

Running by the Numbers

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5 April 2005
(updated 27 January 2009)



Introduction

5k, 10k, 26.2—what do these numbers mean to you? If you have been involved in road running for a while, you probably recognize these as typical road race distances, the first two in kilometers and the last one in miles. There are lots of other numbers involved in running: minutes per mile, how many 800 meter repeats in an interval session at the track, the number of milliliters of water to carry per hour on the trail, and how many extra calories to consume after a long workout, etc.

My professional undergraduate and graduate education is in the field of electrical engineering, so perhaps it is natural for me to think about mathematics and engineering principles when I am out for my daily lunchtime runs. My thoughts lately have turned to the mechanical energy and power necessary to move my body from one place to another. For example, I know that a set of 4 AA-size batteries will power my digital camera for a few hours of use. If I could use those same batteries to power my muscles, I wonder how long those 4 batteries would allow me to run? Such are the things that weirdos like me wonder when out running the roads! But before continuing the geek-o-rama, let's consider a little more background.

An aside: The Treadmill Test

Most of us amateur runners do workouts suggested by friends, magazine articles, books, or perhaps a running club or team coach. The effectiveness of a training routine has a lot to do with



both physiology (nerves, flesh, and bone) and psychology (motivation, attitude, and discipline). Expert trainers and running coaches often specialize in the psychology of running, building the mental toughness necessary to overcome the rigors of competition, along with the obvious physical training regimen.

A few years ago when I was living in Boulder, Colorado, I was a member of a weekly training club for novice runners preparing for the Bolder Boulder, a very popular 10km road race held on Memorial Day every year since 1978. Over 50,000 runners and walkers participate each year. Frank Shorter, U.S. Olympic marathon medalist (gold in 1972, silver in 1976) lives in Boulder and was one of the founders of the race. One benefit for those of us in the training club was that we could sign up to have an aerobic treadmill test done at a greatly reduced cost. I was curious about the testing procedure and what it might reveal about my level of fitness, so I went ahead and signed up.

The test procedure was performed at a sports medicine and cardiac rehabilitation center. After a quick medical questionnaire, weigh-in, pulse count, and blood pressure check, they had me don a

mesh vest with cardiac electrodes and a breathing mask with oxygen uptake sensors attached to a computer. Next, it was on to the motorized treadmill for a warm-up. After a few minutes at a jogging pace the test itself began with a finger prick to obtain a blood sample for lactic acid evaluation. Then the technician set the treadmill to spin at a slightly faster pace than during the warm-up while the computer system kept on keeping track of my heart rate and oxygen usage. Another blood sample was taken, the treadmill was set to go slightly faster, and this "run/take a blood sample/increase speed" sequence was repeated six or seven times until I could barely sprint to keep up at the rate the treadmill was turning. As I started to worry seriously about what would happen if I tripped and fell with all the electrodes and mask cables attached, the technician started shouting "c'mon, c'mon Rob, you've *GOTTA go FASTER now*, C'MON, GO, GO, GO!! Just 60 seconds more, c'MON, GO!!"—I almost expected to feel a whip start lashing at my heels, yet I was already exerting at what must have been my maximum power output! Finally—*finally*—the technician said, "OK, stop!" and the treadmill gradually spun down. I gladly removed the facemask, they took a final blood sample, and the test was over, whew!

Incidentally, when I was finally released from the treadmill, wiping the sweat from my forehead, breathing heavily and feeling my heart still pounding relentlessly, I happened to glance beyond the curtain separating the treadmill equipment from the adjacent exercise area and noticed four or five older men walking gingerly on treadmills and slowly pedaling stationary bikes: the cardiac rehab area. I suddenly felt a bit guilty about indulging myself in "voluntary" sports testing when these gentlemen were obviously fighting their way back from heart attacks, bypass surgery, or some other serious medical condition. I wondered what they must have been thinking when the technician was cracking the whip and eagerly exhorting me to go faster—and I was there voluntarily...

