A Brief Introduction to Musical Acoustics

EE477
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Harmonic and Inharmonic Sounds

- Musical instruments with simple oscillators usually produce periodic waveforms.
- Periodic waveforms have a fundamental frequency, $f_0$, and a *harmonic* spectrum: spectral energy just at frequencies that are integer multiples of $f_0$.
- These harmonic components are called *harmonics*, *overtones*, or *partials*.
- Some musical instruments produce inharmonic sounds: bells, drums, etc.
Pitch

- Musical sounds often have a *pitch* that is related to the sound’s spectral content
- The pitch of a harmonic sound is usually close to the fundamental frequency of that sound
- Inharmonic sounds may have a perceived pitch, but it is not merely the fundamental of some harmonic series
Organization of Western Music

- Two harmonic sounds with different fundamental frequencies can lead to interesting frequency coincidences among their partials.
- When the fundamentals have a low integer ratio relationship, this is a consonant interval.
## Consonant Intervals

<table>
<thead>
<tr>
<th>Unison</th>
<th>3(^{rd})</th>
<th>4(^{th})</th>
<th>5(^{th})</th>
<th>Octave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td>5/4</td>
<td>4/3</td>
<td>3/2</td>
<td>2/1</td>
</tr>
<tr>
<td>100</td>
<td>125</td>
<td>133.33</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
<td>266.67</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>300</td>
<td>375</td>
<td>400</td>
<td>450</td>
<td>600</td>
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<tr>
<td>400</td>
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<tr>
<td>500</td>
<td>625</td>
<td>666.67</td>
<td>750</td>
<td>1000</td>
</tr>
<tr>
<td>600</td>
<td>750</td>
<td>800</td>
<td>900</td>
<td>1200</td>
</tr>
<tr>
<td>700</td>
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<tr>
<td>1000</td>
<td>1250</td>
<td>1333.33</td>
<td>1500</td>
<td>2000</td>
</tr>
<tr>
<td>1100</td>
<td>1375</td>
<td>1466.67</td>
<td>1650</td>
<td>2200</td>
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<tr>
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<td>1600</td>
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<td>2400</td>
</tr>
<tr>
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<td>1625</td>
<td>1733.33</td>
<td>1950</td>
<td>2600</td>
</tr>
<tr>
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<td>1750</td>
<td>1866.67</td>
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<td>2800</td>
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<td>2000</td>
<td>2250</td>
<td>3000</td>
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<tr>
<td>1600</td>
<td>2000</td>
<td>2133.33</td>
<td>2400</td>
<td>3200</td>
</tr>
</tbody>
</table>
Musical Scales and Temperament

- European music is based on the notion of a diatonic pitch scale. The scale specifies the allowable musical pitches: 8 scale steps out of 12.

- Problem: if integer frequency ratios are used (Just intonation), chords only sound in tune if based on fundamental (tonic) pitch. Changing musical “key” is not possible.
Equal Tempered Scale

• To solve the musical “key” problem, keyboard instruments now use *equal-tempered* tuning.

• Note frequencies are distributed uniformly in a logarithmic span:
  \[ f_n = f_0 \times 2^{n/12} \]

• Just vs. equal tempered tuning:

<table>
<thead>
<tr>
<th></th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Octave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unison</td>
<td>125.0000</td>
<td>133.3333</td>
<td>150.0000</td>
<td>200.0000</td>
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<tr>
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<td>125.0000</td>
<td>133.3333</td>
<td>150.0000</td>
<td>200.0000</td>
</tr>
</tbody>
</table>
Rhythm

- Beats per minute
- Beats per measure (time signature)
- Duration of musical notes specified in fractions:
  whole, half, quarter, eighth, sixteenth, 32nd
Musical Notation

- Notation specifies pitches, durations, and time evolution
- Representation is like a spectrogram: frequency vs. time
Standard Tuning Frequencies

C4 261.62Hz
C5 523.25Hz
C6 1046.5Hz
C3 130.81Hz
C2 65.41Hz
A4 440Hz

"middle C" 78cm

λ ≈ 5.2m λ ≈ 2.6m λ ≈ 1.3m λ ≈ 66cm λ ≈ 33cm

C2 65.41Hz
C3 130.81Hz
C4 261.62Hz
A4 440Hz
C5 523.25Hz
C6 1046.5Hz
Musical Timbre

- The relative spectral energy at different frequencies is perceived as a distinct tone color, or timbre (pronounced as either tam-burr or tim-burr)
- Timbre: The combination of qualities of a sound that distinguishes it from other sounds of the same pitch and volume
Musical Instruments

• Almost any object can be considered a musical instrument
• Most *conventional* musical instruments have
  – an *excitation source*
  – a *vibrating element*
  – a *resonant body*
  – a means of *coupling* the vibrations so that they radiate into the air as sound waves
Musical Instruments (cont.)

• The excitation is a motive force
• The vibrating element usually creates many harmonics
• The resonant body emphasizes some frequencies and deemphasizes others
• The coupling means takes energy from the vibrating element and “loses” it (radiates) into an acoustical wave through the air
Example: Singing Voice

- **Lungs**
- **Glottis** (vocal cords)
- Resonance of the throat, nasal passages, and the mouth
- Projection from the mouth
Example: String Instrument

Bow or pluck excitation.

Vibrating string couples energy to the hollow wood body (resonator).

Vibrating body couples sound into the air (radiation).