Abstract

This lab exercise provides the opportunity to practice assembling and solving linear sets of equations.

Introduction and Theory

Refer to the Chapter 7: *Solving Linear Systems of Equations* of the note set for a review of linear equations and basic solution techniques.

Equipment

Matlab to generate two graphs and to solve a system of equations that describes current flow in a resistive circuit.

Procedures

P1: Consider the following set of linear equations:

\[ 2y = 11x \]

\[ 5.5x = y - 12 \]

Rearrange the equations in a form to be used in a matrix description such that the variables are in alphabetical order on the left hand side of the equation and constants are on the right side.

Equation 1: ________________

Equation 2: ________________

The matrix form of the two equation set is (fill in the blanks):

\[
\begin{bmatrix}
\_ \\
\_ \\
\_ \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix} =
\begin{bmatrix}
\_ \\
\_ \\
\end{bmatrix}
\]

Using the space provided on the next page, compute the determinant of the coefficient matrix by hand. You must show all steps to receive credit.
Based on your evaluation of the determinant, does the system of equations have a unique solution?

**ANSWER:** ____________

**P2:** Consider again the same two equations of P1. Solve the equations for y.

Equation 1: _________________________

Equation 2: _________________________

Create a Word document with a heading that includes:

Name
Lab Section Number
Date
Lab 9 Addendum

Use MatLab to plot the two equations on the same graph in the range of \(-10 \leq x \leq 10\) using 0.1 increments for x. Make certain to place a descriptive title on your plot and to label the plot axes. Paste the plot in a Word document. Below the plot, enter the following caption:

“Figure 1: A graph of the two linear equations of part P1.”

Based on your plot, do you believe the system of equations has a unique solution? ____________

Explain the reasoning underlying your response:
P3: Consider the following set of linear equations:

\[
\begin{align*}
1-y &= -2x \\
2y &= -1.6x + 30
\end{align*}
\]  

Equation 1: _________________________
Equation 2: _________________________

The matrix form of the two equation set is (fill in the blanks):

\[
\begin{bmatrix}
-2 & 1 \\
1.6 & -2
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} =
\begin{bmatrix}
c_1 \\ c_2
\end{bmatrix}
\]

Using the space provided on the next page, compute the determinant of the coefficient matrix by hand. You must show all steps to receive credit.

Based on your evaluation of the determinant, does the system of equations have a unique solution?

ANSWER: ____________

P4: Consider again the same two equations of P3. Solve the equations for y.

Equation 3: _________________________
Equation 4: _________________________
Use MatLab to plot the two equations on the same graph in the range of $-10 \leq x \leq 10$ using 0.1 increments for $x$. Make certain to place a descriptive title on your plot and to label the plot axes. Paste the plot into your Word document. Below the plot, enter the following caption:

“Figure 2: A graph of the two linear equations of part P3.”

Based on your plot, do you believe the system of equations has a unique solution? ________________

Explain the reasoning underlying your response:

**P5:** Consider the following electrical circuit.

Using KVL, sum the voltages around the loop containing the battery, $R_1$ and $R_2$. Write your equation in terms of the battery voltage $V$, resistors $R_1$, $R_2$ and $R_3$ and the unknown currents $i_1,i_2$ and $i_3$ as appropriate.

(1)

Using KVL, sum the voltages around the loop containing the voltage source, $R_1$ and $R_3$. Write your equation in terms of the battery voltage $V$, resistors $R_1$, $R_2$ and $R_3$ and the unknown currents $i_1,i_2$ and $i_3$ as appropriate.

(2)

Using KVL, sum the voltages around the loop containing $R_2$ and $R_3$. Write your equation in terms of the battery voltage $V$, resistors $R_1$, $R_2$ and $R_3$ and the unknown currents $i_1,i_2$ and $i_3$ as appropriate.

(3)
Cast your equations into matrix form by filling in the blanks below. Note your equations must be properly ordered!

\[
\begin{bmatrix}
- & - & - \\
- & - & - \\
- & - & - \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\end{bmatrix}
=
\begin{bmatrix}
- \\
- \\
- \\
\end{bmatrix}
\]

Assume that: \( R_1 = 100 \ \Omega \), \( R_2 = 1 \ k\Omega \), \( R_3 = 500 \ \Omega \) and \( V = 5 \ V \). Substitute these values into the matrix arrangement below.

\[
\begin{bmatrix}
- & - & - \\
- & - & - \\
- & - & - \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\end{bmatrix}
=
\begin{bmatrix}
- \\
- \\
- \\
\end{bmatrix}
\]

Calculate the determinant of the coefficient matrix by hand. Show your work.

Use Matlab to verify your results. Attach the Matlab command lines and their result in your Word document. Identify the code with: “P5: Determinant of the original coefficient matrix.”
Based on your calculation of the determinant of the coefficient matrix, does your assembled set of equations have a unique solution?

ANSWER____________________

Perhaps this is a surprising result. Let’s see if we can explain.

Solve equation (1) of part P5 for $i_1$ (do not substitute any numbers, retain expression in terms of variables).

Substitute this result into equation (2) and solve for $i_2$ in terms of $i_3$.

Compare this result to equation (3). Explain the significance of your finding.

Let’s create a new equation, calling it equation (4), by applying KCL at one of the circuit nodes that includes all three unknown currents.

Now replace the third row of your matrix equation (both the third row of the coefficient matrix and the third row of the column vector of constants), thus creating the new matrix equation that follows:

$$
\begin{bmatrix}
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
  i_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
  \_ \\
  \_ \\
  \_ \\
\end{bmatrix}
$$

(4)
Calculate the determinant of the coefficient matrix by hand. Show your work.

Use Matlab to verify your results. Attach the Matlab command lines and their result in your Word document. Identify the code with: “P5: Determinant of the revised coefficient matrix.”

Based on your calculation of the determinant of the coefficient matrix, does your assembled set of equations have a unique solution?

ANSWER____________________

You are now to use Cramer’s method to calculate a value for $i_2$. Using numerical values, assemble the appropriate $3 \times 3$ matrix to determine $i_2$.

Calculate the determinant of this $3 \times 3$ matrix by hand. Show your work.
Calculate $i_2$. Show your work.

Create a Matlab script to calculate $i_1$, $i_2$ and $i_3$ using the “A\B method” described in the note set. Attach your Matlab script, with a descriptive title, and its output (showing the values of $i_1$, $i_2$ and $i_3$) to your Lab 9 addendum.