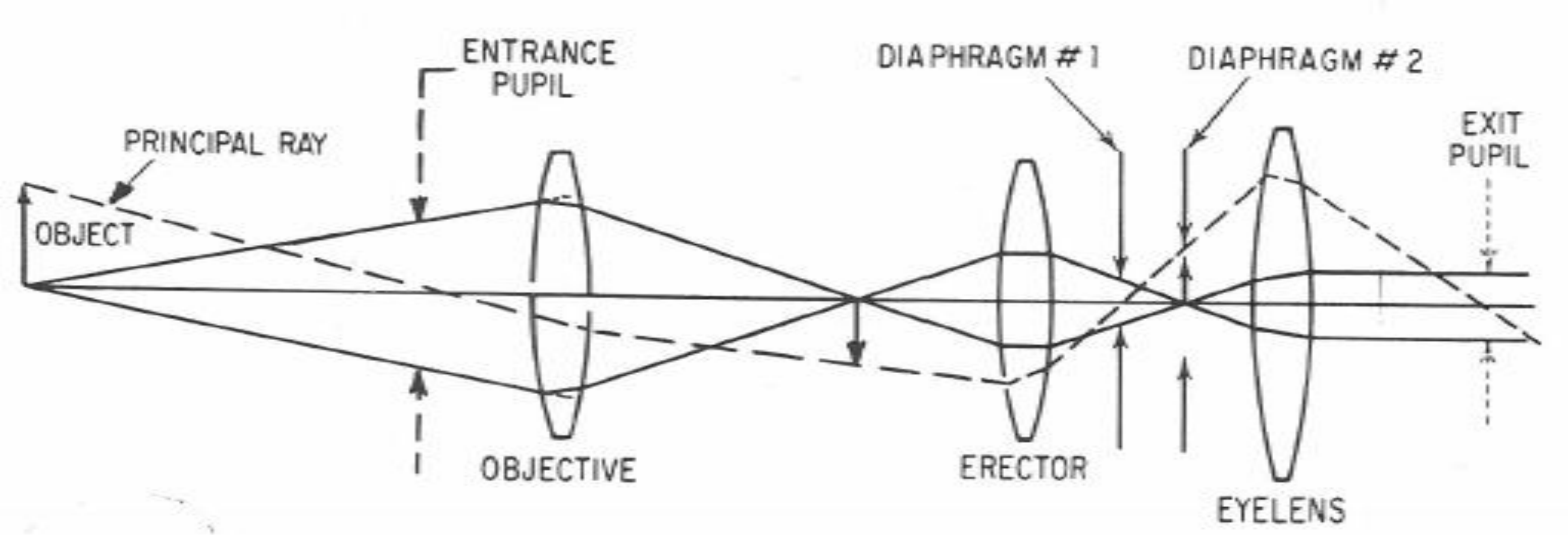
The y-u ray trace method: telescope example

The y-u method is just the y-nu method for thin lenses (ignoring *n* within lenses). This example illustrates the use of paraxial ray tracing for finding cardinal points, identifying aperture and field stops, locating and sizing pupils, etc.



Sfc 1: $f_1 = 10 \ldots D_1 = 2.3 \ldots t_1 = 16.5$

Sfc 2: $f_2 = 2 \ldots D_2 = 1.7 \ldots t_2 = 2.38$

Sfc 3: $\varphi_3 = 0 \ldots D_3 = 0.25 \ldots t_3 = 1.62$

Sfc 4: $\varphi_4 = 0 \ldots D_4 = 0.7 \ldots t_4 = 1.00$

Sfc 5: $f_5 = 1 \ldots D_5 = 1.3 \ldots t_5$ =bfl=?

y-u ray trace: marginal ray to find aperture stop

			Surf #1		Surf #2		Surf #3		Surf #4		Surf #5	
c			(objective lens)		(erector lens)		(diaphragm #1)		(diaphragm #2)		(eye lens)	
t												
n												
$-\phi$			-0.1		-0.5		0		0		-1.0	
t/n = t (thin lenses)		50		16.5		2.38		1.62		1.0		
Ray #1 (initial Marginal ray)												
y	"0"		"1.0"		-0.3200		-0.1296		0.0000		0.0800	
nu		$\frac{1}{50} = 0.02$		-0.0800		$+0.0800$		$+0.0800$		$+0.0800$		0.0000 → image at ∞
u												
$a = \frac{D}{2} =$			1.15"		0.85"		0.125"		0.35"		0.65"	
$\|a/y\|$			1.15		2.6562		0.9645		∞		8.125	⟹ ratio of clear radius to ray ht.
								smallest $\|a/y\|$				
								∴ AS=sfc.#3				
Ray #3 = real marginal ray = ray 1 * 0.9645 to scale ray #1 down so it fits inside AS.												
y	0		0.9645		-0.30864		-0.1250		0		0.07716	
nu		0.01929		-0.07716		0.07716		0.07716		0.07716		0
u												

