Finding Cardinal Points from Paraxial Ray-Trace Values

Chief ray trace

Chief ray angles from entrance and exit pupils give the field of view in object and image space, respectively.

Axis crossings in object and image space indicate locations of pupils (images of AS); if chief ray instead crosses the axis inside the imaging system, then the apparent axis crossings indicate pupil locations.

Entrance pupil location from sfc 1 $x_{ep} = \frac{-\bar{y}_1}{\bar{u}_1}$

Exit pupil location from last sfc (*k*) $x_{xp} = \frac{-\bar{y}_k}{\bar{u}'_k}$

Marginal ray trace

Axis crossings indicate locations of image(s) of the object

Marginal ray height at aperture stop (AS) is equal to the AS semi-diameter

Entrance pupil semi-diameter $y_{ep} = y_1 + x_{ep}u_1$

Exit pupil semi-diameter $y_{xp} = y_k + x_{xp}u'_k$

Ray trace with object at -∞ (u_1=0):

Effective focal length $f_e = \frac{1}{\varphi} = \frac{f'}{n'_k} = \frac{-y_1}{(nu)'_k}$

Second focal length $f' = \frac{n'_k}{\varphi} = \frac{-y_1}{u'_k} = n'_k f_e$

Back focal length $\mathrm{bfl} = \frac{-y_k}{u'_k}$ (called back focal distance by Greivenkamp)

Last surface to 2nd principal plane $\delta' = \mathrm{bfl} - f' = \frac{y_1 - y_k}{u'_k}$

Last surface to 2nd nodal plane $\eta' = \mathrm{bfl} - f = \frac{y_1 - y_k}{u'_k}$

Ray trace with image at ∞ ($u'_k = 0$)

Effective focal length $f_e = \frac{1}{\varphi} = \frac{f}{n_1} = \frac{y_k}{(nu)_1}$

First focal length $f = \frac{n_1}{\varphi} = \frac{y_k}{u_1} = n_1 f_e$

Front focal length $\text{ffl} = \frac{-y_1}{u_1}$

First surface to 1st principal plane $\delta = \text{ffl} + f = \frac{y_k - y_1}{u_1}$

First surface to 1st nodal plane $\eta = \text{ffl} + f'$