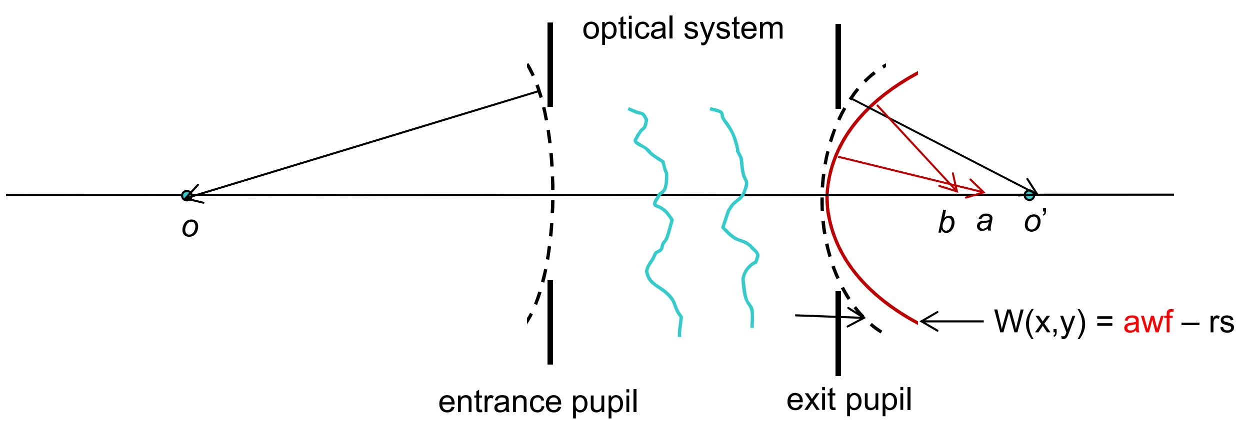
The Wave-Front Aberration Polynomial

Ideal imaging systems perform point-to-point imaging. This requires that a spherical wave front expanding from each object point (*o*) is converted to a spherical wave front converging to a corresponding image point (*o*'). However, real optical systems produce an imperfect "aberrated" image. The aberrated wave front indicated by the solid red line below corresponds to rays near the axis focusing near point *a* and rays near the margin of the pupil focusing near point *b*.

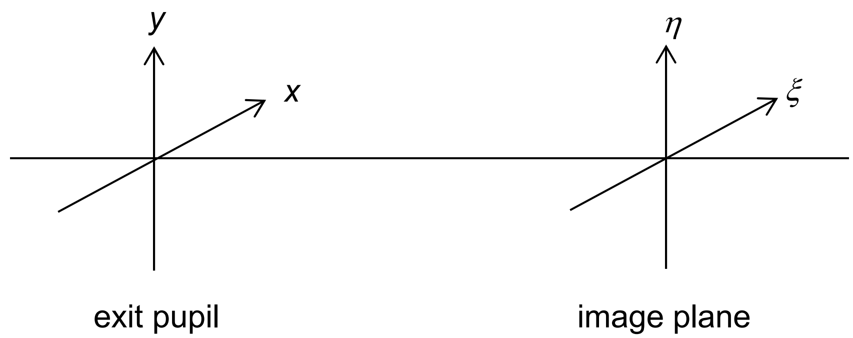


The "wave front aberration function" describes the optical path difference between the aberrated wave front and a spherical reference wave (typically measured in μm or "waves").

$$\boxed{W(x,y) = \text{aberrated wave front} - \text{spherical reference wave front}}$$

Coordinate System

The unique rotationally invariant combinations of the exit-pupil and image-plane coordinates shown below are $x^2 + y^2$, $x\xi + y\eta$, and $\xi^2 + \eta^2$. All others are combinations of these.



