The y-u ray trace method: telescope example

The y-u method is just the y-nu method for thin lenses (ignoring $n$ within lenses). This example illustrates the use of paraxial ray tracing for finding cardinal points, identifying aperture and field stops, locating and sizing pupils, etc.

\[ Sfc\ 1: f_1 = 10 \quad D_1 = 2.3 \quad t_1 = 16.5 \]
\[ Sfc\ 2: f_2 = 2 \quad D_2 = 1.7 \quad t_2 = 2.38 \]
\[ Sfc\ 3: \varphi_3 = 0 \quad D_3 = 0.25 \quad t_3 = 1.62 \]
\[ Sfc\ 4: \varphi_4 = 0 \quad D_4 = 0.7 \quad t_4 = 1.00 \]
\[ Sfc\ 5: f_5 = 1 \quad D_5 = 1.3 \quad t_5 = \text{bfl} = ? \]
y-u ray trace: marginal ray to find aperture stop

<table>
<thead>
<tr>
<th>Surf #1</th>
<th>Surf #2</th>
<th>Surf #3</th>
<th>Surf #4</th>
<th>Surf #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(objective lens)</td>
<td>(ejector lens)</td>
<td>(diaphragm #1)</td>
<td>(diaphragm #2)</td>
<td>(eye lens)</td>
</tr>
<tr>
<td>c</td>
<td>n</td>
<td>-0.1</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>t/n</td>
<td>(thin lenses)</td>
<td>50</td>
<td>16.5</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Ray #1:
- y: 0, 1.0
- u: 0.02
- a/y: 1.15, 0.85, 0.125, 0.35, 0.65
- a: 0.9645
- AS = sfc. #3

Ray #3:
- y: 0, 0.9645, -0.30864, -0.1250, 0, 0.07716
- u: 0.01919, -0.07716, 0.07716, 0.07716, 0.07716, 0

The actual marginal ray just skirts the bottom of the aperture stop (AS).
y-u ray trace: chief ray to find field stop

AS

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<td>(erector lens)</td>
<td>(diaphragm #1)</td>
<td>(diaphragm #2)</td>
<td>(eye lens)</td>
</tr>
<tr>
<td>c</td>
<td>t</td>
<td>n</td>
<td>-φ</td>
<td>t/n = t</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.1</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Ray #1

<table>
<thead>
<tr>
<th>y</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6480</td>
<td>0.07550</td>
</tr>
<tr>
<td>-0.2380</td>
<td>&quot;0&quot;</td>
</tr>
<tr>
<td>0.1620</td>
<td>0.2620</td>
</tr>
<tr>
<td>-0.01145</td>
<td>-0.0190</td>
</tr>
<tr>
<td>Pick...&quot;0.1&quot;</td>
<td>Pick...&quot;0.1&quot;</td>
</tr>
<tr>
<td>0.1000</td>
<td>-0.1620</td>
</tr>
</tbody>
</table>

Ray #3 = chief ray (Ray #1 x 2.16)

<table>
<thead>
<tr>
<th>y</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3997</td>
<td>0.1631</td>
</tr>
<tr>
<td>0.5142</td>
<td>0</td>
</tr>
<tr>
<td>0.35</td>
<td>0.5660</td>
</tr>
<tr>
<td>-0.0247</td>
<td>-0.04105</td>
</tr>
<tr>
<td>0.21605</td>
<td>0.21605</td>
</tr>
<tr>
<td>0.21605</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Maximum object height

(continued on the next page)
Entrance pupil location

We can locate the entrance-pupil plane by finding the distance $x_{ep}$ from surface 1 to where the chief ray appears to cross the axis when viewed from object space.

From the triangle in the figure ... $\bar{u}_1 = \frac{-\bar{y}_1}{x_{ep}}$

(the minus sign is from the sign convention, as the diagram is drawn for a negative angle)

Therefore, the distance from surface 1 to the entrance-pupil plane is

$$x_{ep} = \frac{-\bar{y}_1}{\bar{u}_1}$$

Using values from our y-u ray-trace sheet for surface 1 gives $x_{ep} = \frac{-(0.163)}{-0.0247} = 6.59$

(The exit pupil is located 6.59” to the right of the objective lens ... inside the telescope)
Exit pupil location

We can locate the exit-pupil plane by finding the distance $x_{xp}$ from the last surface $(k)$ to where the chief ray appears to cross the axis when viewed from image space.