The actual marginal ray just skims the bottom of the Aperture stop (AS).

y-u ray trace: chief ray to find field stop

			Surf #1		Surf #2		Surf #3 (AS)		Surf #4		Surf #5	
c			(objective lens)		(erector lens)		(diaphragm #1)		(diaphragm #2)		(eye lens)	
t												
n												
$-\phi$			−0.1		−0.5		0		0		−1.0	
$t/n = \tau$		50		16.5		2.38		1.62		1.0		
Ray #1						← reverse	start here		fwd →			
$\bar{y}$	0.6480		0.07550		−0.2380		"0"		0.1620		0.2620	
$n\bar{u}$		−0.01145		−0.0190		Pick... "0.1"		Pick... "0.1"		0.1000		−0.1620
$\bar{u}$												
$a = D/2$			1.15		0.85		0.125		0.35		0.65	
$\lvert a/\bar{y} \rvert$			15.23		3.57		∞		2.160		2.48	
nu										Smallest $\lvert a/\bar{y} \rvert$		
u										∴ Sfc#4 is FS		

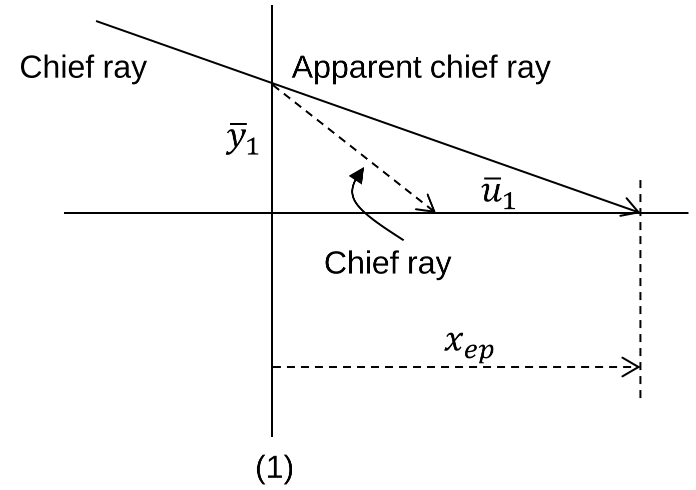
Ray #3 = chief ray (ray #1 × 2.16)

			Surf #1		Surf #2		Surf #3		Surf #4		Surf #5	
$\bar{y}$	1.3997		0.1631		−0.5142		0		0.35		0.5660	
$n\bar{u}$		−0.0247		−0.04105		0.21605		0.21605		0.21605		−0.35
$\bar{u}$												

Maximum object height

Entrance pupil location

We can locate the entrance-pupil plane by finding the distance x_{ep} from surface 1 to where the chief ray appears to cross the axis when viewed from object space.



From the triangle in the figure … $\bar{u}_1 = \frac{-\bar{y}_1}{x_{ep}}$

(the minus sign is from the sign convention, as the diagram is drawn for a negative angle)

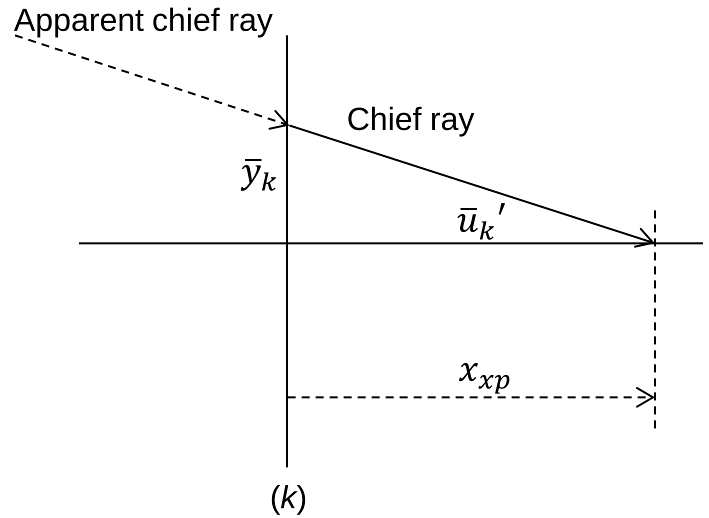
Therefore, the distance from surface 1 to the entrance-pupil plane is $\boxed{x_{ep} = \frac{-\bar{y}_1}{\bar{u}_1}}$