Anyway, the results of the tests identified my "lactate threshold" and corresponding heart rate, my VO_2 MAX (maximum oxygen usage per kilogram of body mass), and several other pieces of information that could be used in a custom training program based on a heart rate monitor. I decided not to pay for the heart rate monitor and the ongoing counseling, but I still have a folder containing the various computer printouts, graphs, and charts.

Now back to the numbers...

Besides the interesting test procedure and training recommendations, the computer printout from the test provided a column of kilocalories-per-minute data obtained every 15 seconds throughout the testing. In my case, the number when I was running at a comfortable 10km "race pace" was 15 dietetic calories per minute.

This means that I need to run 10 minutes or so (150 calories) to burn off the sugar in a can of regular soda pop. If I go out and run, say, 40 minutes at race pace, I use $40 \times 15 = 600$ calories. A 100 calorie light beer? That'll cost me six minutes, thank you very much.

The dietetic calorie (1 *thermal* kilocalorie) is a unit of *energy*. Energy is measured in a variety of units besides calories: your power bill lists electrical energy in "kilowatt-hours," your gas bill in "therms" or "BTU" (British Thermal Unit), and mechanical work is counted in "joules."



The *rate* at which energy is expended (energy *per second*) is called *power*. Common ways to measure power include "watts," and the ever-popular "horsepower."

According to the government, an average adult diet is typically 2000 dietetic calories. That's the energy needed to keep the heart pumping, sustain the electrical activity of the brain, keep the body warm, and so forth. The power represented by 2000 calories per day is $(2000/24) = 83.33$ calories per hour, or $(2000/24)/60 = 1.39$ calories per minute. So just doing regular day-to-day activities should burn about 1.39 calories per minute. That's pretty easy to remember: slightly more than a calorie per minute just minding your own business.

Now compare this regular burn rate to what happens when running: the energy consumption goes from 1.39 calories per minute to 15 calories per minute (in my case), or more than ten times the energy usage rate. Another way of looking at it is that I can burn 83 calories if I work at my desk for an hour, or 900 calories if I go out and run for an hour. That's 45% of a whole day's worth of calories consumed in an hour of running. And just to make an obvious point, it's important to keep in mind that the human body is like an engine in that the energy present in the input raw materials (food) must balance the energy used to keep the body going. If more calories of energy are used in exercise than are consumed in food, the balance must come from the body's reserves (stored glycogen and fat). Conversely, if more calories are consumed than are used by the body, the excess is stored away as a reserve, as we all notice after eating extra holiday goodies.

...and now, at long last, back to the AA batteries

OK, so my trust in mathematics makes me confident that a 180 calorie Clif bar will get me $(180/15) = 12$ minutes of hard race pace running. But the engineer in me is still wondering how far my body would go on the energy contained in an AA-size alkaline battery. In order to find out, we need to do some unit conversions. AA batteries are rated at about 2 amp-hours at 1.5 volts, which corresponds to about 10,800 joules of energy. A key relationship—which I admit that I had to look up in a book—is that one dietetic calorie is 4,187 joules. So, the energy stored in an AA battery is the equivalent of 2.58 dietetic calories. At 1.39 calories per minute, the energy in that AA battery would power the body for less than two minutes.

At a running rate of 15 calories per minute, the energy in the AA battery would be used up in just 10 seconds! Running for one hour would exhaust the power of 350 AA batteries—which would weigh 18.5 pounds. Yes, it's a good thing that our body's power system is more efficient than alkaline batteries.

Finally, it is interesting to consider the body's power consumption when running. Power is the rate at which energy is used, so at 15 calories per minute the body is using $(15 \text{ calories}) \times (4,182 \text{ joules/calorie}) / (60 \text{ seconds/minute}) = 1,046$ watts, or just over 1 kilowatt. This is particularly interesting because one horsepower is defined to be 746 watts! Of course, the horsepower unit is supposed to represent the power a horse can *deliver* continuously all day long, while the calculations above represent the energy *consumption* of the human body, not the power actually delivered. Humans are typically less than 25% efficient, so the actual power delivered is perhaps only a quarter of a horsepower. Nevertheless, a runner should feel good about burning the equivalent of a horsepower, even if it is just for an hour or so on the race course!