For rotationally symmetric optical systems, we can choose the "meridional" plane as our plane of symmetry so that we only need to consider rays that pass through the pupil in the η plane. Then $\xi = 0$ and our variables become the following … $\xi = 0 \rightarrow x^2 + y^2, y\eta, \eta^2$

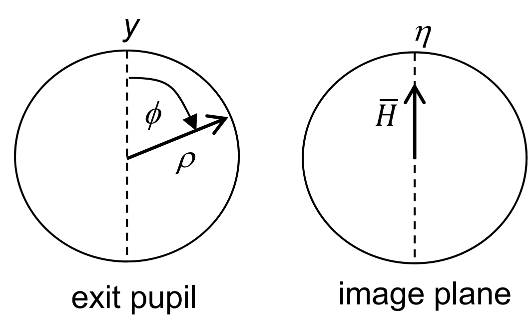
We now convert to circular coordinates in the pupil plane and replace η with $\overline{H}$ to match Geary.

$$x^2 + y^2 \rightarrow \rho^2$$
$$y\eta \rightarrow \rho \overline{H} \cos\phi$$
$$\eta^2 \rightarrow \overline{H}^2$$

ρ = normalized exit-pupil radius

ϕ = exit-pupil angle from meridional plane

$\overline{H}$ = normalized height of ray intersection in image plane



Wave Front Aberration Polynomial

We use a Hamiltonian type of approach and expand the wave front map as a power series in the previously discussed variables.

$$W(\rho^2, \rho\bar{H}\cos\phi, \bar{H}^2) = \sum_{\substack{m,n,k \\ =0,1,2\ldots}} W_{m,n,k}(\rho^2)^m\,(\bar{H}^2)^n\,(\bar{H}\rho\cos\phi)^k = \sum_{\substack{m,n,k \\ =0,1,2\ldots}} W_{m,n,k}\,\bar{H}^{2n+k}\rho^{2m+k}\,(\cos\phi)^k$$

We now rewrite the polynomial using new indices $i = 2n+k$, $j = 2m+k$, *and* k…

$$W(\bar{H},\rho,\cos\phi) = \sum_{\substack{i,j,k \\ =0,1,2\ldots}} W_{i,j,k}\,\bar{H}^i\rho^j\cos^k\phi$$

Remember that this function physically represents the wave front in the exit pupil, with respect to a spherical reference wave front (i.e., the difference between actual and ideal).

Seidel Aberrations

Here is the wave-front aberration polynomial written with terms up to order 4 ($i + j = 4$) …

$W(H,\rho,\cos\phi) =$ W_{000} (piston) + $W_{200}\,\overline{H}^2$ (field-dependent phase)

+ $W_{111}\overline{H}\rho\cos\phi$ (tilt) + $W_{020}\rho^2$ (defocus)

Seidel Aberrations:

+ $W_{040}\rho^4$ (spherical) + $W_{131}\,\overline{H}\,\rho^3\cos\phi$ (coma)

+ $W_{222}\,\overline{H}^2\,\rho^2\cos^2\phi$ (astigmatism) + $W_{220}\,\overline{H}^2\,\rho^2$ (field curvature)

+ $W_{311}\,\overline{H}^3\,\rho\cos\phi$ (distortion) + higher-order terms …

Ray Aberrations

Rays represent the direction of wave-front propagation. Therefore, rays point in the direction of the wave-front surface normal and can be calculated as the wave-front gradient.

The “transverse ray aberration” (TRA) is the distance, orthogonal to the optical axis, between a paraxial ray and its corresponding real ray (i.e., the transverse distance between ideal and real ray locations). The TRA can be calculated as a derivative of the wave front:

$$TRA(y) = -\left(\frac{R}{nr}\right)\frac{\partial W}{\partial y}$$

R = radius of curvature of reference sphere
r = exit pupil height
n = index of refraction in image space
W = wave-front aberration function (OPD)
y = meridional-plane (vertical) coordinate in exit pupil

References
1. W. T. Welford, *Aberrations of optical systems,* Adam Hilger Press (Bristol and Philadelphia), 1986.
2. R. R. Shannon, *The art and science of optical design*, Cambridge Press, 1997.
3. P. Mouroulis and J. Macdonald, *Geometrical optics and optical design,* Oxford Press, 1997.