Using values from our y-u ray-trace sheet for surface 1 gives $x_{ep} = \frac{-(0.163)}{-0.0247} = 6.59$

(The exit pupil is located 6.59” to the right of the objective lens … inside the telescope)

Exit pupil location

We can locate the exit-pupil plane by finding the distance x_{xp} from the last surface *(k)* to where the chief ray appears to cross the axis when viewed from image space.



From the triangle in the figure … $\bar{u}_k{}' = \frac{-\bar{y}_k}{x_{xp}}$

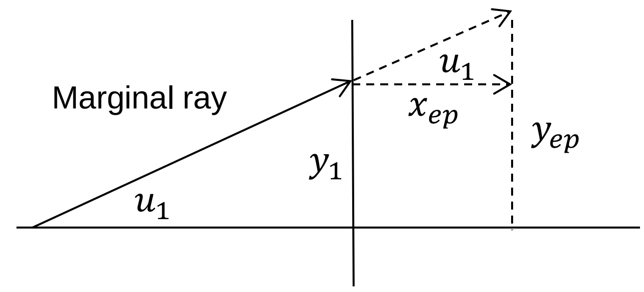
Therefore, the distance from surface *k* to the exit-pupil plane is $\boxed{x_{xp} = \frac{-\bar{y}_k}{\bar{u}_k{}'}}$

Using values from our y-u ray-trace sheet for surface *k* = 5 gives $x_{xp} = \frac{-(0.5660)}{-0.3500} = 1.617$

(The exit pupil is located 1.617” to the right of the eye lens)

Entrance pupil diameter

The apparent marginal ray height at the pupil plane gives the entrance pupil semi-diameter y_{ep}.



From the triangle in the figure … $u_1 = \frac{y_{ep}-y_1}{x_{ep}}$

Therefore, the pupil semi-diameter is $\boxed{y_{ep} = y_1 + x_{ep}u_1}$

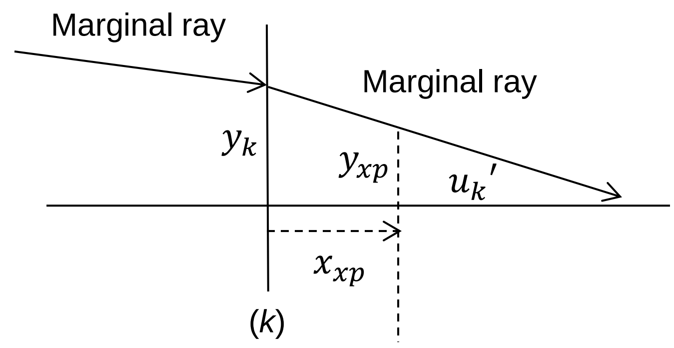
Using values from our y-u ray-trace sheet for surface 1 gives

$$y_{ep} = 0.9645 + (6.59)(0.01929) = 1.0916$$

So the exit pupil diameter is $\boxed{2y_{ep}} = 2.183"$

Exit pupil diameter

The apparent marginal ray height at the pupil plane gives the exit pupil semi-diameter y_{xp}.



From the triangle in the figure … $u_k{}' = \frac{-(y_k - y_{xp})}{x_{xp}}$

Therefore, the pupil semi-diameter is $\boxed{y_{xp} = y_k + x_{xp} u_k{}'}$ (k denotes the last surface)

Using values from our y-u ray-trace sheet for surface k=5 gives

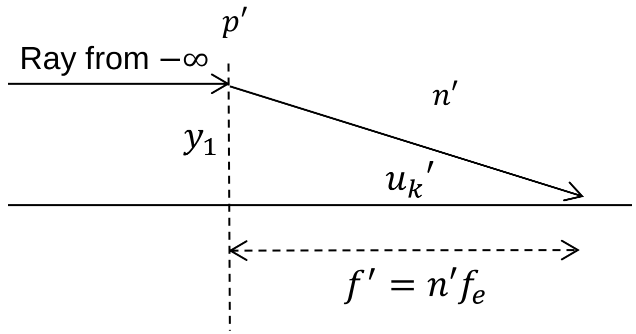
$$y_{xp} = 0.07716 + (1.617)(0) = 0.07716$$