From the triangle in the figure … $\bar{u}_k' = \frac{-\bar{y}_k}{x_{xp}}$

Therefore, the distance from surface $k$ to the exit-pupil plane is:

$$x_{xp} = \frac{-\bar{y}_k}{\bar{u}_k'}$$

Using values from our $y$-$u$ ray-trace sheet for surface $k = 5$ gives $x_{xp} = \frac{-0.5660}{-0.3500} = 1.617$

(The exit pupil is located 1.617” to the right of the eye lens)
Entrance pupil diameter

The apparent marginal ray height at the pupil plane gives the entrance pupil semi-diameter $y_{ep}$.

From the triangle in the figure ... $u_1 = \frac{y_{ep} - y_1}{x_{ep}}$  (1)

Therefore, the pupil semi-diameter is $y_{ep} = y_1 + x_{ep}u_1$

Using values from our y-u ray-trace sheet for surface 1 gives $y_{ep} = 0.9645 + (6.59)(0.01929) = 1.0916$

So the exit pupil diameter is $2y_{ep} = 2.183"$
Exit pupil diameter

The apparent marginal ray height at the pupil plane gives the exit pupil semi-diameter $y_{xp}$.

From the triangle in the figure …

$$u_k' = \frac{-(y_k-y_{xp})}{x_{xp}}$$

Therefore, the pupil semi-diameter is

$$y_{xp} = y_k + x_{xp}u_k'$$

(k denotes the last surface)

Using values from our y-u ray-trace sheet for surface $k=5$ gives

$$y_{xp} = 0.07716 + (1.617)(0) = 0.07716$$

So the exit pupil diameter is $2y_{xp} = 0.154''$
Ray trace from object at $\infty$

Tracing a ray from an object at $(-)\infty$ gives us what we need to find the following:

- effective focal length $f_e$
- 2nd focal length $f'$
- back focal length $bfl$
  (Greivenkamp calls it back focal distance)
- 2nd principal plane distance $d'$

From the triangle formed by the 2nd principal plane, the optical axis, and the ray, the “2nd focal length” is

$$f' = \frac{-y_1}{u_k'} = n'f_e$$  \hspace{1cm} (k = last surface)

Dividing by the image-space index yields the effective focal length ...

```latex
f_e = \frac{-y_1}{(nu)_k'}
```

Replacing the 2nd principal plane in the above figure with the last optical surface gives the back focal length ...

(distance from the last surface to the focal plane)

The distance from the last surface to 2nd principal plane is then

$$d' = bfl - f' = \frac{y_1 - y_k}{u_k'}$$

The distance from the last surface to 2nd nodal plane is

$$\eta' = bfl - f$$
Ray trace from image at $\infty$

Tracing a ray from an image at $\infty$ gives us what we need to find the following:

- effective focal length $f_e$
- $1^{st}$ focal length $f$
- front focal length $ffl$  
  (Greivenkamp calls it front focal distance)
- $1^{st}$ principal plane distance $d$

From the triangle formed by the $1^{st}$ principal plane, the optical axis, and the ray, the “$1^{st}$ focal length” is

$$f = \frac{y_k}{u_1} = nf_e$$  
(k = last surface)

Dividing by the image-space index yields the effective focal length …

$$f_e = \frac{y_k}{(nu)_1}$$

Replacing the $1^{st}$ principal plane in the above figure with the first optical surface gives the front focal length …

(distance from the first surface to the focal plane)

$$ffl = \frac{-y_1}{u_1}$$

The distance from the first surface to $1^{st}$ principal plane is then

$$d = ffl + f = \frac{y_k - y_1}{u_1}$$

The distance from the first surface to $1^{st}$ nodal plane is

$$\eta = ffl + f'$$
The optical (or Lagrange) invariant

Paraxial marginal and chief ray data also can be used to calculate the optical invariant … a quantity that always has the same value, regardless of where in the system it is calculated.

\[ \text{INV} = n(\bar{y}u - y\bar{u}) = n'(\bar{y}'u' - y\bar{u}') \]

I encourage you to carefully consider this simple equation (represented by all sorts of Symbols, including strange Cyrillic characters that resemble \( \aleph \) … it can be very useful!

Ex: object plane \((\bar{y} = h, \ y = 0) \) … \( \text{INV} = nhu = \bar{y}nu \)

Ex: image plane \((\bar{y} = h', \ y = 0) \) … \( \text{INV} = n'h'u' = \bar{y}_{\text{image}}n'u' \)

Transverse magnification: \( m = \frac{h'}{h} = \frac{nu}{n'u'} = \frac{\bar{y}_{\text{image}}}{\bar{y}_{\text{object}}} \)

Can use invariant to calculate image size without tracing chief ray …