So the exit pupil diameter is $\boxed{2y_{xp}} = 0.154"$

Ray trace from object at ∞

Tracing a ray from an object at (-)∞ gives us what we need to find the following:

- effective focal length f_e
- 2nd focal length f'
- back focal length bfl
  (Greivenkamp calls it back focal distance)
- 2nd principal plane distance d'



From the triangle formed by the 2nd principal plane, the optical axis, and the ray, the "**2nd focal length**" is

$$\boxed{f' = \frac{-y_1}{u_k{}'} = n' f_e}$$

(k = last surface)

Dividing by the image-space index yields the **effective focal length** …

$$\boxed{f_e = \frac{-y_1}{(nu)_k{}'}}$$

Replacing the 2nd principal plane in the above figure with the last optical surface gives the **back focal length** … (distance from the last surface to the focal plane)

$$\boxed{bfl = \frac{-y_k}{u_k{}'}}$$

The distance from the last surface to 2nd principal plane is then

$$\boxed{d' = bfl - f' = \frac{y_1 - y_k}{u_k{}'}}$$

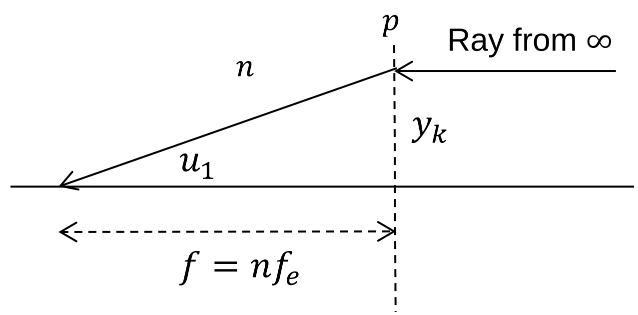
The distance from the last surface to 2nd nodal plane is

$$\boxed{\eta' = bfl - f}$$

Ray trace from image at ∞

Tracing a ray from an image at ∞ gives us what we need to find the following:

- effective focal length f_e
- 1st focal length f
- front focal length ffl
  (Greivenkamp calls it front focal distance)
- 1st principal plane distance d



From the triangle formed by the 1st principal plane, the optical axis, and the ray, the "**1st focal length**" is

$$\boxed{f = \frac{y_k}{u_1} = nf_e}$$

(k = last surface)

Dividing by the image-space index yields the **effective focal length** …

$$\boxed{f_e = \frac{y_k}{(nu)_1}}$$

Replacing the 1st principal plane in the above figure with the first optical surface gives the **front focal length** … (distance from the first surface to the focal plane)

$$\boxed{ffl = \frac{-y_1}{u_1}}$$

The distance from the first surface to 1st principal plane is then

$$\boxed{d = ffl + f = \frac{y_k - y_1}{u_1}}$$

The distance from the first surface to 1st nodal plane is

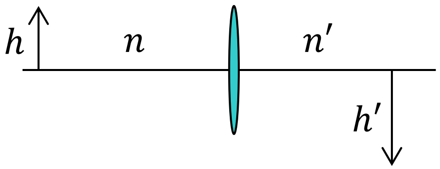
$$\boxed{\eta = ffl + f'}$$

The optical (or Lagrange) invariant

Paraxial marginal and chief ray data also can be used to calculate the optical invariant …
a quantity that always has the same value, regardless of where in the system it is calculated.

$$\boxed{INV = n(\bar{y}u - y\bar{u}) = n'(\bar{y}u' - y\bar{u}')}$$

I encourage you to carefully consider this simple equation (represented by all sorts of Symbols, including strange Cyrillic characters that resemble ℵ … it can be very useful!



<u>Ex: object plane</u> $(\bar{y} = h,\ y = 0)$ … $INV = nhu = \bar{y}nu$

<u>Ex: image plane</u> $(\bar{y} = h',\ y = 0)$ … $INV = n'h'u' = \bar{y}_{image}n'u'$

Transverse magnification: $m = \frac{h'}{h} = \frac{nu}{n'u'} = \frac{\bar{y}_{image}}{\bar{y}_{object}}$

Can use invariant to calculate image size without tracing chief ray …